Final for Statistics 213
Probability and Decision Theory - Fall 2002
Material Covered: Chapters 1–9 of Workbook and text
11th December

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this final. A calculator may also be used. No other aids are permitted.

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1. Consider the following questions on linear functions. Choose three of the following five questions to answer.

(a) [1 point] Calculate the distance between point A(0,0) and point B(300,150). Then calculate the distance between point B(300,150) and point C(-150,200). Then calculate the distance between point C back to point A. The total distance covered is (choose the closest answer)
1017 / **1038** / **1079** / **1123** / **1137**.

(b) [1 point] Determine the slope of line 1 through the pair of points, A(0,4) and B(1,6). Determine the slope of line 2 through the pair of points, A(0,4) and C(2,3). Line 1 and line 2 are

- **(a)** both perpendicular and parallel to one another.
- **(b)** perpendicular but not parallel to one another.
- **(c)** not perpendicular, but parallel to one another.
- **(d)** neither perpendicular nor parallel to one another.
- **(e)** none of these.

(c) [1 point] Wolverine skateboards sell for \$150 a piece, cost \$35 a piece to produce and have a fixed monthly cost of \$10,500. The break–even number of skateboards is (circle closest one)
87 / **91** / **94** / **97** / **100**.

(d) [1 point] Fifty Wolverine skateboards are sold (demanded) per month if they are sold at \$150 a piece; whereas 100 Wolverine skateboards are demanded per month if they are sold at \$100 a piece. Also assume the supply equation is given by

$$p = 2x + 200$$

The equilibrium price is (circle closest one)
143 / **157** / **161** / **167** / **170**.

(e) [1 point] A study is done of bonobos that relates the number of sexual partners, x, to the chance of having this number of partners, y.

x	0	1	2	3	4	5
y	0.004	0.041	0.164	0.329	0.329	0.132

The least squares regression is given by (choose one)

(a) $y = 0.0377x + 0.0473$

(b) $y = 0.0477x + 0.0473$

(c) $y = 0.0577x + 0.0473$

(d) $y = 0.0677x + 0.0473$

(e) $y = 0.0777x + 0.0473$

2. Consider the following questions on the system of linear functions.

(a) [1 point] Consider the following system of linear equations.

$$\begin{array}{ccccc} 2\text{x} & + & 5\text{y} & = & 4 \\ \text{x} & + & \text{ky} & = & 5 \end{array}$$

The value of k for which this system of linear equations has infinitely many solutions is $k =$ (circle closest one)
0.5 / **1.0** / **1.5** / **2.0** / **2.5**.

(b) [1 point] Consider the following system of equations

$$\begin{array}{ccccccc} 3x & - & y & + & 2z & = & 7 \\ 2x & + & 4y & + & 5z & = & 4 \\ x & + & 3y & - & z & = & 10 \end{array}$$

The solution to this system of equations is $(x, y, z) =$ (choose closest one)
$\mathbf{(79/20, 27/20, -38/20)}$ / $\mathbf{(80/20, 27/20, -38/20)}$ /
$\mathbf{(81/20, 27/20, -38/20)}$ / $\mathbf{(82/20, 27/20, -38/20)}$
$\mathbf{(83/20, 27/20, -38/20)}$.

(c) [1 point] Consider the following augmented matrix.

$$A = \left[\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 3 & -2 & 1 & 9 \\ 1 & 1 & 4 & 4 \end{array}\right]$$

The Gauss–Jordan method is used to solve this system of equations. In particular, the row operations necessary to pivot on the element in the first row and first column are (choose TWO)

(i) $R_2 - \frac{1}{3}R_1$

(ii) $R_2 + 3R_1$

(iii) $R_2 - 3R_1$

(iv) $R_1 - R_3$

(v) $R_3 - R_1$

(d) [1 point] Consider the following row–reduced 4×4 system

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

This system has ... (choose one)

(i) no solution where $(x, y, z, w) = (6, -3, 4, 5)$

(ii) no solution

(iii) many solutions where $(x, y, z, w) = (6, -3, 4, t)$

(iv) many solutions where $(x, y, z, w) = (6, -3, 4, t - 5)$

(v) one solution $(x, y, z, w) = (6, -3, 4, 5)$

(e) [1 point] Consider the following row–reduced 2×4 system

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 6 \\ 0 & 1 & 1 & 0 & -3 \end{bmatrix}$$

This system has ... (choose one)

(i) no solution where $(x, y, z, w) = (6, -3, 0, 0)$

(ii) no solution

(iii) many solutions where $(x, y, z, w) = (6 - t, -3 - s, s, t)$

(iv) many solutions where $(x, y, z, w) = (6 - t, -3 - s, t, s)$

(v) one solution $(x, y, z, w) = (6, -3, 0, 0)$

(f) [1 point] Consider the following system of equations.

$$\begin{bmatrix} 3 & \text{2 - x} \\ \text{y} & 5 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 4 & w \end{bmatrix} = \begin{bmatrix} v & 5 \\ -3 & 2 \end{bmatrix}$$

Then $(x, y, w, v) =$ (choose closest one)
$\mathbf{(1, -7, 4, -3)}$ / $\mathbf{(1, 7, 4, -3)}$ / $\mathbf{(-1, -7, 4, 3)}$
$\mathbf{(-1, -7, 4, -3)}$ / $\mathbf{(1, 7, 3, 3)}$.

(g) [1 point] **True** / **False** If A is a matrix and c is a scalar, then

$$(cA)^T = (1/c)A^T$$

(h) [1 point] **True** / **False** If A and B are both square matrices and

$$AB = I$$

then A is the inverse matrix of B.

