Chapter 19

Simulation

It is often difficult to calculate probabilities from sampling distributions, probability models of statistics such as the mean or SD. *Simulation* (previously briefly discussed in chapter 3), repeatedly drawing numbers from random numbers table, allows approximation of these probabilities. If the outcome of one draw does not depend on outcome of another draw, draws are *independent*; otherwise they are *dependent*. The method of simulation is introduced in this chapter and applied to approximate the mean (expected value) in the next chapter.

Exercise 19.1 (Simulation)

1. *Simulation of number of seizures.* Chance of number of seizures epileptic has in a hour modeled below. Two seizures occurs with 60% chance for example.

number seizures, x	$P(x)$
0	0.1
1	0.3
2	0.6

   (a) *Assign digits to represent outcomes.*

      i. Digits 0, 1 represents zero (0) seizures since 2 of 10 digits is 10%; digits 2, 3 represent one seizure since 2 of 10 digits is 30%; digits 4, 5, 6, 7, 8, 9 represent two seizures since 6 of 10 digits is 60%.

      ii. Digit 0 represents zero (0) seizures since 1 of 10 digits is 10%; digits 1, 2, 3 represent one seizure since 3 of 10 digits is 30%; digits 4, 5, 6, 7, 8, 9 represent two seizures since 6 of 10 digits is 60%.

      iii. Digit 0 represents zero (0) seizures since 1 of 10 digits is 10%; digits 1, 2 represent one seizure since 2 of 10 digits is 30%; digits 3, 4, 5, 6, 7, 8, 9 represent two seizures since 7 of 10 digits is 60%.

(b) *Simulate chance of 2 seizures using 10 repetitions.*

i. *Start line 101*

random number	1	9	2	2	3	9	5	0	3	4
no. seizures	1	2	1	1	1	2	2	0	1	2

So approximate chance 2 seizures = $\frac{4}{10}$ = **0.1** / **0.4** / **0.6**

ii. *Start line 102*

random number	7	3	6	7	6	4	7	1	5	0
no. seizures	2	1	2	2	2	2	2	1	2	0

So approximate chance 2 seizures = $\frac{7}{10}$ = **0.1** / **0.4** / **0.7**

iii. *Start line 103*

random number	4	5	4	6	7	7	1	7	0	9
no. seizures										

So approximate chance 2 seizures = **0.1** / **0.4** / **0.8**

iv. Different simulations give **same** / **different** answers. Simulated answers may or may not equal actual probability.

2. *Simulation of roulette payoff.*
Roulette table has 38 numbers: numbers are 1 to 36, 0 and 00. A ball is spun on a roulette wheel. After a time, ball drops into one of 38 slots which correspond to 38 numbers on roulette table.

(a) *Betting on even.* If $1 bet on even: $1 lost if ball drops on odd or 0 or 00, $1 won (added to $1 bet) if even.

payoff	probability
-$1	$\frac{20}{38} \approx 0.526$
$1	$\frac{18}{38} \approx 0.474$

i. *Assign digits to represent outcomes.*

A. Digits 01, 02, ..., 20 represent -$1;
digits 21, 22, ..., 38 represent $1; ignore (skip) all other digits.

B. Digits 01, 02, ..., 21 represent -$1;
digits 22, 23, ..., 38 represent $1; ignore (skip) all other digits.

ii. *Simulate chance of $1 payoff.*

A. *Start line 101, simulate 5 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-$1	$1	skip	skip	$1	-$1	skip	skip	skip	-$1

So approximate chance of $1 payoff = $\frac{2}{5}$ = **0.1** / **0.4** / **0.6**

B. *Start line 101, simulate 8 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-$1	$1	skip	skip	$1	-$1	skip	skip	skip	-$1
random number	96	40	91	25	31	42	54	48	28	53
payoff	skip	skip	skip	$1	$1	skip	skip	skip	$1	skip

Approximate chance $1 payoff = $\frac{5}{8}$ = **0.525** / **0.625** / **0.725**

(b) *Betting on a section.* If $1 bet on section (with 12 numbers): $1 lost if ball drops on one of 24 numbers not in section or 0 or 00, $2 won (added to $1 bet) if number in section.

