# Chapter 1

# Linear Functions

Linear functions and their slopes are discussed in this chapter; *non*linear functions are discussed in the next chapter. One part of calculus, the *derivative*, is introduced in the chapter after this: a derivative describes the slopes of tangents to points along functions.

## 1.1 Slopes and Equations of Lines

After discussing points and the *Cartesian coordinate system*, *linear functions* (lines) are discussed. In particular, the *slope* of a line is,

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \quad x_1 \neq x_2.$$

Linear functions can take different forms:

- slope-intercept: $y = mx + b$, slope $m$, $y$-intercept $b$
- point-slope form: $y - y_1 = m(x - x_1)$, slope $m$, line passes through $(x_1, y_1)$
- general form: $ax + by = c$, $a, b, c$ integers with no common factor, $x \geq 0$
- vertical line: $x = k$, $x$-intercept $k$, undefined slope
- horizontal line: $y = k$, $y$-intercept $k$, zero slope

Lines are *parallel* if and only if slopes equal or all are vertical. Two lines are *perpendicular* if and only if product of slopes are $-1$, $m_1 \cdot m_2 = -1$ (or $m_2 = -\frac{1}{m_1}$) or one is vertical and the other is horizontal. Many real-world situations can be modeled by linear functions.

**Exercise 1.1 (Slopes and Equations of Lines)**

1. *Points, graphs and tables: reading ability versus level of illumination* Consider the following graph of the set of points which describe reading ability versus level of illumination. Use your calculator to plot these ordered points.

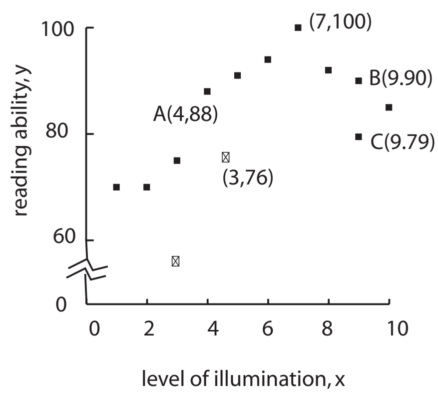


Figure 1.1 (Points on the Cartesian Coordinate System: Reading Ability Versus Level of Illumination)

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| illumination, $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 9 | 10 |
| ability to read, $y$ | 70 | 70 | 76 | 88 | 91 | 94 | 100 | 92 | 79 | 90 | 85 |

(Type $x$ into $L_1$and $y$ into $L_2$ by STAT EDIT. Set up an appropriate viewing window by WINDOW 0 11 1 60 110 10. Choose an appropriate graph choice by 2nd STAT PLOT.)

(a) At a level of illumination of 3, the reading ability is
(i) **70** (ii) **76** (iii) **80**.

(b) Point (9,90) means

i. at a level of illumination of 90, the reading ability is 9.

ii. at a level of illumination of 9, the reading ability is 90.

(c) Point $B$ is (i) $\mathbf{(3,75)}$ (ii) $\mathbf{(4,88)}$ (iii) $\mathbf{(9,90)}$

(d) The *x-coordinate* in the point (7,100) is (i) **7** (ii) **100** (iii) $\mathbf{(7,100)}$.
And so the $y$-coordinate is 100.

(e) Points $B$ and $C$ have the same (i) $x$-**coordinate** (ii) $y$-**coordinate**.

(f) Points $(1, 70)$ and $(2, 70)$ have the same
(i) $x$-**coordinate** (ii) $y$-**coordinate**.

(g) The *origin* is the point (i) $\mathbf{(1,1)}$ (ii) $\mathbf{(0,1)}$ (iii) $\mathbf{(0,0)}$.

(h) This example shows *quadrant I* of Cartesian coordinate system which has
(i) **2** (ii) **3** (iii) **4** quadrants.

2. *Slope of a line.* Consider the following lines on the reading ability versus level of illumination set of points.

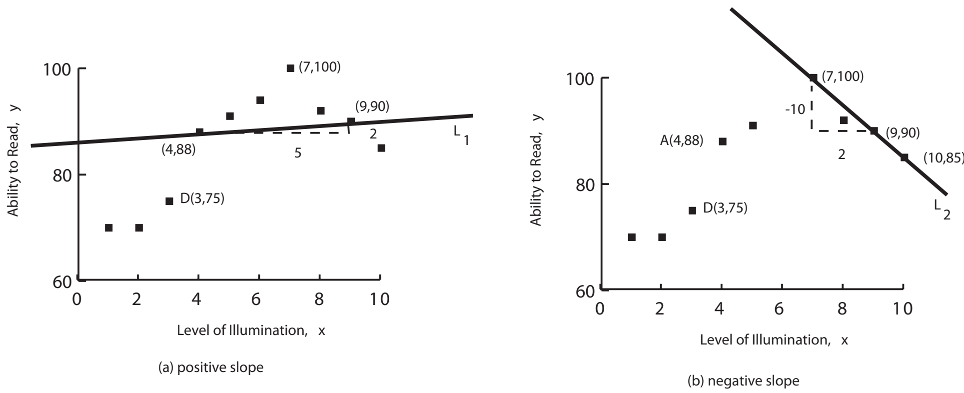


Figure 1.2 (Slope of a Line: Reading Ability Versus Level of Illumination)