(i) [1 point] Consider the following two matrices.

$$A = \begin{bmatrix} 3 & 2 \\ \text{-2} & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 4 & 9 \end{bmatrix}$$

Then $AB =$ (choose closest one)

(i) matrix

$$\begin{bmatrix} 11 & 30 \\ 26 & 55 \end{bmatrix}$$

(ii) matrix

$$\begin{bmatrix} 21 & 30 \\ 26 & 55 \end{bmatrix}$$

(iii) matrix

$$\begin{bmatrix} 31 & 30 \\ 26 & 55 \end{bmatrix}$$

(iv) matrix

$$\begin{bmatrix} 41 & 30 \\ 26 & 55 \end{bmatrix}$$

(v) matrix

$$\begin{bmatrix} 51 & 30 \\ 26 & 55 \end{bmatrix}$$

(j) [1 point] Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Then the inverse, A^{-1}, does NOT exist if (choose closest one)

(i) $a - c = 0$

(ii) $a - d = 0$

(iii) $ad - bc = 0$

(iv) $cd - ab = 0$

(v) $bd - ac = 0$

(k) [1 point] Let

$$A = \begin{bmatrix} 0.3 & 0.1 \\ 0.4 & 0.2 \end{bmatrix}$$

and

$$D = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

Solve the matrix equation $(I - A)X = D$ for X (choose closest one)

(i) $(152/13, 175/13)^T$

(ii) $(153/13, 175/13)^T$

(iii) $(154/13, 175/13)^T$

(iv) $(155/13, 175/13)^T$

(v) $(156/13, 175/13)^T$

3. Consider the following questions on linear programming problems, which are solved using a geometric approach.

(a) [1 point] Consider the following system of inequalities.

$$\begin{array}{rcrcr} 2x & + & 2y & \leq & 5 \\ 3x & + & y & \geq & 4 \\ x & & & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

The solution set is (circle one) **bounded** / **unbounded**.

(b) [1 point] Consider the following system of inequalities.

$$\begin{array}{rcrcr} 2x & + & 2y & \leq & 5 \\ 3x & + & y & \geq & 4 \\ x & & & \geq & 0 \end{array}$$

The solution set is (circle one) **bounded** / **unbounded**.

(c) [1 point] A lake is to be stocked with trout and perch, which feed on two types of food (A and B) that grow in the lake at rates of 30 units and 24 units per day, respectively. Each trout consumes 10 units of the first food and 5 units of the second food per day, and each perch consumes 9 units of the first food and

3 units of the second food per day. An average trout weighs 1.5 pound and an average perch weighs 0.8 pounds. The lake is stocked so that the total weight of fish in the lake is as great as possible. If there are x number of trout and y number of perch, the LP problem for this situation is given by (circle one)

(i) LP Candidate 1

$$\begin{array}{lcccccc} \text{Minimize} & 1.5x & + & 0.8y & & \\ \text{subject to} & 10x & + & 9y & \leq & 30 \\ & 5x & + & 3y & \leq & 24 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(ii) LP Candidate 2

$$\begin{array}{lcccccc} \text{Maximize} & 1.5x & + & 0.8y & & \\ \text{subject to} & 10x & + & 9y & \leq & 30 \\ & 5x & + & 3y & \leq & 24 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(iii) LP Candidate 3

$$\begin{array}{lcccccc} \text{Maximize} & 1.5x & + & 0.8y & & \\ \text{subject to} & 10x & + & 5y & \leq & 30 \\ & 9x & + & 3y & \leq & 24 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(iv) LP Candidate 4

$$\begin{array}{lcccccc} \text{Maximize} & 1.5x & + & 0.8y & & \\ \text{subject to} & 10x & + & 9y & \geq & 30 \\ & 5x & + & 3y & \geq & 24 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(v) LP Candidate 5

$$\begin{array}{lcccccc} \text{Maximize} & 30x & + & 24y & & \\ \text{subject to} & 10x & + & 9y & \leq & 1.5 \\ & 5x & + & 3y & \leq & 0.8 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(d) [1 point] Use the graphical procedure to solve the following linear programming problem.

$$\begin{array}{lccccc} \text{Maximize} & 8x & - & 4y & & \\ \text{subject to} & x & + & y & \leq & 4 \\ & 2x & + & y & \geq & 10 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

The solution is (choose one)

(i) unbounded

(ii) inconsistent

(iii) $(x, y) = (1, 2)$

(iv) $(x, y) = (2, 4)$

(v) $(x, y) = (3, 2)$

(e) [1 point] Use the graphical procedure to solve the following linear programming problem.

$$\begin{array}{lccccc} \text{Maximize} & 3x & + & 4y & & \\ \text{subject to} & x & + & y & \leq & 4 \\ & 2x & + & y & \leq & 10 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

The solution is (choose one)

(i) unbounded

(ii) inconsistent

(iii) $(x, y) = (4, 2)$

(iv) $(x, y) = (0, 4)$

(v) $(x, y) = (3, 2)$

(f) [1 point] Use the graphical procedure to solve the following linear programming problem.

$$\begin{array}{lccccc} \text{Maximize} & 2x & + & 9y & & \\ \text{subject to} & x & + & y & \leq & 4 \\ & 2x & + & y & \geq & 10 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

The solution is (choose one)

(i) unbounded

(ii) inconsistent

(iii) $(x, y) = (1, 3)$

(iv) $(x, y) = (0, 2)$

(v) $(x, y) = (3, 1)$

4. Consider the following questions on linear programming problems, which are solved using the simplex method.

(a) [1 point] Consider the following simplex tableau.

x	y	z	u	v	w	P	
1	3	2	1	0	0	0	3
2	2	2	0	1	0	0	5
3	1	1	0	0	1	0	4
-4	-2	-1	0	0	0	1	0

Identify the pivot point then use this information to identify the next simplex tableau.

