Internet Distance Learning On A Small Campus

Jonathan R. D. Kuhn Mathematics and Physics Purdue University North Central 1401 S. US 421 Westville, Indiana 46391–9528 Phone: (219) 785–5563 E-mail: jkuhn@purduenc.edu

ABSTRACT

This paper describes how the author was able to start and develop a successful online course at the small campus of Purdue University North Central. Previous to this online course, there were no online courses and, more importantly, little or no support for online teaching at this campus. This paper describes what can be done to overcome the lack of support: the interactive workbook set of Portable Document Format (pdf) files. These kind of files can be posted on essentially any web page. They require little, if any, administrative, technical or teaching assistant support. A statistical analysis indicates the academic performance of the internet students is comparable to the academic performance of the classroom students, in spite of the circumstances.

KEYWORDS

rudimentary asynchronous learning networks; interactive workbook; mixed internet and classroom; Portable Document Format (pdf); WebCT; rank–sum test

I. INTRODUCTION

The author teaches an introductory algebra–based non–calculus statistics course called Elementary Statistical Methods. Sampling, descriptive statistics, correlation and regression, probability, probability distributions, confidence intervals and tests of significance are covered. The graphical TI-83 (or TI-83 plus) calculator is used extensively.

This course is taught to a small mixed class of distance learning internet students and classroom students at Purdue University North Central. Purdue University North Central is a 3500–student, 90–faculty, branch campus of Purdue University located near Westville, Indiana. The main campus of Purdue University is in West Lafayette which is about 80 miles south of the North Central campus. The online version of this course, the first online course at Purdue University North Central, was first given by the author in the fall of 1998.

The author has received funding and support from the following sources for this online course: the Indiana Higher Education Telecommunication System, the Multimedia Instructional Development Center at Purdue University West Lafayette and Information Services and the Learning Center at Purdue University North Central. The asynchronous learning network at Purdue University North Central was not and is still not systematic or institutionalized as described in, for example, Hartman et. al [1] or Fredericksen et. al [2]. It is rudimentary and consists of, essentially, a basic web browser, Adobe Reader, basic technical support and a no-teaching-assistants environment.

Nonetheless, the author was and is still able to successfully provide an online introductory statistics course for the students at Purdue University North Central. One small but important teaching technique described in the paper involves a three–step start–up procedure, which is used to simply get internet students going in the online course. The main teaching technique described in this paper, though, involves an interactive workbook set of Portable Document Format (pdf) files, developed by the author, which is used to successfully get internet students through the semester with minimal support. Also, recently, the WebCT [3] software package has been used for quizzes and will be used for homework assignments.

The interactive workbook consists of the pdf files, "week1.pdf", "week2.pdf", ..., "week14.pdf", posted on the web page ESM web page [4]. These kind of files can be posted on any web page. They require little, if any, administrative, technical or teaching assistant support. An online student is able to download and print off these files either one week at a time throughout the semester or all at once.

The following section gives a description of the rudimentary nature of the asynchronous learning network at Purdue University North Central. A discussion is provided in the section after of the interactive workbook set of pdf files. Then, two other components of the online course are described. Next, comments and a statistical analysis are given. An appendix gives greater detail for the rank–sum test used in the statistical analysis and describes an example exercise out of the interactive workbook.

II. RUDIMENTARY ASYNCHRONOUS LEARNING NETWORK

Although there has been some support for online teaching, the asynchronous learning network at Purdue University North Central was and is still rudimentary.

The Indiana Higher Education Telecommunication System was created in 1967 by the Indiana General Assembly to facilitate sharing of educational resources via technology. Its primary members are Indiana's colleges and universities, but K12 schools, public libraries, state government, and public broadcasting are also included as partners [5]. The Indiana Partnership for Statewide Education is an important part of the Indiana Higher Education Telecommunication System. The Indiana College Network is run by the Indiana Partnership for Statewide Education. The Indiana College Network is a web-based service which enables distance education students to take courses from any of the participating institutions in Indiana.

