Chapter 13

Normal Distributions

Histograms of sampled data often *approximated* (or "idealized") by graphs of distributions. Area of a portion of a histogram often approximated by area of equivalent portion of a graph of related distribution. Normal distribution is, by far, most important used in statistical analysis. Many statistical studies related to, say, psychological experiments, economic indicators or scientific measurements are assumed to possess (or, at least, can be fairly well approximated by) a Normal distribution.

We look at percentages and percentiles for Normal distribution. The *68-95-99.7 rule* tells us

- 68% of distribution falls within one SD of mean,
- 95% of distribution falls within two SDs of mean,
- 99.7% of distribution falls within three SDs of mean.

Furthermore, we look at the *standard score* given by

$$\text{standard score} = \frac{\text{observation} - \text{mean}}{\text{standard deviation}}$$

Exercise 13.1 (Normal Distributions)

1. *Temperatures.*
   In Westville, in January, temperature, Z, assumed *standard* normally distributed, mean 0^o and standard deviation (SD) 1^o.

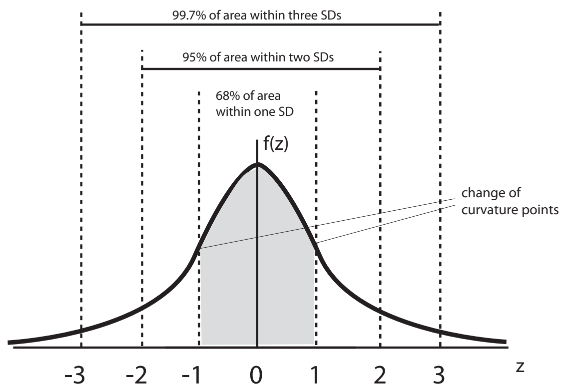

Figure 13.1 (Graph of normal: temperatures.)

(a) Shape (name) of distribution of January temperatures is **triangular / bell-shaped (normal) / rectangular**.

(b) Distribution centered at mean temperature $\mathbf{0^o}$ / $\mathbf{1^o}$ / $\mathbf{5^o}$.

(c) Distribution has SD in temperature of $\mathbf{0^o}$ / $\mathbf{1^o}$ / $\mathbf{5^o}$.
SD determines width of distribution, center to points of curvature.

(d) Total area (probability) under distribution **50% / 75% / 100% / 150%**.

(e) Using 68-95-99.7 rule, 68% of temperatures falls within *one* SD of mean,

i. between -1^o and 1^o
ii. between -2^o and 2^o
iii. between -3^o and 3^o

(f) Using 68-95-99.7 rule, 95% of temperatures falls within *two* SDs of mean,

i. between -1^o and 1^o
ii. between -2^o and 2^o
iii. between -3^o and 3^o

(g) Using 68-95-99.7 rule, 99.7% of temperatures falls within *three* SDs of mean,

i. between -1^o and 1^o
ii. between -2^o and 2^o
iii. between -3^o and 3^o

2. *More temperatures.*
In Westville, in February, temperature, X, is assumed normally distributed with mean 5^o and standard deviation 0.5^o

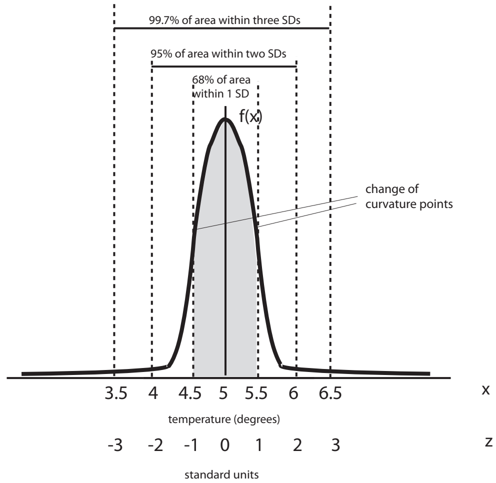

Figure 13.2 (Graph of normal: more temperatures.)

(a) Shape (name) of distribution of January temperatures is **triangular** / **bell-shaped (normal)** / **rectangular**.