(i) Simplex tableau candidate A.

x	y	z	u	v	w	P	
0	8/3	5/3	1	0	-4/3	0	5/3
0	5/3	4/3	0	1	-2/3	0	7/3
1	1/3	1/3	0	0	1/3	0	4/3
0	-2/3	1/3	0	0	2/3	1	16/3

(ii) Simplex tableau candidate B.

x	y	z	u	v	w	P	
0	8/3	5/3	1	0	-1/3	0	5/3
0	4/3	4/3	0	1	-2/3	0	7/3
1	1/3	1/3	0	0	1/3	0	4/3
0	-2/3	1/3	0	0	4/3	1	16/3

(iii) Simplex tableau candidate C.

x	y	z	u	v	w	P	
0	8/3	7/3	1	0	-1/3	0	5/3
1	4/3	4/3	0	1	-2/3	0	7/3
0	1/3	1/3	0	0	2/3	0	4/3
0	-4/3	1/3	0	0	4/3	1	16/3

(iv) Simplex tableau candidate D.

x	y	z	u	v	w	P	
0	4/3	5/3	1	0	-1/3	0	5/3
0	4/3	4/3	0	1	-2/3	0	9/3
1	2/3	1/3	0	0	1/3	0	4/3
0	-2/3	1/3	0	0	4/3	1	16/3

(v) Simplex tableau candidate E.

x	y	z	u	v	w	P	
1	8/3	7/3	1	0	-1/3	0	5/3
0	1/3	4/3	0	1	-1/3	0	7/3
0	1/3	2/3	0	0	1/3	0	4/3
0	-2/3	1/3	0	0	6/3	1	16/3

(b) [1 point] Which of the following conditions applies in a standard maximization problem? Choose none, one or more.

(i) The objective function is maximized.

(ii) All variables are nonnegative.

(iii) Each linear constraint is written as an inequality which is greater than or equal to a constant.

(iv) Each linear constraint is written as an inequality which is less than or equal to a constant.

(v) Each linear constraint is written as an inequality which is greater than or equal to a nonnegative constant.

(c) [1 point] Consider the following linear programming problem.

$$\begin{array}{lcccccc} \text{Minimize} & 1.5\text{x} & + & 0.8\text{y} & & \\ \text{subject to} & 10\text{x} & + & 9\text{y} & \geq & 30 \\ & 5\text{x} & + & 3\text{y} & \geq & 24 \\ & \text{x} & & & \geq & 0 \\ & & & \text{y} & \geq & 0 \end{array}$$

The dual of this problem is (choose one)

(i) LP Candidate 1

$$\begin{array}{lcccccc} \text{Maximize} & 1.5\text{u} & + & 0.8\text{v} & & \\ \text{subject to} & 10\text{u} & + & 5\text{v} & \leq & 30 \\ & 9\text{u} & + & 3\text{v} & \leq & 24 \\ & \text{u} & & & \geq & 0 \\ & & & \text{v} & \geq & 0 \end{array}$$

(ii) LP Candidate 2

$$\begin{array}{lcccccc} \text{Maximize} & 30\text{u} & + & 24\text{v} & & \\ \text{subject to} & 10\text{u} & + & 9\text{v} & \leq & 1.5 \\ & 5\text{u} & + & 3\text{v} & \leq & 0.8 \\ & \text{u} & & & \geq & 0 \\ & & & \text{v} & \geq & 0 \end{array}$$

(iii) LP Candidate 3

$$\begin{array}{lrcrcl} \text{Maximize} & 30u & + & 24v & & \\ \text{subject to} & 10u & + & 5v & \geq & 1.5 \\ & 9u & + & 3v & \geq & 0.8 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

(iv) LP Candidate 4

$$\begin{array}{lrcrcl} \text{Maximize} & 1.5u & + & 0.8v & & \\ \text{subject to} & 10u & + & 9v & \leq & 30 \\ & 5u & + & 3v & \leq & 24 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

(v) LP Candidate 5

$$\begin{array}{lrcrcl} \text{Maximize} & 30u & + & 24v & & \\ \text{subject to} & 10u & + & 5v & \leq & 1.5 \\ & 9u & + & 3v & \leq & 0.8 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

(d) [1 point] Consider the following linear programming problem.

$$\begin{array}{lrcrcl} \text{Minimize} & 1.5x & + & 0.8y & & \\ \text{subject to} & 10x & + & 9y & \geq & 30 \\ & 5x & + & 3y & \geq & 24 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

Use the simplex method to find the solution of the dual of this problem (choose one)

(i) unbounded

(ii) inconsistent

(iii) $(x, y) = \left(0, \frac{4}{15}\right)$ where $P = \frac{32}{5}$

(iv) $(u, v) = \left(0, \frac{4}{15}\right)$ where $P = \frac{32}{5}$

(v) $(x, y) = (0, 8)$ where $P = \frac{32}{5}$

(e) [1 point] Consider the following linear programming problem.

$$\begin{array}{lcccccc}
\text{Minimize} & 1.5x & + & 0.8y & & \\
\text{subject to} & 10x & + & 9y & \geq & 30 \\
& 5x & + & 3y & \geq & 24 \\
& x & & & \geq & 0 \\
& & & y & \geq & 0
\end{array}$$

Use the simplex method to find the solution of the primal of this problem (choose one)