The author received an Indiana Higher Education Telecommunication System grant to implement the Elementary Statistical Methods online course, the first online course at Purdue University North Central. This course is now one of the many courses offered by the Indiana College Network. Staff at the Learning Center at Purdue University North Central coordinate a liaison between distance learning students and the author, for the Indiana College Network. However, most students are local internet students registered at Purdue University North Central.

The author has attended seminars and training sessions and used both the information resources and technical expertise provided by the Multimedia Instructional Development Center at Purdue University West Lafayette. The software WebCT, which runs on the Multimedia Instructional Development Center's computer server, has been recently used in the Elementary Statistical Methods course for online quizzes.

Nonetheless, the asynchronous learning network within Purdue University North was and is still rudimentary for the following reasons.

- There is little incentive to teach distance learning internet courses. The author has always taught the introductory statistics online course as an overload fifth course, since its inception in the fall of 1998.
- There is no faculty development for online courses at Purdue University North Central. The author is now one of only two faculty members at Purdue University North Central who give online distance learning courses through the Indiana College Network. Both faculty members have developed their internet online courses independently under their own initiative.
- There is no online teaching software based at Purdue University North Central. In particular, WebCT is based on a server 80 miles south at Purdue University West Lafayette. All support staff are located at Purdue University West Lafayette. Consequently, neither the author nor students can easily talk face-to-face with any technical staff who support WebCT.
- There was no technical staff in Information Services to support online teaching. However, a staff member has just recently been hired for this purpose.
- There are no teaching assistants. The author is responsible for the marking, online and offline, of all homework assignments, workbook assignments, quizzes and finals. He is responsible for the email related to the internet course.
- Academic advising at Purdue University North Central does not provide students with adequate information with regard to online courses.

III. INTERACTIVE WORKBOOK

One tool that has been found to be indispensable to the success of the online course is the interactive workbook. The interactive workbook of pdf files is used as a supplement to the text used in the introductory statistics course called <u>Understandable Statistics</u> [6]. This interactive workbook, and a number of other interactive workbooks for other courses, can be found on the internet at <u>Interactive Workbooks</u> [7]. All interactive workbooks can be

posted on essentially any web page and, other than Adobe Reader, do not require elaborate software support.

The interactive workbook is an attempt to mimic classroom discussion. As demonstrated in the appendix to this paper, the exercises in the interactive workbook are created in a careful question-by-question fashion for a student to "discover" the appropriate statistical ideas, definitions or formulas. Each question is not supposed to be, in and of itself, an earth-shattering thought-provoking question. Rather, each question is meant to be simple and lead naturally to the next question in the string of questions in the exercise. Although each individual question is important, the string of questions which make up an exercise is more important. The interactive workbook now contains around 1500 individual questions organized into 50 exercises which, in turn, have been collected into 14 weeks of instruction. The interactive workbook continues to evolve from semester to semester, due to feedback from internet and classroom students.

Mathematical symbols are easily handled by the interactive workbook. The text in the interactive workbook is typed using the LATEX word processing language. Using Adobe Acrobat [8], the encapsulated postscript output of the LATEX files are converted into pdf files. The HTML format is generally avoided in favor of the pdf files for the interactive workbook because the HTML format does not allow for the easy display of mathematical symbols. Microsoft Word could be used, but the author finds the Equation Editor for mathematical notation awkward to use. WebCT could be used but, again, WebCT does not handle mathematical notation very well. The interactive workbook does not yet include video, sound bites or animation. However, it does contain many figures, created using Adobe Illustrator, similar to the one given in the binomial formula exercise in the appendix.

The interactive workbook allows a *mixed* class of internet students and classroom students to be treated *fairly*.

On the one hand, the interactive workbook is printed out by the *classroom* students and brought to class. The interactive workbook is used as a way for the instructor to guide class and group discussion. Classroom students fill in the interactive workbook as the lecture proceeds. To aid in directing class or group discussion, the interactive workbook is displayed, via a portable computer and an overhead projector, onto a screen for class viewing purposes. The classroom students obtain answers based on class or group discussion. The use of the interactive workbook in class time has proven so successful for the introductory statistics course, that interactive workbooks have been created and are used by the author in all of the courses he now teaches.

