Chapter 11

Engineering Process Control and SPC

We look at *engineering process control* in this chapter.

11.1 Process Monitoring and Process Regulation

On the one hand, engineering process control (EPC) involves using *feedback* from a process to *correct* the process, to force the process back towards a target value. The previously discussed stochastic process control (SPC), on the other hand, emphasizes *monitoring* a process and assessing whether or not the process has changed. From a statistical point of view, EPC is analogous to parameter estimation, whereas SPC is analogous to hypothesis testing.

11.2 Process Control by Feedback Adjustment

SAS program: att10-11-2-icecream-integralcontrol

We look at an example of an *integral feedback controller*, which is an example of engineering process control. We also mention other possible feedback controllers.

Exercise 10.1 (Process Control by Feedback Adjustment)
Thirty (30) *individual* ice cream treats are weighed (in ounces).

5.4	5.6	5.8	5.7	6.2	6.2	6.5
7.8	7.3	7.8	7.9	7.8	7.9	9.8
9.1	9.4	9.0	9.2	8.1	7.7	7.6
7.0	7.0	7.0	7.2	7.0	6.9	6.0
6.9	6.4					

Calculate and plot various *integral control charts*, where the target value is $T = 7$ ounces and the associated exponentially weighted moving average (*ewma*), z_t, process is assumed to be

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1} = 0.2x_i + (1 - 0.2) z_{i-1}$$

1. *Bounded adjustment,* $L = 1$, $T = 7$, $\lambda = 0.2$ *and* $g = 0.1$
   From SAS,

obs	y_t	y_t^{adj}	z_t^{adj}	adj_t	adj_t^{cum}
1	5.4	5.4	6.68	0.0	0.0
2	5.6	5.6	6.47	0.0	0.0
3	5.8	5.8	6.33	0.0	0.0
4	5.7	5.7	6.20	0.0	0.0
5	6.2	6.2	6.20	0.0	0.0
6	6.2	6.2	6.20	0.0	0.0
7	6.5	6.5	6.26	0.0	0.0
8	7.8	7.8	5.57	0.0	0.0
9	7.3	7.3	6.72	0.0	0.0
10	7.8	7.8	6.93	0.0	0.0
11	7.9	7.9	7.13	0.0	0.0
12	7.8	7.8	7.26	0.0	0.0
13	7.9	7.9	7.39	0.0	0.0
14	9.8	9.8	7.87	0.0	0.0
15	9.1	9.1	8.12	-4.2	-4.2
16	9.4	5.2	6.64	0.0	-4.2
17	9.0	4.8	6.27	0.0	-4.2
18	9.2	5.0	6.02	0.0	-4.2
19	8.1	3.9	5.59	6.2	2.0
20	7.7	9.7	7.54	0.0	2.0
21	7.6	9.6	7.95	0.0	2.0
22	7.0	9.0	8.16	-4.0	-2.0
23	7.0	5.0	6.60	0.0	-2.0
24	7.0	5.0	6.28	0.0	-2.0
25	7.2	5.2	6.06	0.0	-2.0
26	7.0	5.0	5.85	4.0	2.0
27	6.9	8.9	7.38	0.0	2.0
28	6.0	8.0	7.50	0.0	2.0
29	6.9	8.9	7.78	0.0	2.0
30	6.4	8.4	7.91	0.0	2.0

   (a) *Ice cream weights in control?*
   From the SAS graph, the ice cream weight process, y_t, is clearly (choose one) **in** / **out of** control.

(b) *Target value, T*
True / **False**
The target value, $T = 7$ ounces, is the weight we would like the ice cream treats to achieve. We will adjust the weight process, y_t, periodically so that this target value is achieved as best as is possible.

(c) *Two processes: observed y_t and z_t*
Two processes are recorded,

- the observed ice cream weight process, y_t,
- and the associated ewma z_t process.

Of these two processes, the one *most* sensitive to going out of control is (choose one) $\boldsymbol{y_t}$ / $\boldsymbol{z_t}$. Because of this, the z_t and not the y_t process is used to decide when the process goes out of control.

(d) *The z_t process*
True / **False**
From the SAS graph, it is clear the z_t process is *adjusted* to stay within the $T \pm L = 7 \pm 1 = (6, 8)$ control limits.

(e) *The y_t process*
True / **False**
From the SAS graph, it is clear the y_t process is *adjusted* to stay within the $T \pm L = 7 \pm 1 = (6, 8)$ control limits.

(f) *Bounded adjustment: control limits, L, z_t and y_t*
If the z_t process deviates more than $L = 1$ ounce either above or below the target value, $T = 7$, then this triggers an adjustment. Although z_t *triggers* an adjustment, an adjustment is first made on the y_t process and then on the z_t process. For example, from the table above, the first chain of adjustments occurs at observation (choose one) **14** / **15** / **16**

(g) *Calculation of z_t^{adj}*
The z_t^{adj} process at observation $t = 15$ is calculated,

$$\begin{aligned} z_{15}^{adj} &= (\lambda) y_{15}^{adj} + (1 - \lambda) z_{14}^{adj} \\ &= (0.2)(9.1) + (1 - 0.2)(7.87) \approx \end{aligned}$$

(choose one) **8.01** / **8.11** / **8.31**
Since $z_{15}^{adj} = 8.11 > L = 8$, this triggers an adjustment for the *next* step. Notice, that, up to this point, neither the z_t nor y_t processes have actually been adjusted; that is, up to this point, $z_t^{adj} = z_t$ and $y_t^{adj} = y_t$.
Also notice that z_t^{adj} depends on y_t^{adj}; that is, *first* y_t^{adj} is calculated, *then* z_t^{adj} is calculated.