payoff	probability
-$1	$\frac{26}{38} \approx 0.684$
$2	$\frac{12}{38} \approx 0.316$

i. *Assign digits to represent outcomes.*

A. Digits 01, 02, ..., 20 represent -$1;
digits 21, 22, ..., 38 represent $2; ignore (skip) all other digits.

B. Digits 01, 02, ..., 26 represent -$1;
digits 27, 28, ..., 38 represent $2; ignore (skip) all other digits.

ii. *Simulate chance of $2 payoff.*

A. *Start line 101, simulate 5 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-$1	-$1	skip	skip	$2	-$1	skip	skip	skip	-$1

Simulated chance of $2 payoff = $\frac{1}{5}$ = **0.2** / **0.4** / **0.6**

B. *Start line 101, simulate 8 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-$1	-$1	skip	skip	$2	-$1	skip	skip	skip	-$1
random number	96	40	91	25	31	42	54	48	28	53
payoff										

Complete the simulation table, count $2 payoffs, divide number by 8.

Simulated chance $2 payoff **0.375** / **0.625** / **0.715**

C. Increasing number of simulations **improves** / **worsens** answer.

3. *Dependence and Independence: marbles.* Two red marbles are chosen at random from a bucket containing 5 red, 4 blue and 4 green marbles.

(a) *Chance of choosing two red marbles if sampling without replacement.*
$P(RR) = P(R)P(R|R) = \frac{5}{13} \times \frac{4}{12} =$
$\mathbf{\frac{12}{156}}$ / $\mathbf{\frac{20}{44}}$ / $\mathbf{\frac{20}{156}}$ / $\mathbf{\frac{25}{169}}$;
so choosing 2nd marble

depends on / is independent of
of choosing 1st marble

(b) *Chance of choosing two red marbles if sampling with replacement.*
$P(RR) = P(R)P(R) = \frac{5}{13} \times \frac{5}{13} =$
$\mathbf{\frac{12}{156}}$ / $\mathbf{\frac{20}{44}}$ / $\mathbf{\frac{20}{156}}$ / $\mathbf{\frac{25}{169}}$;
so choosing 2nd marble
depends on / is independent of
of choosing 1st marble

4. *Independent airplane engines.*
Each engine on an airplane fails 10% of time, independent of any other engine.

   (a) *Chance two independent engines fail.*
   $P(\bar{E}\bar{E}) = P(\bar{E})P(\bar{E}) = 0.1 \times 0.1 =$
   0.01 / 0.09 / 0.20 / 0.81

   (b) *Chance two independent engines are okay.*
   $P(EE) = P(E)P(E) = 0.9 \times 0.9 =$
   0.01 / 0.09 / 0.20 / 0.81

   (c) *Chance one engine fails and another is okay.*
   $P(\bar{E}E) = P(\bar{E})P(E) = 0.1 \times 0.9 =$
   0.01 / 0.09 / 0.20 / 0.81

5. *Simulation of number of independent airplane engine failures.*
Each engine of five on an airplane fails 10% of time.

   (a) *Assign digits to represent outcomes.*

      i. Digit 0 represents engine failure;
      digits 1, 2, . . . , 9 represent functioning engine.

      ii. Digits 0, 1 represents engine failure;
      digits 2, 3, . . . , 9 represent functioning engine.

   (b) *Chance none of five engines fail, start line 101, simulate 10 times.*

random number	19223	95034	05756	28713	96409	12531	42544	82853	73676	47150
no failures?	yes	no	no	yes	no	yes	yes	yes	yes	no
no failures?	1	0	0	1	0	1	1	1	1	0

   Simulated chance no engines fail $= \frac{6}{10} =$ **0.2 / 0.4 / 0.6**

   (c) *Chance 1 of five engines fails, start line 101, simulate 10 times.*

random number	19223	95034	05756	28713	96409	12531	42544	82853	73676	47150
1 failure?	no	yes	yes	no	yes	no	no	no	no	yes
1 failure?	0	1	1	0	1	0	0	0	0	1

   Simulated chance of 1 engine failure $= \frac{4}{10} =$ **0.2 / 0.4 / 0.6**

(d) *Chance at least one of five engines fails, start line 101, simulate 10 times.*

random number	19223	95034	05756	28713	96409	12531	42544	82853	73676	47150
1, 2, 3, 4 or 5 failures?	no									
1, 2, 3, 4 or 5 failures?	0									

Complete the simulation table, count number of times "0" appears at least once, divide number by 10.