On the other hand, the interactive workbook for the introductory statistics course could be viewed online or printed out by the distance learning *internet* students. Internet students fill in the interactive workbook on their own. On a weekly basis, they are required, as part of an "attendance" mark, to email their answers to the instructor. Although the instructor does not comment or grade a student's answers to the week's worth of interactive exercises, the answer key is emailed back to the students. Also, students are encouraged, when submitting

their answers, to ask any questions they might have. The interactive workbook is used as a teaching assistant in this case.

The interactive workbook is designed in such a way that a distance learning internet student take the introductory statistics course at the specified weekly pace, although they are free to proceed at a faster pace. By keeping on pace with the classroom students, internet students, that are within commuting distance of the campus and who are having difficulty, are then able to drop by and physically attend class during the semester from time to time. Both distance learning internet students and classroom students are encouraged to spend at least three hours per week on the interactive workbook and six hours outside of this either doing homeworks or preparing for quizzes.

Other classroom-based (not online) interactive workbooks are available, including one by Aliaga and Gunderson [9] and one by Mathew and Mathew [10]. In both these cases, the instructors circulate throughout the classroom and help students work their way through a variety of carefully designed exercises given in their interactive workbook. However, there are no online versions of these interactive workbooks.

Like any other teaching aid, how effective the interactive workbook method is depends on what an instructor is used to and how creatively this instructor can use this teaching device. On the one hand, poor instructors may use the interactive workbook to just make their way through the material, in the same way an instructor might overuse pre-taped video instruction instead of teaching. However, on the other hand, it is possible to use the interactive workbook as a basis for class and group discussion. At the very worst, the workbooks provides a very detailed lesson plan for each class in the semester. More than this, the interactive workbooks method is extremely flexible. It is entirely up to the instructor as to which questions appear in each exercise and in what order.

IV. OTHER COMPONENTS IN THE ONLINE COURSE

In addition to the main interactive workbook component to the online course, there are two other important components: a three–step start–up procedure and the homework assignments and quizzes. When the online course was first started, both of these components were, like the interactive workbook, simple uses of the internet and email. Recently, though, greater use of WebCT has been made for the quizzes and homework assignments.

A. Three–Step Start–Up Procedure

It is important to get both the classroom and internet students up and running quickly and efficiently with minimal administrative support. Classroom students, as usual, are lead through a syllabus on the first day of the semester. Distance learning internet students, on the other hand, work through a three–step procedure on the internet. This self–directed procedure provides not only syllabus information but, also, more importantly, academic advice on taking the online course. With regard to academic advice, the author found that it was very important internet students have a very good idea up front what they were getting into, to prevent discouragement down the road.

The author goes over a syllabus with the *classroom* students on the first day of the semester. Such things as contact information, texts, calculator usage, grading, course outline, homework assignments, quizzes, final exam and attendance policy are all briefly discussed. The syllabus, all of the weekly workbook installments, homework assignments, practice quiz questions and weekly calculator instructions are all available on the author's web site for this introductory statistics course *before* the semester begins.

The *internet* students use the same introductory statistics web page as the classroom students. Internet students are expected to click on the distance learning link found on the introductory statistics web page at or near the beginning of the semester. The distance learning web page is linked to the three web pages: Am I really ready?, How do I get started? and Frequently Asked Questions.

The point of three web pages is not only to provide syllabus information but also to alert students to the under-developed nature of the asynchronous learning network at Purdue University North Central. It not only lists the benefits, but also, more importantly, lists the *drawbacks* of distance learning for the introductory statistics internet course. The email from a previously *disappointed* internet student is posted here. Less-committed student are *dis*couraged from proceeding. As the asynchronous learning network is improved, these improvements will be reflected in these start-up web pages.

B. Homework Assignments, Quizzes and the Interactive Workbook

Presently, the classroom students write all homework assignments and quizzes by hand, whereas the internet students do these things over the internet. However, in the fall 2001 semester, both groups of students will submit homeworks and quizzes on the internet using a software product called WebCT. WebCT is fairly sophisticated and both the author and students will make use of the technical support available from the Multimedia Instructional Development Center at the Purdue West Lafayette main campus.