(b) Distribution centered at mean temperature $\mathbf{0^o}$ / $\mathbf{1^o}$ / $\mathbf{5^o}$.
Previous distribution centered at 0^o, moves to 5^o.

(c) Distribution has SD in temperature of $\mathbf{0.5^o}$ / $\mathbf{1^o}$ / $\mathbf{5^o}$.
Current distribution narrower because SD decreases from 1^o to 0.5^o.

(d) Total area (probability) under distribution **50%** / **75%** / **100%** / **150%**.

(e) Using 68-95-99.7 rule, 68% of temperatures falls within one SD of mean,

  i. between -1^o and 1^o
  ii. between -0.5^o and 0.5^o
  iii. between 4.5^o and 5.5^o

(f) Since 68% of temperatures falls between 4.5^o and 5.5^o, what percentage falls *outside* this range? **16%** / **32%** / **68%**

(g) 95% of temperatures falls within two SDs of mean,

  i. between 4.5^o and 5.5^o
  ii. between 4^o and 6^o
  iii. between 3.5^o and 6.5^o

(h) Percentage of temperatures more than 6^o: **2.5%** / **5%** / **16%**

(i) 99.7% of temperatures falls within three SDs of mean,

  i. between 4.5^o and 5.5^o
  ii. between 4^o and 6^o
  iii. between 3.5^o and 6.5^o

(j) **True** / **False** *Family* of normal distributions all bell-shaped, centered at mean (which can be any negative or positive number) with width given by the SD (which can only be nonnegative).

3. *Heights of women.* Consider histogram and overlapping approximate normal distribution below for heights of 1000 women. Mean height is 63.5 inches and standard deviation in height is 2.5 inches, SD = 2.5.

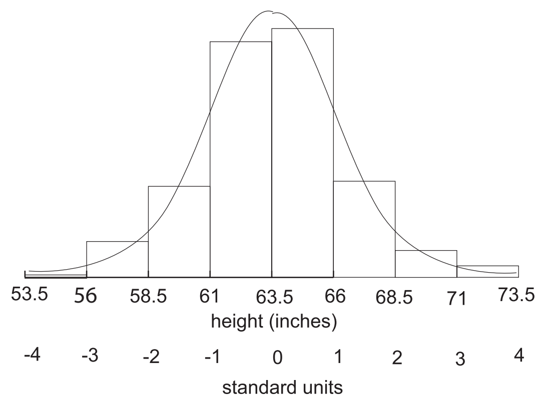

Figure 13.3 (Heights of women)

(a) Using 68-95-99.7 rule, 68% of heights falls within one SD of mean,
   i. between 61 and 66 inches
   ii. between 58.5 and 68.5 inches
   iii. between 58 and 71 inches

(b) Percentage of women taller than 66 inches **16%** / **32%** / **68%**

(c) Chance a woman shorter than 61 inches **16%** / **32%** / **68%**

(d) Chance a woman shorter than 66 inches **32%** / **68%** / **84%**

(e) Chance a woman between 63.5 and 66 inches tall **16%** / **32%** / **34%**

(f) 95% of heights falls within two SDs of mean,
   i. between 61 and 66 inches
   ii. between 58.5 and 68.5 inches
   iii. between 58 and 71 inches

(g) Chance a women between 58.5 and 63.5 inches tall **32%** / **47.5%** / **95%**

(h) Tallest 2.5% of woman taller than **58.5** / **68.5** / **71** inches.

(i) Shortest 2.5% of woman shorter than **58.5** / **68.5** / **71** inches.

4. *IQs.* IQ scores for 16 year olds and 20 year olds described by normal distributions.