Simulated chance of at least 1 engine failure **0.4 / 0.6 / 0.8**

6. *Simulation of number of dependent airplane engine failures.*
Any engine of five on an airplane fails 10% of time. Once first engine fails, second engine fails 60% of time. Once second engine fails, third engine fails 80% of time. Plane crashes if third engine fails. What is chance plane crashes?

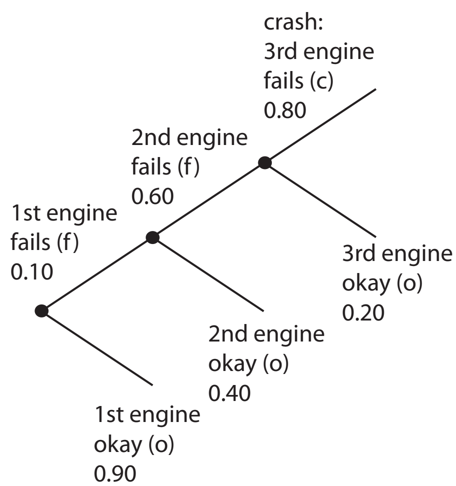

Figure 19.1 (Tree diagram for probability of engine failure)

(a) *Assign digits to represent outcomes.*

i. **Digit assignment A.**

scan at most three digits at a time; start line 101

if 1st of three digits 0 "first engine fails" and check next digit; otherwise plane OK

if 2nd of three digits 0 to 5, "second engine fails" and check next digit; otherwise, plane OK;

if 3rd of three digits 0 to 7, "crash"; otherwise plane OK

proceed to next group of digits if "plane OK" occurs or if three digits read; repeat 10 times

"f" stands for "engine fails", "o" stands for "plane OK" and "c" stands for "plane crash"

ii. **Digit assignment B.**

scan at most three digits at a time; start line 101

if 1st of three digits 0, "first engine fails" and check next digit; otherwise plane OK

if 2nd of three digits 0, "second engine fails" and check next digit; otherwise, plane OK;

if 3rd of three digits 0, 1 or 2, "crash"; otherwise plane OK

proceed to next group of digits if "plane OK" occurs or if three digits read; repeat 10 times

"f" stands for "engine fails", "o" stands for "plane OK" and "c" stands for "plane crash"

(b) *Start line 101, simulate 10 times.*

random numbers	1	9	2	2	3	9	5	034	057	5
engine failures?	o	o	o	o	o	o	o	ffc	ffc	o
crash?	no	no	no	no	no	no	no	crash	crash	no

Simulated chance plane crashes = $\frac{2}{10}$ = **0 / 0.2 / 0.6**

(c) *Start line 102, simulate 10 times.*

random numbers	7	3	6	7	6	4	7	1	5	09
engine failures?	o	o	o	o	o	o	o	o	o	fo
crash?	no	no	no	no	no	no	no	no	no	no

Simulated chance plane crashes = $\frac{0}{10}$ = **0 / 0.4 / 0.6**

(d) *Start line 101, simulate 10 times.*

random numbers	1	9	2	2	3	9	5	034	057	5
engine failures?	o	o	o	o	o	o	o	ffc	ffc	o
crash?	no	no	no	no	no	no	no	crash	crash	no

Simulated chance plane okay **0.7 / 0.8 / 1.0**

(e) *Start line 102, simulate 10 times.*

random numbers	7	3	6	7	6	4	7	1	5	09
engine failures?	o	o	o	o	o	o	o	o	o	fo
crash?	no	no	no	no	no	no	no	no	no	no

Simulated chance plane okay **0.7 / 0.8 / 1.0**

Chapter 20

The House Edge: Expected Values

We discuss *expected value*, or "center", of probability distributions; for example, expected winnings of betting on a roulette game. If exact calculation of expected value not possible, it is often possible to obtain an approximate simulated expected value.

