Final for Statistics 213
Probability and Decision Theory - Spring 1998
Material Covered: entire course
Wednesday, 6th May

This is a 2 hour final, worth 27% and marked out of 27 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator and appropriate statistical tables may also be used. No other aids are permitted.

Name (please print): ______________________________ . **ID Number:** ______.
last first

1[1]**.** A drug treatment program is known to be 80% effective in curing a certain disease. Twelve patients are placed in this program.

(a) [2] This problem could be described in terms of the four conditions which make up a binomial experiment. Give the interpretation of these 4 conditions which make up a binomial experiment *for this problem*[2].

Condition 1. ______________________________.

Condition 2. ______________________________.

Condition 3. ______________________________.

Condition 4. ______________________________.

(b) [1] The probability that at least 10 of the 12 patients in the drug treatment program is cured is closest to:

(i) 0.44 (ii) 0.24 (iii) 0.56 (iv) 0.28 (v) 0.73

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2. It was found in 1997 the brain weights of a certain population of adult chimps follow a normal distribution with mean 135 gm and standard deviation 32 gm.

(a) [1] The fraction of adult chimps with brains weighing between 135 gm and 142 gm is closest to:

(i) 0.000 (ii) 0.499 (iii) 0.087 (iv) 0.913 (v) 0.318

(b) [1] A brain weight of 122 gm, expressed as a percentile, is closest to:

(i) 12th (ii) 35th (iii) 23rd (iv) 55th (v) 45th

3[3]**.** In a certain city, property is zoned as single–family residential (1), multi–family residential (2), commercial (3) or industrial (4). In this city, once property is zoned 1 or 4, it cannot be rezoned.

[1]Grosnick, Final, Math 124, May 1996.
[2]It is *not* enough just to state the four general conditions which make up a binomial experiment.
[3]based on Grosnick, 4, Final, Math 124, May 1996.

Property zoned 2 or 3 can be rezoned with approval of the City Council. The City Council hears rezoning appeals in January of each year. The probability that during a given year land zoned 2 will be rezoned to 3 is $\frac{1}{3}$, and the probability it will remain 2 is $\frac{2}{3}$. The probability land zoned 3 will be rezoned to 1 is $\frac{1}{8}$; the probability it will be rezoned to 2 is $\frac{1}{4}$; the probability it will be rezoned to 4 is $\frac{1}{8}$; and the probability it will remain 3 is $\frac{1}{2}$.

(a) [1] Write the transition matrix for this Markov chain.

(b) [1] Rearrange the transition matrix into the form,

$$A = \left[\begin{array}{c||c} I & O \\ \hline\hline R & S \end{array}\right] = \left[\begin{array}{c|c||c|c} \hline \quad & \quad & \quad & \quad \\ \hline\hline \quad & \quad & \quad & \quad \\ \hline \quad & \quad & \quad & \quad \end{array}\right]$$

(c) [1] The probability a multi–family residential is rezoned to

single–family residential is ______________________________.

4[4]**.** A game is played where each player is given one red, one white and one blue chip. Each player will lay down one chip. The loser pays the winner by the following rules: a red chip wins over a white chip with a payoff of \$2; a red chip wins over a blue chip with a payoff of \$5; white wins over blue with payoff of \$5. If two red chips are played, the column (C) player pays the row (R) player \$10. If two white chips are played, the row player pays the column player \$10. If two blue chips are played, it is a draw with no payoff.

(a) [1] For this two–person game, construct the payoff matrix and determine both the row minima and column maxima.

Player C → Player R ↓	red (1)	blue (2)	white (3)	row minima
red (1)				
blue (2)				
white (3)				
column maxima				

(b) [1] If player R uses the maximin strategy, s/he will

[4]based on Grosnick, 6, Final, Math 124, May 1996.

play chip color ____.

(c) [1] **True** / **False** The minimax strategy is optimal for player C and the maximin strategy is optimal for player R in this case. This game is said to be *strictly determined.*

5[5]**.** Of 450 students asked to identify the leaders of North America,

- 300 correctly identified the Prime Minister of Canada,
- 308 correctly identified the President of the United States,
- 145 correctly identified the President of Mexico,
- 95 correctly identified everyone except the US President,
- 98 correctly identified everyone except the Canadian Prime Minister,
- 195 correctly identified everyone except the Mexican President,
- 75 correctly identified all three leaders.

(a) [1] The number who identified only one leader is closest to:

(i) 200 (ii) 305 (iii) 145 (iv) 22 (v) 202

(b) [1] The number who did not know any of the leaders is closest to:

(i) 43 (ii) 10 (iii) 72 (iv) 95 (v) 55

(c) [1] The number who knew the US President, but not the Mexican President, is closest to:

(i) 110 (ii) 196 (iii) 210 (iv) 253 (v) 252

6[6]**.** Consider the following problems.