(i) $(u, v) = \left(8, \frac{1}{15}\right)$ where $P = \frac{32}{5}$

(ii) $(u, v) = \left(0, \frac{4}{15}\right)$ where $P = \frac{32}{5}$

(iii) $(x, y) = \left(0, \frac{3}{15}\right)$ where $P = \frac{32}{5}$

(iv) $(x, y) = (8, 0)$ where $P = \frac{32}{5}$

(v) $(x, y) = (0, 8)$ where $P = \frac{32}{5}$

(f) [1 point] Consider the following nonstandard linear programming problem.

$$\begin{array}{lcccccccc}
\text{Minimize} & 2x & - & y & + & 4z & & \\
\text{subject to} & 2x & + & y & + & z & \geq & 2 \\
& x & + & 3y & + & z & \geq & 6 \\
& 2x & + & y & + & 2z & \leq & 12 \\
& x & & & & & \geq & 0 \\
& & & y & & & \geq & 0 \\
& & & & & z & \geq & 0
\end{array}$$

The solution to this problem is (choose one)

(i) $(x, y, z) = (0, 11, 0)$ where $P = 11$

(ii) $(x, y, z) = (0, 11, 0)$ where $P = -11$

(iii) $(x, y, z) = (0, 12, 0)$ where $P = 12$

(iv) $(x, y, z) = (0, 12, 0)$ where $P = -12$

(v) $(x, y, z) = (0, 13, 0)$ where $P = 13$

5. Consider the following questions on the mathematics of finance.

(a) [1 point] Professor Bee put \$26,000 into a blue–chip investment and, two years later, sold this investment for a \$32,300. Determine the effective annual rate of return on this investment. (circle closest one)
0.1146 / **0.1156** / **0.1166** / **0.1176** / **0.1186**.

(b) [1 point] Find the future amount of the following ordinary annuity: \$2300 per quarter for 6 years at 8% per year compounded quarterly. Circle closest one.
\$29, 970.28 / \$39, 970.28 / \$49, 970.28 / \$59, 970.28 / \$69, 970.28.

(c) [1 point] Find the present value of the following ordinary annuity: \$2300 per quarter for 6 years at 8% per year compounded quarterly. Circle closest one.
\$13, 502.03 / \$23, 502.03 / \$33, 502.03 / \$43, 502.03 / \$53, 502.03.

(d) [1 point] Find the periodic payment R required to amortize a loan of \$2300 over 36 periods with interest earned at the rate of 0.03 per period. Circle closest one.
\$104.35 / \$105.35 / \$106.35 / \$107.35 / \$108.35.

(e) [1 point] Find the periodic payment R required to accumulate a sum of \$2300 over 36 periods with interest earned at the rate of 0.03 per period. Use the sinking fund formula. Circle closest one.
\$36.35 / \$37.35 / \$38.35 / \$39.35 / \$40.35.

(f) [1 point] Find the ninth term in the arithmetic progression,

$$x + y, 2x - y, 3x - 3, \dots.$$

Circle closest one.
$\boldsymbol{9x - 11y}$ / $\boldsymbol{9x - 12y}$ / $\boldsymbol{9x - 13y}$ / $\boldsymbol{9x - 14y}$ / $\boldsymbol{9x - 15y}$.

(g) [1 point] Find the sum of the first nine terms in the arithmetic progression,

$$\frac{1}{3}, \frac{4}{3}, \frac{7}{3}, \frac{10}{3}, \dots.$$

Circle closest one.
38 / 39 / 40 / 41 / 42.

(h) [1 point] Find the sum of the first nine terms in the geometric progression,

$$1, -\frac{1}{3}, -\frac{1}{9}, \dots.$$

Circle closest one.
$\mathbf{\frac{78733}{59049}}$ / $\mathbf{\frac{78734}{59049}}$ / $\mathbf{\frac{78735}{59049}}$ / $\mathbf{\frac{78736}{59049}}$ / $\mathbf{\frac{78737}{59049}}$.

(i) [1 point] Find the ninth term in the geometric progression,

$$1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \dots.$$

Circle closest one.
$\mathbf{\frac{1}{6559}}$ / $\mathbf{\frac{1}{6560}}$ / $\mathbf{\frac{1}{6561}}$ / $\mathbf{\frac{1}{6562}}$ / $\mathbf{\frac{1}{6563}}$.

6. Try the following counting questions.

(a) [1 point] Let $U = \{-2,-1,0,1,2\}$, $A = \{-2,1\}$, $B = \{0,1,2\}$ and $C = \{0\}$.

$$(A \cap B) \cup C =$$

(choose closest one)
$\mathbf{\{-2,1\}}$ / $\mathbf{\{-1,1\}}$ / $\mathbf{\{0,1\}}$
$\mathbf{\{1,1\}}$ / $\mathbf{\{2,1\}}$.

(b) [1 point] Let $U = \{-2,-1,0,1,2\}$, $A = \{-2,1\}$, $B = \{0,1,2\}$ and $C = \{0\}$.

$$(A \cap B)^c \cup C =$$

(choose closest one)
$\mathbf{\{-2,-1,0,2\}}$ / $\mathbf{\{-2,-1,1,2\}}$ / $\mathbf{\{-1,0,1,2\}}$ / $\mathbf{\{-2,0,1,2\}}$ / $\mathbf{\{-2,-1,0,1\}}$.

(c) [1 point] Let A and B be subsets of the universal set U, where $n(U) = 350$, $n(A) = 50$, $n(B) = 150$, $n(A \cap B) = 25$.

$$n(A^c \cap B^c) =$$

(circle closest one) **150** / **175** / **200** / **225** / **250**.