Exercise 20.1 (The House Edge: Expected Values)

1. *Expected value of number of seizures.* Chance of number of seizures epileptic has in a hour modeled below. Two seizures occurs with 60% chance for example.

number seizures, x	$P(x)$
0	0.1
1	0.3
2	0.6

   (a) *Exact expected value.* Expected number of seizures in an hour is

$$0 \times 0.1 + 1 \times 0.3 + 2 \times 0.6 =$$

   1.1 / 1.4 / 1.5

   (b) *Simulations of expected value, using 10 repetitions.*
   Let digit 0 represents zero (0) seizures since 1 of 10 digits is 10%;
   digits 1, 2, 3 represent one seizure since 3 of 10 digits is 30%;
   digits 4, 5, 6, 7, 8, 9 represent two seizures since 6 of 10 digits is 60%.

      i. *Start line 101*

random number	1	9	2	2	3	9	5	0	3	4
no. seizures	1	2	1	1	1	2	2	0	1	2

      Simulation $\bar{x} = \frac{1+2+1+1+1+2+2+0+1+2}{10}$ = **1.1 / 1.3 / 1.5**

      ii. *Start line 102*

random number	7	3	6	7	6	4	7	1	5	0
no. seizures	2	1	2	2	2	2	2	1	2	0

Simulation $\bar{x} = \frac{2+1+2+2+2+2+2+1+2+0}{10} =$ **1.1 / 1.3 / 1.6**

iii. *Start line 103*

random number	4	5	4	6	7	7	1	7	0	9
no. seizures										

Simulation $\bar{x} =$ **1.1 / 1.5 / 1.7**

2. *Expected value of roulette payoffs.*
Roulette table has 38 numbers: numbers are 1 to 36, 0 and 00. A ball is spun on a roulette wheel. After a time, ball drops into one of 38 slots which correspond to 38 numbers on roulette table.

(a) *Betting on even.* If $1 bet on even: $1 lost if ball drops on odd or 0 or 00, $1 won (added to $1 bet) if even.

payoff	probability
-$1	$\frac{20}{38} \approx 0.526$
$1	$\frac{18}{38} \approx 0.474$

i. *Exact expected value.* Expected payoff of bet on even is

$$-\$1 \times \frac{20}{38} + \$1 \times \frac{18}{38} =$$

−0.045 / −0.053 / −0.072

ii. *Simulations of expected payoff.*
Digits 01, 02, ..., 20 represent -$1;
digits 21, 22, ..., 38 represent $1; ignore (skip) all other digits.

A. *Start line 101, simulate 5 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-$1	$1	skip	skip	$1	-$1	skip	skip	skip	-$1

Simulation $\bar{x} = \frac{-\$1+\$1+\$1-\$1-\$1}{5} =$ **−$0.10 / −$0.20 / −$0.30**

B. *Start line 101, simulate 8 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-$1	$1	skip	skip	$1	-$1	skip	skip	skip	-$1
random number	96	40	91	25	31	42	54	48	28	53
payoff	skip	skip	skip	$1	$1	skip	skip	skip	$1	skip

Simulation $\bar{x} = \frac{3\times -\$1+5\times\$1}{8} =$ **$0.15 / $0.25 / $0.35**

(b) *Betting on a section.* If $1 bet on section (with 12 numbers): $1 lost if ball drops on one of 24 numbers not in section or 0 or 00, $2 won (added to $1 bet) if number in section.

payoff	probability
-\$1	$\frac{26}{38} \approx 0.684$
\$2	$\frac{12}{38} \approx 0.316$

i. *Exact expected value.* Expected payoff of bet on section is

$$-\$1 \times \frac{26}{38} + \$2 \times \frac{12}{38} =$$

−0.045 / −0.053 / −0.072

ii. *Simulations of expected payoff.*
Digits 01, 02, ..., 26 represent -\$1;
digits 27, 28, ..., 38 represent \$2; ignore (skip) all other digits.

A. *Start line 101, simulate 5 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-\$1	-\$1	skip	skip	\$2	-\$1	skip	skip	skip	-\$1

Simulation $\bar{x} = \frac{-\$1-\$1+\$2-\$1-\$1}{5} =$ **−\$0.20 / −\$0.30 / −\$0.40**

B. *Start line 101, simulate 8 times*

random number	19	22	39	50	34	05	75	62	87	13
payoff	-\$1	-\$1	skip	skip	\$2	-\$1	skip	skip	skip	-\$1
random number	96	40	91	25	31	42	54	48	28	53
payoff										

Simulation $\bar{x} =$ **−\$0.125 / \$0.125 / \$0.250**

3. *Expected value of number of independent airplane engine failures.*
Each engine of five on an airplane fails 10% of time.