Homework assignments and quizzes in WebCT and the interactive workbook set of pdf files are all similar in that a student is asked questions in a similar kind of way by one instructor. On the one hand, this consistency between homework assignment/quizzes and the interactive workbook would seem to benefit a student. On the other hand, since WebCT is essentially an HTML-based presentation method, whereas the interactive workbook is a IAT_EX/PDF -based presentation method, there is enough difference between the two presentations to prevent a student from being bored with this consistency.

IV. DISCUSSION AND STATISTICAL ANALYSIS

In spite of the rudimentary nature of the asynchronous learning network at Purdue University North Central, it is a pleasant surprise that a statistical analysis indicates the

academic performance of the internet students is comparable to the academic performance of the classroom students in this mixed class.

The final exam grades for the 16 internet and 111 classroom students taught the introductory statistics course using the interactive workbook over the last six semesters, from the fall 1998 semester to the spring 2001 semester, is summarized in the table below.

internet	f98	s99	f99	s00	f00	s01	totals	%	
A (4)	1	0	2	1	0	1	5	31.25%	
B(3)	1	0	0	0	0	0	1	6.25%	
C(2)	1	0	1	0	0	1	3	18.75%	
D (1)	1	0	0	2	1	2	6	37.5%	
F(0)	0	0	1	0	0	0	1	6.25%	
average	2.5	0	2.5	2	1	2	2.19		
median	2.5	0	3	1	1	1.5	2		
S.E.	0.65	0	0.96	1.00	0	0.71	0.36		
sample	4	0	4	3	1	4	16		
classroom	f98	s99	f99	s00	f00	s01	totals	%	
A(4)	9	4	4	4					
(-)	3	4	4	4	3	8	26	23.4%	
B(3)	3 5	4 4	$\frac{4}{5}$	$\frac{4}{3}$	$\frac{3}{4}$	8 8	$\frac{26}{29}$	23.4% 26.1%	
$ B (3) \\ C (2) $	3 5 5	4 4 5	$4 \\ 5 \\ 6$	$\begin{array}{c} 4\\ 3\\ 10 \end{array}$	$egin{array}{c} 3 \ 4 \ 4 \ 4 \end{array}$		$\begin{array}{c} 26 \\ 29 \\ 36 \end{array}$	23.4% 26.1% 32.4%	
$ \begin{array}{c} \text{B} (3) \\ \text{C} (2) \\ \text{D} (1) \end{array} $		$\begin{array}{c} 4\\ 4\\ 5\\ 3\end{array}$	$4 \\ 5 \\ 6 \\ 2$	$ \begin{array}{c} 4 \\ 3 \\ 10 \\ 2 \end{array} $	$egin{array}{c} 3 \\ 4 \\ 4 \\ 6 \end{array}$	$egin{array}{c} 8 \ 8 \ 6 \ 3 \end{array}$	26 29 36 18	$\begin{array}{c} 23.4\% \\ 26.1\% \\ 32.4\% \\ 16.2\% \end{array}$	
$ \begin{array}{c} \text{B (3)} \\ \text{C (2)} \\ \text{D (1)} \\ \text{F (0)} \end{array} $			$ \begin{array}{c} 4 \\ 5 \\ 6 \\ 2 \\ 1 \end{array} $	$ \begin{array}{c} 4 \\ 3 \\ 10 \\ 2 \\ 0 \end{array} $	$ \begin{array}{c} 3 \\ 4 \\ 4 \\ 6 \\ 0 \end{array} $		$26 \\ 29 \\ 36 \\ 18 \\ 2$	$\begin{array}{c} 23.4\% \\ 26.1\% \\ 32.4\% \\ 16.2\% \\ 1.8\% \end{array}$	
B (3) C (2) D (1) F (0) average	$ \begin{array}{r} 3 \\ 5 \\ 5 \\ 2 \\ 1 \\ 2.44 \end{array} $	$ \begin{array}{r} 4 \\ 4 \\ 5 \\ 3 \\ 0 \\ 2.56 \end{array} $			$ \begin{array}{r} 3 \\ 4 \\ 4 \\ 6 \\ 0 \\ 2.24 \end{array} $		$26 \\ 29 \\ 36 \\ 18 \\ 2 \\ 2.53$	$23.4\% \\ 26.1\% \\ 32.4\% \\ 16.2\% \\ 1.8\%$	
B (3) C (2) D (1) F (0) average median	$ \begin{array}{r} 3 \\ 5 \\ 2 \\ 1 \\ 2.44 \\ 2.5 \\ \end{array} $	$ \begin{array}{r} 4 \\ 4 \\ 5 \\ 3 \\ 0 \\ 2.56 \\ 2.5 \\ \end{array} $	$ \begin{array}{r} 4 \\ 5 \\ 6 \\ 2 \\ 1 \\ 2.5 \\ 2.5 \\ 2.5 \\ \end{array} $		$ \begin{array}{r} 3 \\ 4 \\ 4 \\ 6 \\ 0 \\ 2.24 \\ 2 \end{array} $		$26 \\ 29 \\ 36 \\ 18 \\ 2 \\ 2.53 \\ 2.5$	$23.4\% \\ 26.1\% \\ 32.4\% \\ 16.2\% \\ 1.8\%$	
B (3) C (2) D (1) F (0) average median S.E.	$ \begin{array}{r} 3 \\ 5 \\ 2 \\ 1 \\ 2.44 \\ 2.5 \\ 0.29 \\ \end{array} $	$ \begin{array}{r} 4 \\ 4 \\ 5 \\ 3 \\ 0 \\ 2.56 \\ 2.5 \\ 0.27 \\ \end{array} $	$ \begin{array}{r} 4 \\ 5 \\ 6 \\ 2 \\ 1 \\ 2.5 \\ 2.5 \\ 0.27 \\ \end{array} $	$ \begin{array}{r} 4 \\ 3 \\ 10 \\ 2 \\ 0 \\ \hline 2.47 \\ 2 \\ 0.22 \\ \end{array} $	$ \begin{array}{r} 3 \\ 4 \\ 4 \\ 6 \\ 0 \\ 2.24 \\ 2 \\ 0.28 \\ \end{array} $		$26 \\ 29 \\ 36 \\ 18 \\ 2 \\ 2.53 \\ 2.5 \\ 0.10$	$23.4\% \\ 26.1\% \\ 32.4\% \\ 16.2\% \\ 1.8\%$	