(d) [1 point] Let A and B be subsets of the universal set U, where $n(U) = 350$, $n(A) = 50$, $n(B) = 150$, $n(A \cap B) = 25$.

$$n(A^c \cup B) =$$

(circle closest one) **25** / **50** / **75** / **100** / **125**.

(e) [1 point] The number of four–digit numbers that can be formed from the digits 1, 2 and 3, if each four–digit number must be odd is
(circle closest one) **27** / **35** / **44** / **54** / **67**.

(f) [1 point] Evaluate $C(23,3)$.
(circle closest one) **227** / **564** / **1083** / **1323** / **1771**.

(g) [1 point] Evaluate $C(23,b)$.
(circle closest one) $\frac{b!}{(23-b)!b!}$ / $\frac{b!}{(b-23)!23!}$ / $\frac{23!}{(23-b)!23!}$ / $\frac{23!}{(23+b)!b!}$ / $\frac{23!}{(b-23)!b!}$.

(h) [1 point] The number of different arrangements that can be made from the letters WESTVILLE is
(circle closest one) **90720** / **91720** / **92720** / **93720** / **94720**.

(i) [1 point] From a group of 6 women and 9 men, how many different committees consisting of 4 women and 3 men can be formed?
(circle closest one) **960** / **1060** / **1160** / **1260** / **1360**.

7. Try the following probability questions.

(a) [1 point] Describe the sample space associated with flipping a coin until either heads or tails occurs twice. Choose one.

(i) $\{HHT, THH, HTH, TT, HTT, THT\}$

(ii) $\{HH, THH, HTHT, TT, HTT, THT\}$

(iii) $\{HH, THH, HTH, TT, HTT, THT\}$

(iv) $\{HH, THH, HTH, TT, TTH, THT\}$

(v) $\{HH, HHT, HTH, TT, TTH, THT\}$

(b) [1 point] In two rolls of a fair die, let event A be the event that no fours, fives or sixes are rolled. Then, $P(A) =$
(circle closest one) $\mathbf{\frac{8}{36}}$ / $\mathbf{\frac{9}{36}}$ / $\mathbf{\frac{10}{36}}$ / $\mathbf{\frac{11}{36}}$ / $\mathbf{\frac{13}{36}}$.

(c) [1 point] In ten rolls of a fair die, let event A be the event that no fours, fives or sixes are rolled. Then, $P(A) =$
(circle closest one) $\mathbf{\frac{59048}{60466176}}$ / $\mathbf{\frac{59049}{60466176}}$ / $\mathbf{\frac{59050}{60466176}}$ / $\mathbf{\frac{59051}{60466176}}$ / $\mathbf{\frac{59052}{60466176}}$.

(d) [1 point] Let E and F be two events of an experiment where $P(E) = 0.35$, $P(F) = 0.15$ and $P(E \cap F) = 0.03$. Then $P(E \cup F) =$
(choose closest one) **0.46** / **0.47** / **0.48** / **0.49** / **0.50**.

(e) [1 point] Let E and F be two events of an experiment where $P(E) = 0.35$, $P(F) = 0.15$ and $P(E \cap F) = 0.03$. Then $P(E^c \cup F^c) =$
(choose closest one) **0.96** / **0.97** / **0.98** / **0.99** / **1.00**.

(f) [1 point] We randomly pick eight journalists from a pack of 240 of which 15 are also photographers. The probability that three of the eight picked are photographers is (choose closest one)
$\mathbf{\frac{C(8,3)C(232,5)}{C(240,8)}}$ / $\mathbf{\frac{C(15,3)C(210,5)}{C(225,8)}}$ / $\mathbf{\frac{C(15,3)C(225,5)}{C(240,8)}}$
$\mathbf{\frac{C(15,5)C(225,3)}{C(240,8)}}$ / $\mathbf{\frac{C(15,3)C(5,5)}{C(15,8)}}$.

(g) [1 point] Poker dice are played by simultaneously rolling 6 dice. $P\{\text{no two alike}\} =$ (choose closest one)
$\mathbf{\frac{718}{46656}}$ / $\mathbf{\frac{719}{46656}}$ / $\mathbf{\frac{720}{46656}}$ / $\mathbf{\frac{721}{46656}}$ / $\mathbf{\frac{722}{46656}}$.

(h) [1 point] Urn A has 7 red and 9 blue marbles; urn B has 5 red and 10 blue marbles. A fair coin is tossed. If the coin comes up heads, a marble from urn A

is chosen, otherwise a marble from urn B is chosen. The chance a red marble is chosen is (choose closest one)
$\mathbf{\frac{37}{96}}$ / $\mathbf{\frac{38}{96}}$ / $\mathbf{\frac{39}{96}}$ / $\mathbf{\frac{40}{96}}$ / $\mathbf{\frac{41}{96}}$.

(i) [1 point] Urn A has 10 red and 9 blue marbles; urn B has 10 red and 10 blue marbles. A fair coin is tossed. If the coin comes up heads, a marble from urn A is chosen, otherwise a marble from urn B is chosen. The chance a red marble is chosen is (choose closest one)
$\mathbf{\frac{76}{152}}$ / $\mathbf{\frac{77}{152}}$ / $\mathbf{\frac{78}{152}}$ / $\mathbf{\frac{79}{152}}$ / $\mathbf{\frac{80}{152}}$.