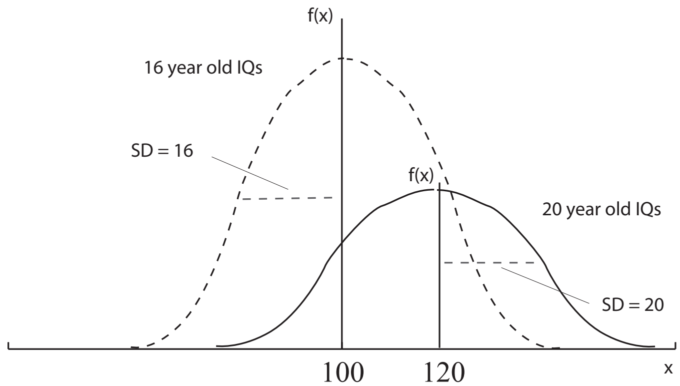

Figure 13.4 (IQ scores of 16 year olds and 20 year olds)

(a) Mean IQ score for 20 year olds **100** / **120** / **124** / **136**.

(b) Standard deviation in IQ score for 20 year olds **16** / **20** / **24** / **36**.

(c) Using 68-95-99.7 rule, 95% of 20 year old IQs fall within two SDs of mean,

  i. between 100 and 140
  ii. between 80 and 160
  iii. between 60 and 180

(d) Mean IQ score for 16 year olds **100** / **120** / **124** / **136**.

(e) Standard deviation in IQ score for 16 year olds **16** / **20** / **24** / **36**.

(f) 99.7% of 16 year old IQs fall within three SDs of mean,

  i. between 84 and 116
  ii. between 68 and 132
  iii. between 52 and 148

(g) Chance 16 year old IQs between 52 and 100 **49.50%** / **49.85%** / **49.99%**

(h) Smartest 0.15% of 16 year olds have at least **116** / **132** / **148** IQ.

(i) Chance 16 year old IQ is less than 148 **0.15%** / **49.85%** / **99.85%**

(j) Normal distribution for 20 year old IQ scores is
broader than / **as wide as** / **narrower than**
normal for 16 year old IQ scores.

(k) Normal distribution for 20 year old IQ scores is
shorter than / **as tall as** / **taller than**
normal for 16 year old IQ scores.

(l) Total area (probability) under normal for 20 year old IQ scores is
smaller than / **same as** / **larger than**
area (probability) under normal for 16 year old IQ scores.

5. *Percentages and percentiles (Table B): temperatures.* In Westville, in January, temperature, Z, assumed *standard* normally distributed, mean 0^o and standard deviation (SD) 1^o.

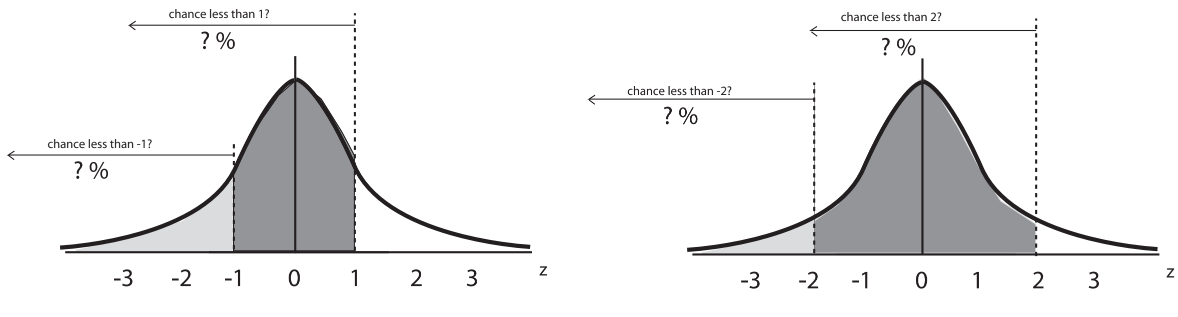

Figure 13.5 (Graph of normal: temperatures.)

(a) Chance temperatures less than -1^o: **13.87%** / **15.87%** / **16.46%**

(b) Chance temperatures less than 1^o: **84.13%** / **87.13%** / **88.21%**

(c) Chance temperatures between -1^o and 1^o
$84.13 - 15.87 =$ **68.26%** / **73.13%** / **84.26%**

(d) Chance temperatures less than -2^o: **2.27%** / **8.49%** / **10.32%**

(e) Chance temperatures less than 2^o: **92.18%** / **95.13%** / **97.73%**

(f) Chance temperatures between -2^o and 2^o
$97.73 - 2.27 =$ **92.41%** / **95.46%** / **96.69%**

6. *Percentages and percentiles (Table B): temperatures.* In Westville, in January, temperature, Z, assumed *standard* normally distributed, mean 0^o and standard deviation (SD) 1^o.