Table 1. Internet and Classroom Final Exam Statistics

Notice, for example, four internet students wrote the final exam in the fall of 1999, f99, where two received an A, zero received a B, one received a C, zero received a D and one received an F. Since each letter grade is assigned the following values,

$$A = 4, \quad B = 3, \quad C = 2, \quad D = 1, \quad F = 0,$$

the average grade on the final for these four students is then given by,

$$\frac{(2 \times 4) + (0 \times 3) + (1 \times 2) + (0 \times 1) + (1 \times 0)}{4} = 2.5$$

The median and standard error (S.E.) (standard deviation divided by the square root of the sample size) are calculated assigning the same values as above to each letter grade.

The totaled final exam grades for internet and classroom students are *about the same*, even though the internet final exam average, 2.19, is lower than the classroom final exam

average, 2.53, according to a nonparametric rank–sum statistical test. This test is explained in greater detail in the appendix to this paper. Notice that the difference in the totaled final exam grade averages, 2.53 - 2.19 = 0.34, is within one internet standard error (0.36) of no (zero) difference, which also indicates this difference is *not* significant.

A similar analysis, not shown here, reveals the difference in *total* (semester work plus final exam mark) grade point averages, 2.56 - 2.19 = 0.37, is about the same as the internet standard error of 0.34. Again, this indicates the *total* grades for internet and classroom students are about the same.

Are the comparable scores between internet students and classroom students entirely due to the interactive workbook? No. There may be other reasons to explain the comparability of scores. For example, it may be the case better students take the internet course and, in spite of the workbook, do as well as the classroom students. Or, it may be the case that it is the text on which the interactive workbook is based that is strongly influencing the test scores of both groups of students. Whatever the case, it is clear that the interactive workbook is *one* possible reason that there are comparable scores between internet students and classroom students.