(j) [1 point] Urn A has 10 red and 9 blue marbles; urn B has 10 red and 10 blue marbles. A fair coin is tossed. If the coin comes up heads, a marble from urn A is chosen, otherwise a marble from urn B is chosen. The chance the coin is flipped heads given that a red marble is chosen is (choose closest one)
$\mathbf{\frac{17}{39}}$ / $\mathbf{\frac{18}{39}}$ / $\mathbf{\frac{19}{39}}$ / $\mathbf{\frac{20}{39}}$ / $\mathbf{\frac{21}{39}}$.

(k) [1 point] Urn A has 10 red and 9 blue marbles; urn B has 10 red and 10 blue marbles. A fair coin is tossed. If the coin comes up heads, a marble from urn A is chosen, otherwise a marble from urn B is chosen. The chance the coin is flipped tails given that a red marble is chosen is (choose closest one)
$\mathbf{\frac{17}{39}}$ / $\mathbf{\frac{18}{39}}$ / $\mathbf{\frac{19}{39}}$ / $\mathbf{\frac{20}{39}}$ / $\mathbf{\frac{21}{39}}$.

(l) [1 point] **True** / **False** Two events independent of one another are necessarily also mutually exclusive of one another.

8. Try the following questions on probability distributions and statistics.

(a) [1 point] A weighted coin is flipped where heads comes up twice as often as tails. The chance that two or more heads comes up in three flips is
(choose closest one) $\mathbf{\frac{16}{27}}$ / $\mathbf{\frac{17}{27}}$ / $\mathbf{\frac{18}{27}}$ / $\mathbf{\frac{19}{27}}$ / $\mathbf{\frac{20}{27}}$.

(b) [1 point] A weighted coin is flipped where heads comes up twice as often as tails. The chance that one head comes up in three flips is
(choose closest one) $\mathbf{\frac{2}{9}}$ / $\mathbf{\frac{3}{9}}$ / $\mathbf{\frac{4}{9}}$ / $\mathbf{\frac{5}{9}}$ / $\mathbf{\frac{6}{9}}$.

(c) [1 point] The chance of marrying at ages 20, 25, 30, 35 and 40 is 0.10, 0.50, 0.15, 0.15 and 0.10, respectively. In this case, the expected age to be married is (choose closest one) **27.75** / **28.00** / **28.25** / **28.50** / **28.75**.

(d) [1 point] The chance of marrying at ages 20, 25, 30, 35 and 40 is 0.10, 0.50, 0.15, 0.15 and 0.10, respectively. In this case, the variance in the age to be married is
(choose closest one) **33.0875** / **33.1875** / **33.2875** / **33.3875** / **33.4875**.

(e) [1 point] The chance of marrying at ages 20, 25, 30, 35 and 40 is 0.10, 0.50, 0.15, 0.15 and 0.10, respectively. The odds in favor of getting married at age 30 is (choose closest one) **2 to 17** / **3 to 17** / **4 to 17** / **5 to 17** / **6 to 17**.

(f) [1 point] The chance of marrying at ages 20, 25, 30, 35 and 40 is 0.10, 0.50, 0.15, 0.15 and 0.10, respectively. The odds in favor of getting married at age 20 is (choose closest one) **1 to 9** / **2 to 9** / **3 to 9** / **4 to 9** / **5 to 9**.

(g) [1 point] The odds in favor of getting married at age 20 is 2 to 19; in other words, the chance of getting married at age 20 is
(choose closest one) **0.0652** / **0.0752** / **0.0852** / **0.0952** / **0.1052**.

(h) [1 point] A probability distribution has a mean of -23 and standard deviation of 5. Use Chebychev's inequality to estimate the probability that a randomly chosen outcome of the experiment lies between -33 and -13 is at least
(choose closest one) **0.75** / **0.78** / **0.81** / **0.83** / **0.86**.

(i) [1 point] On a multiple choice exam with 5 possible answers for each of the 10 questions, what is the probability that a student should get 8 or more correct answers just by guessing? [Hint: binomial.] (choose closest one)
$\mathbf{5.7926 \times 10^{-5}}$ / $\mathbf{6.7926 \times 10^{-5}}$ / $\mathbf{7.7926 \times 10^{-5}}$
$\mathbf{8.7926 \times 10^{-5}}$ / $\mathbf{9.7926 \times 10^{-5}}$.

(j) [1 point] **True** / **False** One of the properties of a binomial experiment is that the number of trials in the experiment varies.

(k) [1 point] Let Z be a standard normal variable. $P(-2.3 < Z < 0.14) =$
(choose closest one) **0.4449** / **0.5449** / **0.6449** / **0.7449** / **0.8449**.

(l) [1 point] Let X be a normal variable where $\mu = -10$ and $\sigma = 2$. $P(-13 < X < -10) =$
(choose closest one) **0.4332** / **0.4432** / **0.4532** / **0.4632** / **0.4732**.

(m) [1 point] Let Z be a standard normal variable and let X be a normal variable where $\mu = -10$ and $\sigma = 2$. Then, $P(-13 < X < -10) =$
(choose closest one) $\boldsymbol{P(-0.5 < Z < 0)}$ / $\boldsymbol{P(-1.0 < Z < 0)}$ / $\boldsymbol{P(-1.5 < Z < 0.5)}$ / $\boldsymbol{P(-2.0 < Z < 0.5)}$ / $\boldsymbol{P(-1.5 < Z < 0)}$.

(n) [1 point] Let X be a binomial variable where $n = 100$ and $p = 0.35$. Approximating, without a continuity correction, this binomial variable with a normal variable, $P(X < 32) \approx$
(choose closest one) **0.2347** / **0.2447** / **0.2547** / **0.2647** / **0.2747**.