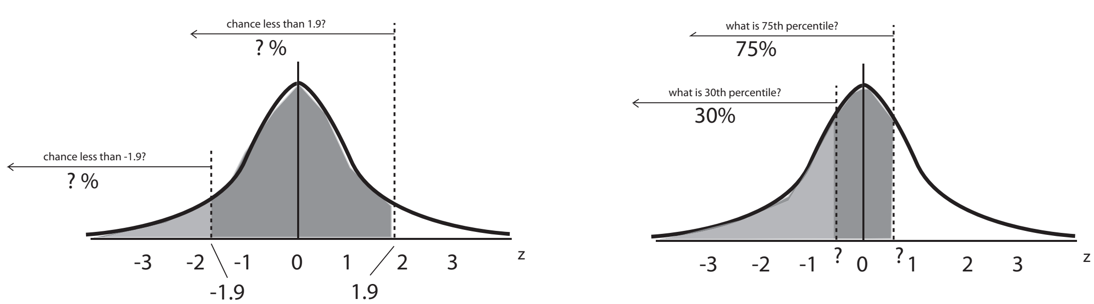

Figure 13.6 (Graph of normal: temperatures.)

(a) Chance temperatures less than -1.9^o: **2.87%** / **3.59%** / **4.46%**

(b) Chance temperatures less than 1.9^o: **96.41%** / **97.13%** / **98.21%**

(c) Chance temperatures between -1.9^o and 1.9^o
$97.13 - 2.87 =$ **92.41%** / **93.13%** / **94.26%**

(d) Chance temperatures less than 1.2^o: **86.43%** / **88.49%** / **90.32%**

(e) Chance temperatures less than 2.4^o: **95.18%** / **97.13%** / **99.18%**

(f) Chance temperatures between 1.2^o and 2.4^o
$99.18 - 88.49 =$ **8.41%** / **9.13%** / **10.69%**

(g) 30th percentile temperature: $\mathbf{-1.5^o}$ / $\mathbf{-1^o}$ / $\mathbf{-0.5^o}$

(h) 75th percentile temperature: $\mathbf{0^o}$ / $\mathbf{0.3^o}$ / $\mathbf{0.7^o}$

7. *Percentages and percentiles (Table B): heights of women.* Mean height is 63.5 inches and standard deviation in height is 2.5 inches, SD = 2.5.

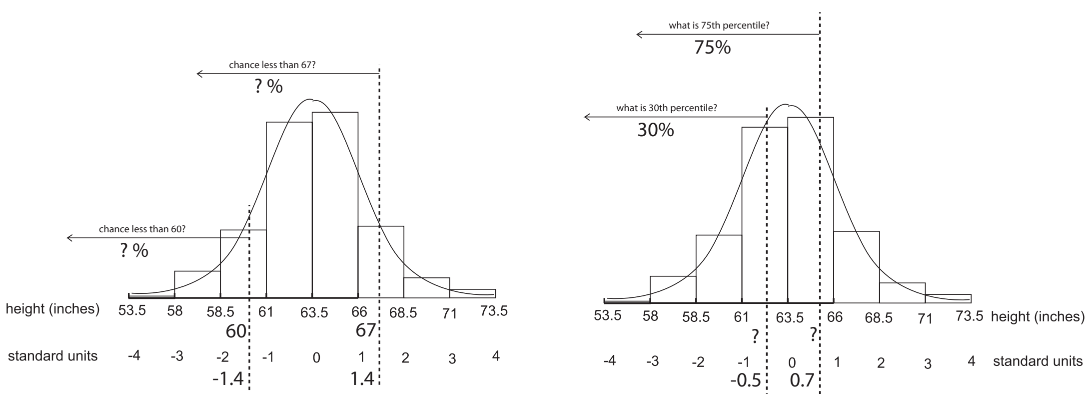

Figure 13.7 (Heights of women)