Other online courses [11], which use elaborate asynchronous learning networks, have shown the internet and classroom students have about the same test scores. Combining these results with the results in this paper, it is tempting to claim the interactive workbook online teaching technique described in this paper that requires only a rudimentary asynchronous learning network, works as well as other online teaching techniques that require elaborate asynchronous learning networks. However, there are too many confounding factors such as different students, teachers, courses, texts and so on between the different online courses to be able to support this conjecture.

The comparable test scores between internet students and classroom students do come at a cost. Both the author and internet students find the time demands for the introductory statistics course to be severe, probably more so than those on a campus with a fully– developed asynchronous learning network. From comments made by students and from the little formal feedback received, internet student evaluations for the online course do appear to be lower than classroom evaluations. Internet students rate the course "3 out of 5", whereas class students rate the course "4 out of 5", say. Having said this, the fall 2001 semester enrollment of students is the largest yet with 20 internet students, compared with 0-4 in previous semesters.

V. SUMMARY AND CONCLUSIONS

This paper has described how the author is able to successfully give an online course at Purdue University North Central with a rudimentary asynchronous learning network. In other words, even though the author and students have much less support and fewer resources than faculty and students have at larger campuses, this paper describes what can be done to overcome these shortcomings. One tool that has been found to be indispensable to the success of the online course is the interactive workbook. The interactive workbook is organized into a number of multiple–question exercises. A student works through the sequence of questions in each exercise to "discover" appropriate statistical ideas, definitions or formulas. It is used to teach both internet students and classroom students who share the same introductory statistics class. It mimics classroom discussion, acts as a teaching assistant and allows a easy transition between course work and homework assignments/quizzes for internet students. This is all accomplished within a rudimentary asynchronous learning network which consists of a basic web browser, Adobe Reader, little technical support and no-teaching-assistants environment.

In addition to the main interactive workbook component to the online course, there are two other important components: a three–step start–up procedure and the homework assignments and quizzes. The three–step procedure, Am I really ready?, How do I get started? and Frequently Asked Questions, which is used to quickly get internet students up and running in the online course, with minimal administrative support. The homework assignments and quizzes will be given using WebCT software. WebCT requires more technical support than the interactive workbook and is awkward when dealing with mathematical symbols.

The interactive workbook is one reason why a statistical analysis indicates the academic performance of the internet students is comparable to the academic performance of the classroom students.

It is hoped the simple technical solution of the interactive workbook of pdf files suggested in this paper is of interest to other individuals at campuses who also have little online support. In addition, although used for an online course on a campus with an underdeveloped asynchronous learning network, it would seem this method could be easily applied for courses on campuses with more fully-developed asynchronous learning networks.

APPENDIX

A. Nonparametric Rank–Sum Statistical Test

A nonparametric rank–sum statistical test is given here to show the totaled final exam grades for internet and classroom students given in table 1 above are about the same.

Assume each letter grade is *ranked* in the following way,

$$A = 1, \quad B = 2, \quad C = 3, \quad D = 4, \quad F = 5,$$

where, notice, the letter grades are assigned *different* numbers from those used to calculate the summary statistics in table 1 above. So, for example, three students graded A, B and C would be ranked 1, 2 and 3, respectively. In the case of ties, the ranks are averaged. For three students graded A, A and C, for instance, the ranks would be $\frac{1+2}{2} = 1.5$, 1.5 and 3, respectively. From table 1, five (5) internet students and 26 classroom students, for a total of 31 students, all achieved an A standing on their final exams. Since they are all "tied" with the same A grade, they are all assigned the same rank of

$$\frac{1+2+\dots+31}{31} = 16$$

In a similar way, the 30 students who all received a B grade are all ranked

$$\frac{31+32+\dots+61}{30} = 46.5$$

The other ranks for other students are given in the table below.

grades	Α	В	С	D	F
internet	5	1	3	6	1
classroom	26	29	36	18	2
totals	31	30	39	24	3
rank	16	46.5	81	112.5	126