9. Try the following questions on Markov chains and the theory of games.

(a) [1 point] Consider the following matrices.

$$T = \begin{bmatrix} 0.4 & 0.2 & 0.7 & 0.2 & 0.1 \\ 0.4 & 0.2 & 0.1 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 & 0.2 & 0.4 \\ 0 & 0.2 & 0 & 0.2 & 0.3 \end{bmatrix}, \quad X_0 = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}.$$

Then $T^5 X_0 =$
Choose one
$\mathbf{(0.104, 0.268, 0.114, 0.185, 0.130)^T}$
$\mathbf{(0.204, 0.268, 0.114, 0.185, 0.130)^T}$
$\mathbf{(0.304, 0.268, 0.114, 0.185, 0.130)^T}$
$\mathbf{(0.404, 0.268, 0.114, 0.185, 0.130)^T}$
$\mathbf{(0.504, 0.268, 0.014, 0.185, 0.130)^T}$.

(b) [1 point] Consider the following matrices.

$$T = \begin{bmatrix} 0.4 & 0.2 & 0.7 & 0.2 & 0.1 \\ 0.4 & 0.2 & 0.1 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 & 0.2 & 0.4 \\ 0 & 0.2 & 0 & 0.2 & 0.3 \end{bmatrix}, \quad X_0 = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}.$$

Choose none, one or more.

(i) Matrix T is a regular matrix.

(ii) Matrix T converges to a steady–state distribution vector.

(iii) Matrix T diverges to a steady–state matrix.

(iv) The sequence $T^n X_0$, $n = 1, 2, \ldots$ converges to a steady state distribution vector.

(v) The sequence $T^n X_0$, $n = 1, 2, \ldots$ diverges to a steady state distribution vector.

(c) [1 point] Consider the following matrices.

$$T = \begin{bmatrix} 0.4 & 0.2 & 0.7 & 0.2 & 0.1 \\ 0.4 & 0.2 & 0.1 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0 \\ 0.1 & 0.2 & 0.1 & 0.2 & 0.4 \\ 0 & 0.2 & 0 & 0.2 & 0.3 \end{bmatrix}, \quad X_0 = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}.$$

The conditions $TX = X$ and the sum of the components of X must equal 1 can be described by the following augmented matrix. Choose one.

(i) Matrix

$$\left[\begin{array}{ccccc|c} 0.4 & 0.2 & 0.7 & 0.2 & 0.1 & 0 \\ 0.4 & 0.2 & 0.1 & 0.3 & 0.2 & 0 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0 & 0 \\ 0.1 & 0.2 & 0.1 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0 & 0.2 & 0.3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

(ii) Matrix

$$\left[\begin{array}{ccccc|c} -0.6 & 0.2 & 0.7 & 0.2 & 0.1 & 0 \\ 0.4 & -0.8 & 0.1 & 0.3 & 0.2 & 0 \\ 0.1 & 0.2 & -0.9 & 0.1 & 0 & 0 \\ 0.1 & 0.2 & 0.1 & -0.8 & 0.4 & 0 \\ 0 & 0.2 & 0 & 0.2 & -0.7 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

(iii) Matrix

$$\left[\begin{array}{ccccc|c} -0.6 & 0.2 & 0.7 & 0.2 & 0.1 & 1 \\ 0.4 & -0.8 & 0.1 & 0.3 & 0.2 & 1 \\ 0.1 & 0.2 & -0.9 & 0.1 & 0 & 1 \\ 0.1 & 0.2 & 0.1 & -0.8 & 0.4 & 1 \\ 0 & 0.2 & 0 & 0.2 & -0.7 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

(iv) Matrix

$$\left[\begin{array}{ccccc|c} -0.6 & 0.2 & 0.7 & 0.2 & 0.1 & 0 \\ 0.4 & -0.7 & 0.1 & 0.3 & 0.2 & 0 \\ 0.1 & 0.2 & -0.9 & 0.1 & 0 & 0 \\ 0.1 & 0.2 & 0.1 & -0.8 & 0.4 & 0 \\ 0 & 0.2 & 0 & 0.2 & -0.7 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

(v) Matrix

$$\left[\begin{array}{ccccc|c} -0.6 & 0.2 & 0.7 & 0.2 & 0.1 & 0 \\ 0.4 & -0.8 & 0.1 & 0.3 & 0.2 & 0 \\ 0.1 & 0.2 & -0.8 & 0.1 & 0 & 0 \\ 0.1 & 0.2 & 0.1 & -0.8 & 0.4 & 0 \\ 0 & 0.2 & 0 & 0.2 & -0.7 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

(d) [1 point] Consider the following matrices.

$$A = \begin{bmatrix} 0 & 0.2 & 0.7 & 0.2 & 0.1 \\ 0 & 0.2 & 0.1 & 0.3 & 0.2 \\ 0 & 0.2 & 0.1 & 0.1 & 0 \\ 0 & 0.4 & 0.1 & 0.4 & 0.4 \\ 1 & 0 & 0 & 0 & 0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & 0.4 \\ 0 & 0 & 0 & 0.2 & 0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.1 \\ 0 & 1 & 0 & 0.3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0.4 \\ 0 & 0 & 0 & 0.4 & 0.5 \end{bmatrix}.$$

Identify which of the following matrices are absorbing stochastic matrices. Choose none, one or more.

(i) Matrix A only.

(ii) Matrix B only.

(iii) Matrix C only.

(iv) Matrices A and B.

(v) Matrices A, B and C.