(a) [1] If $P(A \cap B) = \frac{5}{7}$ and $P(B) = \frac{5}{6}$, then $P(A|B)$ is closest to:

(i) $\frac{2}{3}$ (ii) $\frac{1}{7}$ (iii) $\frac{7}{6}$ (iv) $\frac{25}{42}$ (v) $\frac{6}{7}$

(b) [1] If A and B are independent, where $P(A) =$ and $P(B) = \frac{1}{6}$, then $P(A \cap B)$ is closest to:

(i) $\frac{2}{18}$ (ii) $\frac{3}{9}$ (iii) $\frac{1}{6}$ (iv) $\frac{6}{7}$ (v) $\frac{2}{9}$

(c) [1] If A and B are independent, where $P(A|B) = \frac{3}{4}$ and $P(B|A) = \frac{1}{3}$, then $P(A \cap B)$ is closest to:

(i) $\frac{3}{4}$ (ii) $\frac{1}{4}$ (iii) $\frac{3}{7}$ (iv) $\frac{1}{3}$ (v) $\frac{4}{7}$

[5]based on Grosnick, 3, Chapter 5 Exam, Math 124, March 1996.

[6]based on Brela, 35–37, Test 3, Statistics 213, Spring 1997.

7[7]**.** [3] A post office requires different numbers of full–time employees on different days of the week. The number of full–time employees required on each day is given in the table below.

Day	Mon (1)	Tues (2)	Wed (3)	Thur (4)	Fri (5)	Sat (6)	Sun (7)
required employees	17	13	15	19	14	16	11

Union rules states that each full–time employees must work five consecutive days and then receive two days off. For example, an employee who works Monday through Friday must be off on Saturday and Sunday. The post office wants to meet its daily requirements using only full–time employees. If x_i is the number of full–time employees used on day i, where $i = 1, \ldots, 7$, what should be x_i such that the total number of full–time employees is minimized? Set up (but do *not* solve) this linear programming problem by completing the following table.

Minimize	$\_\_\_x_1 + \_\_\_x_2 + \_\_\_x_3 + \_\_\_x_4 + \_\_\_x_5 + \_\_\_x_6 + \_\_\_x_7$
subject to	$x_1 \quad\quad x_4 + x_5 + x_6 + x_7 > 17$

and, of course, $x_i > 0$, where $i = 1, \ldots, 7$.

8[8]**.** [2] Solve the following linear program using the graphical procedure. Remember to graph the feasible region and to mark all the coordinates of all the vertices (corner points) on this region.

$$\begin{array}{lcccccc} \text{Minimize} & 7x & - & 15y & & \\ \text{subject to} & x & + & y & \leq & 2 \\ & 3x & - & y & \leq & 0 \\ & x & & & \geq & 0 \\ & & & y & \geq & 1 \end{array}$$

[7]based on Lauer, 4, Bonus LP Problems, Statistics 213.
[8]based on Lauer, V, Final, Statistics 213, May 1997.

9[9]**.** Determine whether the following augmented matrices (which are *not* necessarily row–reduced) are unique, inconsistent or dependent and, then, where possible, give their solution.

(a) [1] The matrix,

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

is (circle one) **unique** / **inconsistent** / **dependent** with

(possible) solution(s) given by: ____.

(b) [1] The matrix,

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

is (circle one) **unique** / **inconsistent** / **dependent** with

(possible) solution(s) given by: ____.

10[10]**.** A manufacturer produces a product for sale at a retail price of \$5 per unit. For this product, the manufacturer's fixed cost is \$1600 and each unit costs \$3 to produce.

(a) [1] The (linear) total cost function is given by ______________________________.

(b) [1] To break–even, the manufacturer must produce ____________________________.

(c) [1] To make a profit of \$3000, the manufacturer must produce ______________________.

[9]related to Tan, p 108, 1997.
[10]based on Lauer, I, Final, Statistics 213, May 1997.

1. (a) 12 patients, cured or not, independence, 80% cured; (b) iii
2. (a) iii; (b) ii
3. (a) 1,0,0,0;0,$\frac{2}{3}$,$\frac{1}{3}$,0; $\frac{1}{8}$,$\frac{1}{4}$,$\frac{1}{2}$,$\frac{1}{8}$;0,0,0,1;
   (b) 1,0,0,0;0,1,0,0;0,0,$\frac{2}{3}$,$\frac{1}{3}$, $\frac{1}{8}$,$\frac{1}{8}$,$\frac{1}{24}$,$\frac{1}{2}$;
   (c) 0.5
4. (a) 10,5,$\boxed{2}$;-5,0,-5;-2,5,-10; rows: $\boxed{2}$,-5,-10; columns: 10,5,$\boxed{2}$;
   (b) red; (c) True
5. (a) v; (b) ii; (c) iii
6. (a) v; (b) i; (c) ii
7. 17,13,15,19,14,16,11; $1,2,5,6,7 \geq 13$; $1,2,3,6,7 \geq 15$; $1,2,3,4,7 \geq 19$; $1,2,3,4,5 \geq 14$; $2,3,4,5,6 \geq 16$; $3,4,5,6,7 \geq 11$
8. (0,2) gives -30; (0.5,1.5) gives -19; (0.33,1) gives -12.7; (0,1) gives -15 so minimum point is (0,2) with -30
9. (a) inconsistent; (b) dependent with solution $(-3, 1, 3 - t, t)$
10. (a) $C(x) = 1600 + 3x$; (b) $x = 800$; (c) $x = 1400$