Table 2. Ranks For Internet and Classroom Exam Grades

Consequently, the summed rank of the internet students is given by

$$R = (5 \times 16) + (1 \times 46.5) + (3 \times 81) + (6 \times 112.5) + (1 \times 126) = 1170.5,$$

and the average rank and standard deviation of the rank, assuming no difference in the ranks of internet and classroom student final exam grades, are

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} = \frac{16(16 + 111 + 1)}{2} = 1024$$

$$\sigma_R = \sqrt{\frac{n_1n_2(n_1 + n_2 + 1)}{12}} = \sqrt{\frac{16(111)(16 + 111 + 1)}{12}} \approx 137.64$$

where n_1 is 16, the number of internet students, and n_2 is 111, the number of classroom students. Since the z statistic,

$$z = \frac{R - \mu_R}{\sigma_R} \approx \frac{1170.5 - 1024}{137.64} \approx 1.06,$$

is small, this implies there is no difference in the ranks of internet and classroom student final exam grades.

B. Example Interactive Exercise: Binomial Formula

This example of an interactive exercise leads a student in a question-by-question fashion to an initial understanding of the binomial formula.

A lawyer estimates she wins 40% (p = 0.4) of her cases. Assume each trial is independent of one another and, in general, this problem obeys the conditions of a binomial experiment. The lawyer presently represents 10 (n = 10) defendants.

1. Since her chance of winning each time she presents a case is 40%, and each case is independent of one another, the chance of her winning all ten cases must be

 $\overbrace{0.40 \times 0.40 \times \cdots \times 0.40}^{10} \approx (\text{circle closest one}) \ \mathbf{10^{-4}} \ / \ \mathbf{10^{-5}} \ / \ \mathbf{10^{-6}}.$

Use your calculator: type $0.4 \land (10)$, not $0.4 \land 10$; remember 1.04...E-4 means $1.04... \times 10^{-4}$.

Comments: This is an introductory arithmetic question, to give the student confidence to proceed. It reminds the student of previous exercises where it is explained that "independence" means "multiply" (rather than add, say) the ten 0.40 probabilities. It encourages the student to start using the calculator, which is used heavily in this exercise.

2. Since her chance of winning each time she presents a case is 40%, (and so her chance of losing must be 60%) and each case is independent of one another, the chance of her *losing all ten* cases must be

$$\underbrace{0.60 \times 0.60 \times \cdots \times 0.60}^{10} \approx \text{(circle one) } \mathbf{0.004} \ / \ \mathbf{0.005} \ / \ \mathbf{0.006}.$$

Comments: The similarity of this question with the first question reinforces the ideas of the first question. It also reminds the student of the previously explained complementation rule of probability: the chance of losing is one minus the chance of winning.

3. The chance of her winning *one* of the ten cases, assuming she wins the first case (and loses the next nine), is

$$0.40 \times 0.60 \times \cdots \times 0.60 \approx (\text{circle one}) \ \mathbf{0.004} \ / \ \mathbf{0.005} \ / \ \mathbf{0.006}.$$

This is one possible sequence of wins and loses that can take place, if there is only one win.

4. The chance of her winning one of the ten cases, assuming she wins the *second* case (and loses the first case, as well as the third to tenth cases) is

$$\overbrace{0.60}^{1} \times 0.40 \times \overbrace{0.60 \times \cdots \times 0.60}^{8} \approx \text{(circle closest one) } \mathbf{0.004} / \mathbf{0.005} / \mathbf{0.006}$$

This is a second way the lawyer can win one case. In addition, the chance here is exactly equal to the chance calculated before, $0.40(0.60)^9 \approx 0.004$.

5. The chance of her winning one of the ten cases, assuming she wins the *third* case (and loses the first two cases, as well as the fourth to tenth cases) is

$$\underbrace{0.60 \times 0.60}^{2} \times 0.40 \times \underbrace{0.60 \times \cdots \times 0.60}^{7} \approx \text{(circle one) } \mathbf{0.004} / \mathbf{0.005} / \mathbf{0.006}.$$

This is a third way the lawyer can win one case and, again, $0.40(0.60)^9 \approx 0.004$.