(e) [1 point] Consider the following matrix.

$$A = \begin{bmatrix} 0 & 0.2 & 0.7 & 0.2 & 0.1 \\ 0 & 0.2 & 0.1 & 0.3 & 0.2 \\ 0 & 0.2 & 0.1 & 0.1 & 0 \\ 0 & 0.4 & 0.1 & 0.4 & 0.4 \\ 1 & 0 & 0 & 0 & 0.3 \end{bmatrix}$$

Then $S(I - R)^{-1} =$
Choose one
$\mathbf{(0.1, 0.2, 0.3, 0.4)}$ /
$\mathbf{(1, 0, 0.5, 0.8)}$ /
$\mathbf{(1, 1, 1, 1)}$ /
$\mathbf{(1, 0.5, 0, 1)}$ /
$\mathbf{(0.5, 0.5, 0.5, 0.5)}$.

(f) [1 point] Consider the following payoff matrix.

$$\begin{bmatrix} -3 & -2 & 3 & 2 & -1 \\ 2 & 2 & -1 & 3 & 2 \\ 0 & 3 & -1 & 1 & 0 \\ 1 & 4 & -1 & 4 & -4 \\ 5 & 5 & 4 & 9 & 5 \end{bmatrix}$$

Choose none, one or more.

(i) This game is not fair, both players have a fixed optimal strategy and the saddlepoint has a value of 4.

(ii) This game is strictly determined, both players have mixed optimal strategies and the saddlepoint has a value of 0.

(iii) This game is strictly determined, both players have a fixed optimal strategy and the saddlepoint has a value of 3.

(iv) This game is strictly determined, both players have a fixed optimal strategy and the saddlepoint has a value of 4.

(v) This game is not strictly determined, both players have a fixed optimal strategy and the saddlepoint has a value of 4.

(g) [1 point] Consider the following payoff matrix and mixed strategies

$$\begin{bmatrix} -3 & -2 & 3 & 2 & -1 \\ 2 & 2 & -1 & 3 & 2 \\ 0 & 3 & -1 & 1 & 0 \\ 1 & 4 & -1 & 4 & -4 \\ 5 & 0 & 4 & 9 & 3 \end{bmatrix}, \quad \begin{bmatrix} 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}. \quad \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.6 \end{bmatrix}.$$

The expected value of the game is
(choose closest one) **2.04** / **2.05** / **2.06** / **2.07** / **2.08**.

(h) [1 point] Consider the following payoff matrix and mixed strategies

$$\begin{bmatrix} -3 & -2 & 3 & 2 & -1 \\ 2 & 2 & -1 & 3 & 2 \\ 0 & 3 & -1 & 1 & 0 \\ 1 & 4 & -1 & 4 & -4 \\ 5 & 0 & 4 & 9 & 3 \end{bmatrix}, \quad \begin{bmatrix} 0.1 \\ 0.3 \\ 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}. \quad \begin{bmatrix} 0.1 & 0.2 & 0.1 & 0.1 & 0.5 \end{bmatrix}.$$

The expected value of the game is
(choose closest one) **1.88** / **1.89** / **1.90** / **1.91** / **1.92**.

(i) [1 point] Consider the following payoff matrix

$$\begin{bmatrix} 5 & 6 \\ 6 & 3 \end{bmatrix}$$

The optimal mixed strategy for the row player is given by Choose closest one.
$\mathbf{(0.75, 0.25)}$ / $\mathbf{(0.65, 0.35)}$ / $\mathbf{(0.55, 0.45)}$ / $\mathbf{(0.45, 0.55)}$ / $\mathbf{(0.35, 0.65)}$.

(1) (a) $(\mathbf{\frac{5}{4}, -23})$ (b) $(x, y, z) = (135/28, 81/56, -16/7) = (4.82, 1.45, -2.29)$ (c) **False**

(2) (a) **inconsistent** (b) $\boldsymbol{(x, y) = (0, 4)}$ (c) **unbounded**

(3) simplex tableau problem

(a) initial simplex tableau.

x	y	z	u	v	w	P	
1	3	2	1	0	0	0	3
2	2	2	0	1	0	0	5
[3]	1	1	0	0	1	0	4
-4	-2	-1	0	0	0	1	0

(b) next simplex tableau.

x	y	z	u	v	w	P	
0	8/3	5/3	1	0	-1/3	0	5/3
0	4/3	4/3	0	1	-2/3	0	7/3
1	1/3	1/3	0	0	1/3	0	4/3
0	-2/3	1/3	0	0	4/3	1	16/3

(c) *one more* simplex tableau is required, and then $(x, y, z) = (9/8, 5/8, 0)$ with $P = 23/4$

(4) (a) **123,097.43** (b) **34,533.99**

(5) (a) **144** (b) **432** (c) **360**

(6) (a) **0.48** (b) $\mathbf{\frac{7}{12}}$ (c) $\mathbf{\frac{21}{26}}$

(7) binomial distribution

(a) $\mathbf{\frac{10}{3}}$

(b) probability distribution

X	0	1	2	3	4	5
$P(X = x)$	1/243	10/243	40/243	80/243	80/243	32/243
	0.004	0.041	0.164	0.329	0.329	0.132

(c) **4**

(8) (a) $\boldsymbol{B}$, $\boldsymbol{C}$ (b) **1** (c) **1**

(9) (a) **a unique** (b) $\mathbf{90^o}$

(10) linear programming problem statement

Maximize	0.15x	+	0.10y	+	0.05z		
subject to	x	+	y	+	z	$\leq$	100,000
	0.90x	-	0.10y	-	0.10z	$\leq$	0
	-0.35x	+	0.65y	-	0.35z	$\geq$	0
	-0.40x	-	0.40y	+	0.60z	$\geq$	0
	x					$\geq$	0
			y			$\geq$	0
					z	$\geq$	0