(h) *Calculation of y_t^{adj}*
At observation $t = 15$, an adjustment is calculated by

$$adj_{15} = -\left(\frac{\lambda}{g}\right)(y^{adj_{15}-T}) = -\left(\frac{0.2}{0.1}\right)(9.7-7) \approx$$

(choose one) **−4.0 / −4.1 / −4.2**
The *accumulated* adjustment at the *next* observation $t = 16$, is

$$adj_{16}^{cum} = adj_{15} + adj_{16}^{cum} = -4.2 + 0 = -4.2$$

Consequently, at this *next* observation 16, y_t is adjusted by,

$$y_{16}^{adj} = y_{16} - adj_{16}^{cum} = 9.4 - 4.2 = 5.2$$

Notice that we want to be within *zero* of the target $T = 7$ and so $T_{zero} = 0$.

2. *Bounded adjustment, $L = 0.5$, $T = 7$, $\lambda = 0.2$ and $g = 0.1$*
From SAS,

obs	y_t	y_t^{adj}	z_t^{adj}	adj_t	adj_t^{cum}
1	5.4	5.4	6.68	0.0	0.0
2	5.6	5.6	6.47	2.8	2.8
3	5.8	8.6	7.32	0.0	2.8
⋮	⋮	⋮		⋮	⋮
29	6.9	4.7	6.54	0.0	-2.2
30	6.4	4.2	6.07	5.6	3.4

(a) *Consequence of lowering $L = 1$ to $L = 0.5$*
From the SAS graphs, it appears that a (choose one) **lesser** / **greater** number of out–of–bound signals are caused by lowering $L = 1$ to $L = 0.5$.

(b) *Why is the $L = 0.5$ process more volatile than the $L = 1$ process?*
True / **False** The $L = 0.5$ process is more volatile than the $L = 1$ process because the adjustments made to the y_t^{adj} process are so big (and the control limits so narrow) that, at every next step, y_{t+1}^{adj} is pushed through the in–control region and back outside of the control limits, triggering another immediate adjustment.

3. *Bounded adjustment, $L = 0.5$, $T = 7$, $\lambda = 0.2$ and $g = 0.5$*
From SAS,

obs	y_t	y_t^{adj}	z_t^{adj}	adj_t	adj_t^{cum}
1	5.4	5.4	6.68	0.0	0.0
2	5.6	5.6	6.47	0.6	0.6
3	5.8	6.4	6.87	0.0	0.6
⋮	⋮	⋮		⋮	⋮
29	6.9	6.3	6.53	0.0	-0.6
30	6.4	5.8	6.39	0.5	-0.1

(a) *Consequence of increasing process gain $g = 0.1$ to $g = 0.5$*
From the SAS graphs, it appears that a (choose one) **lesser** / **greater** number of out–of–bound signals are caused by increasing $g = 0.1$ to $g = 0.5$. This is because the adjustment is larger for $g = 0.1$ than it is for $g = 0.5$.

(b) *What is the process gain, g?*
True / **False** The process gain, g, is *initially* defined by

$$y_{t+1} - T = gx_t$$

and so is like a regression coefficient and could be thought of as the amount the *output* y_{t+1} process changes for a unit change in the *input* x_t process. Ultimately, though, g appears in the adjustment,

$$adj_t = -\left(\frac{\lambda}{g}\right)(z_t^{adj} - T_{zero})$$

and so a smaller g causes a larger adjustment.

(c) *Influence of ewma process weight, λ?*
If the ewma process weight, λ, is made larger then adj_t is made (choose one) **smaller** / **larger**.

4. *Integral controller, $T = 7$, $\lambda = 0.2$ and $g = 0.5$*
From SAS,

obs	y_t	y_t^{adj}	z_t^{adj}	adj_t	adj_t^{cum}
1	5.4	5.4	6.68	0.6	0.6
2	5.6	6.2	6.85	0.3	0.9
3	5.8	6.7	6.95	0.1	1.0
⋮	⋮	⋮		⋮	⋮
29	6.9	7.3	7.06	-0.1	0.3
30	6.4	6.7	6.93	0.1	0.4

Whereas an adjustment for the previous bounded charts occurred only if the ewma z_t process exceeded predetermined control limits, L, *integral controller* charts make an adjustment *for every single step in the process*. Consequently, the bounded charts are

(a) more volatile

(b) less volatile

(c) neither more nor less volatile

than the integral controller charts. This is because although the *number* of adjustments is larger, the *size* of the adjustments, regulated by λ and g, are not.

5. *Integral controller,* $T = 7$*,* $\lambda = 0.2$ *and* $g = 0.8$
From SAS,

obs	y_t	y_t^{adj}	z_t^{adj}	adj_t	adj_t^{cum}
1	5.4	5.4	6.68	0.4	0.4
2	5.6	6.0	6.80	0.3	0.7
3	5.8	6.5	6.89	0.1	0.9
⋮	⋮	⋮		⋮	⋮
29	6.9	7.0	7.00	0.0	0.1
30	6.4	6.5	6.90	0.1	0.2

Although difficult to tell, from the SAS graphs and tables, the adjustments to the z_t and y_t processes for $g = 0.8$ are (choose one) **smaller** / **larger** than they were for when $g = 0.5$.

11.3 Combining SPC and EPC

The main point made in this section is that it is possible to combine stochastic process control (SPC) techniques such as Shewhart charts with the engineering process control (EPC) techniques described in this chapter.