- 6. The chance the lawyer wins one of ten cases occurs in a *total* of (circle one) **two** / **five** / **ten** different ways.
- 7. The chance of her winning one of the ten cases, adding the ten possible ways she can do this, is

 $10 \times 0.40 \times (0.60)^9 = (circle one) 0.040 / 0.050 / 0.060.$

- 8. The chance of her winning one of the ten cases, 0.040, can be calculated using the formula (circle none, one or more)
 - (a) $10 \times 0.4 \times (0.60)^9$.
 - (b) $\frac{10!}{1!9!} \times 0.4^1 \times 0.6^9$.
 - (c) $\frac{10!}{1!(10-1)!} \times 0.4^1 \times 0.6^9$.
 - (d) $\frac{n!}{r!(n-r)!} \times p^r \times (1-p)^{n-r}$ (the binomial formula) where n = 10, r = 1 and p = 0.4.

All of the answers should have been circled here; notice how the example here of the lawyer winning one case has been transformed into a special case of the use of the binomial formula.

Comments: Although this example demonstrates the use of the binomial formula, it does not formally derive the binomial from the binomial experiment conditions, which is in keeping with the scope of the course. This example also reminds students of the previously explained factorial notation, denoted by the exclamation mark, "!".

- 9. The chance of her winning two of the ten cases is (circle none, one or more)
 - (a) $\frac{n!}{r!(n-r)!} \times p^r \times (1-p)^{n-r}$ where n = 10, r = 2 and p = 0.4.
 - (b) $\frac{10!}{2!(10-2)!} \times 0.4^2 \times 0.6^8$.
 - (c) $\frac{10!}{2!8!} \times 0.4^2 \times 0.6^8$.
 - (d) $45 \times 0.16 \times 0.017 \approx 0.121$.

Use your calculator: 2nd DISTR 0: binompdf(10, 0.4, 2) ENTER.

10. **True** / **False** The chance of her winning *seven* of the ten cases is $\frac{n!}{r!(n-r)!} \times p^r \times (1-p)^{n-r}$ where n = 10, r = 7 and p = 0.4.

Comment: The last two questions provide more reinforcement of the binomial formula.

11. The tabular form of this distribution is given by,

number won, r	0	1	2	3	4	5	6	7	8	9	10
chance	0.006	0.040	0.121	0.215	0.251	0.201	0.111	0.043	0.011	0.002	0.000

where, for example, the chance of her winning 6 of 10 cases is 0.111. In a similar way, the chance of her winning 4 of 10 cases is (circle one) 0.121 / 0.215 / 0.251.

(Use your calculator to construct the entire distribution table: STAT EDIT ENTER; type 0,1, ...,10 into L_1 ; define $L_2 = 2$ nd DISTR 0:binompdf(10, 0.4, L_1) ENTER.)

Comments: This question points out that the binomial formula generates more than one (eleven, in fact) probabilities. The tabular display of the probability information reminds students about previous exercises where discrete probability distributions (such as the binomial distribution, in this case) can be displayed in more than one way. The usefulness of the calculator to be able to generate all eleven probabilities is demonstrated.

12. The graphical form of the probability distribution is given by,



Figure 1. Example Binomial Distribution

The lawyer has the most chance of winning (circle one) **one** / **two** / **three** / **four** cases of the next 10 cases.

(Use your calculator to draw the distribution: STAT EDIT ENTER; type 0,1, ...,10 into L_1 ; define $L_2 = 2$ nd DISTR 0:binompdf(10, 0.4, L_1) ENTER; set up window: WINDOW 0 10 1 -0.1 0.3 0.1 1; pick histogram: 2nd STAT PLOT ENTER pick histogram, make Xlist L_1 and Freq L_2 ; then GRAPH TRACE.)

Comments: The graphical display of the probability information remind students once again that discrete probability distributions such as the binomial distribution can be displayed in more than one way. The usefulness of the calculator to graph the eleven probabilities is demonstrated. This exercise is the first of a number of exercises on the binomial formula. The other exercises mostly repeat important aspects of this formula, but also would bring out various other different aspects of the binomial formula.

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