(a) *Exact expected value?*
Exact expected number of airplane engine failures **known / unknown**.
Since formula required to answer this question is not given in this course, answer is "unknown", but, having said this, exact expected number is, in fact, $5 \times 0.10 = 0.5$ failures out of five engines.

(b) *Simulations of expected value.*
Digit 0 represents engine failure;
digits 1, 2, ..., 9 represent functioning engine.

i. *Start line 101, simulate 10 times.*

random number	19223	95034	05756	28713	96409	12531	42544	82853	73676	47150
no. failures	0	1	1	0	1	0	0	0	0	1

Simulation $\bar{x} =$ **0.3 / 0.4 / 0.5**

ii. *Start line 102, simulate 10 times.*

random number	73676	47150	99400	01927	27754	42648	82425	36290	45467	71709
no. failures										

Simulation $\bar{x}$ = **0.4 / 0.5 / 0.6**

4. *Expected value of number of dependent airplane engine failures.*
Any engine of five on an airplane fails 10% of time. Once first engine fails, second engine fails 60% of time. Once second engine fails, third engine fails 80% of time. Plane crashes if third engine fails. What is expected number of engine failures?

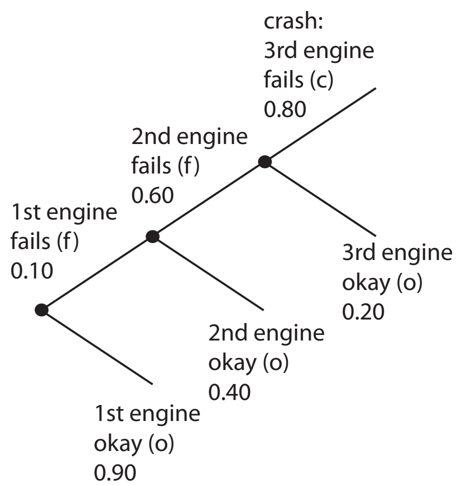

Figure 20.1 (Tree diagram for probability of engine failure)

(a) *Exact expected value?*
Exact expected number of airplane engine failures **known / unknown**.
Since formula required to answer this question is not given in this course, answer is "unknown", but, having said this, exact expected number is, in fact, $1 \times 0.10 \times 0.40$ (first engine fails but second okay) + $2 \times 0.10 \times 0.60 \times 0.20$ (two engines fail, but third okay) + $3 \times 0.10 \times 0.60 \times 0.80$ (three engines fail) = 0.208 failures out of five engines.

(b) *Simulations of expected value.*
scan at most three digits at a time; start line 101
if 1st of three digits 0 "first engine fails" and check next digit; otherwise plane OK
if 2nd of three digits 0 to 5, "second engine fails" and check next digit; otherwise, plane OK;
if 3rd of three digits 0 to 7, "crash"; otherwise plane OK
proceed to next group of digits if "plane OK" occurs or if three digits read; repeat 10 times
"f" stands for "engine fails", "o" stands for "plane OK" and "c" stands for "plane crash"

i. *Start line 101, simulate 10 times.*

random numbers	1	9	2	2	3	9	5	034	057	5
engine failures?	o	o	o	o	o	o	o	ffc	ffc	o
no. failures	0	0	0	0	0	0	0	3	3	0

Expected number of engine failures of five: $\bar{x}$ = **0 / 0.2 / 0.6**
Only up to three of five can fail because, on the third, the plane crashes.

ii. *Start line 102, simulate 10 times.*

random numbers	7	3	6	7	6	4	7	1	5	09
engine failures?	o	o	o	o	o	o	o	o	o	fo
no. failures	0	0	0	0	0	0	0	0	0	1

Expected number of engine failures out of five: $\bar{x} =$ **0 / 0.1 / 0.4**

iii. *Start line 103, simulate 10 times.*

random numbers	4	5	4	6	7	7	1	7	09	7
engine failures?	o	o	o	o	o	o	o	o	fo	o
no. failures										

Expected number of engine failures out of five: $\bar{x} =$ **0 / 0.1 / 0.4**