(a) Chance height less than 60, since $\frac{60-63.5}{2.5} = -1.4$: **5.87%** / **8.08%** / **9.46%**

(b) Chance height less than 67, since $\frac{67-63.5}{2.5} = 1.4$: **85.87%** / **91.92%**

(c) Chance height between 60 inches and 67 inches tall,
$91.92 - 8.08 =$ **82.41%** / **83.84%** / **84.26%**

(d) 30th percentile height is $63.5+(-0.5)(2.5) =$ **61.25** / **62.25** / **63.25** inches

(e) 75th percentile height is $63.5+(0.7)(2.5) =$ **61.25** / **62.25** / **65.25** inches

8. *Percentages and percentiles (Table B): IQs.* IQ scores for 16 year olds and 20 year olds described by normal distributions.

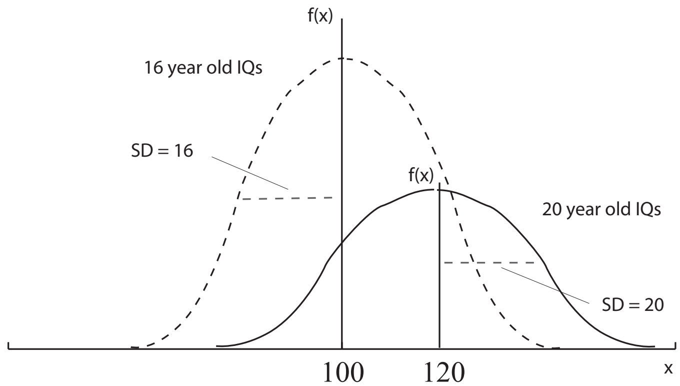

Figure 13.8 (IQ scores of 16 year olds and 20 year olds)

(a) Chance 16 year old IQ less than 110,
since $\frac{110-100}{16} \approx 0.6$: **4.46%** / **72.58%**

(b) Chance 20 year old IQ less than 110,
since $\frac{110-120}{20} = -0.5$: **0.26%** / **30.85%**

(c) 8th percentile 16 year old IQ: $100 + (-1.4)(16) =$ **76.6** / **77.6** / **78.6**

(d) 58th percentile 20 year old IQ: $120 + (0.2)(20) =$ **120** / **124** / **128**

9. *Uniform: temperatures.* Assume temperatures vary anywhere from 0^o to 1^o.

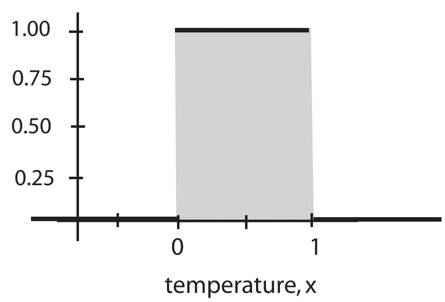

Figure 13.9 (Uniform: temperatures)

(a) Chance temperature less than 0.7^o is **0.3** / **0.5** / **0.7** / **1**.

(b) Chance temperature between 0.3^o and 0.7^o is **0.3** / **0.4** / **0.7** / **1**.

(c) Chance temperature more than 0.4^o is **0.3** / **0.4** / **0.6** / **1**.

10. *Uniform: weight of potatoes.* An automated process fills one bag after another with Idaho potatoes. Although each filled bag should weigh 50 pounds, in fact, because of differing shapes and weights of each potato, each bag weighs anywhere from 50 pounds to 51 pounds.

Figure 13.10 (Uniform: weights of potatoes)

(a) Chance a bag chosen at random weighs between 50.5 and 51 pounds (choose one) **0.25** / **0.50** / **0.75** / **1**.

Notice, *probability* of 0.50 equals *area* of 0.50.

(b) Chance a bag chosen at random weighs between 49.5 and 50.5 pounds (choose one) **0.25** / **0.50** / **0.75** / **1**.

(c) Chance a bag chosen at random weighs more than 49.5 pounds (choose one) **0.25** / **0.50** / **0.75** / **1**.

(d) Chance a bag chosen at random weighs less than 49.5 pounds (choose one) **0** / **0.50** / **0.75** / **1**.