(a) Slope of line $L_1$ through points (4,88) and (9,90) in figure (a) above:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 88}{9 - 4} =$$

(i) **0.2** (ii) **0.3** (iii) **0.4**

(b) Slope of line $L_2$ through points (7, 100) and (9,90) in figure (b) above:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 100}{9 - 7} =$$

(i) $\mathbf{-4}$ (ii) $\mathbf{-5}$ (iii) $\mathbf{-6}$

(c) Slope of line $L_2$ through points (7, 100) and (10,85) in figure (b) above:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{85 - 100}{10 - 7} =$$

(i) $\mathbf{-4}$ (ii) $\mathbf{-5}$ (iii) $\mathbf{-6}$
This shows the slope of a *particular* line ($L_2$, say) is
**dependent** / **independent**
of two distinct points on the line used to compute it.

(d) When $m > 0$, the line **rises** / **falls** and when $m < 0$, the line **rises** / **falls**.

(e) The steeper the slope, the **larger** / **smaller** the absolute value of $m$; for example, line $L_2$ with slope $|-5| = 5$ is steeper than line $L_1$ with slope 0.4.

(f) The slope of the line $L_1$, $m = 0.4$, says

i. ability to read *in*creases by 0.4 units for a unit increase in illumination
ii. ability to read *de*creases by 0.4 units for a unit increase in illumination

(g) The slope of the line $L_2$, $m = -5$, says

i. ability to read *in*creases by 5 units for a unit increase in illumination

ii. ability to read *de*creases by 5 units for a unit increase in illumination

(h) **True / False** The slope measures the *rate of change* of $y$ with respect to $x$ or, another way of saying it, the slope measures the amount by how much $y$ changes for a unit change in $x$.

(i) Consider two lines given below.

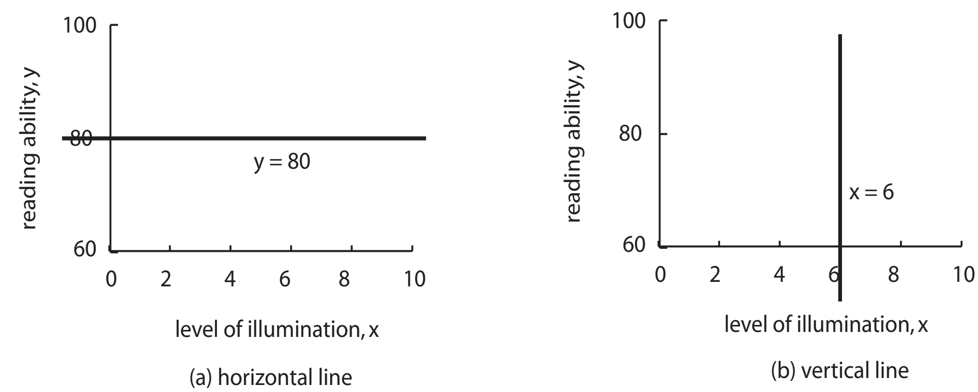


Figure 1.3 (Horizontal and Vertical Lines: Reading Ability Versus Level of Illumination)

*Slope* of horizontal line $y = 80$ is (i) **zero** (ii) **undefined**
*Slope* of vertical line $x = 6$ is (i) **zero** (ii) **undefined**

(j) Slope of line *parallel* to line $y = mx + b = 3x + 2$ is
$m =$ (i) $-\frac{1}{3}$ (ii) $-3$ (iii) $3$
whereas slope of line *perpendicular* to $y = mx + b = 3x + 2$ is
$-\frac{1}{m} =$ (i) $-\frac{1}{3}$ (ii) $-3$ (iii) $3$

(k) Slope of line through $(x, x+3)$ and $(x+h, (x+h)+3)$ is

$$\frac{[(x+h)+3]-(x+3)}{(x+h)-x} = \frac{x+h+3-x-3}{x+h-x} = \frac{h}{h} =$$

(i) **1** (ii) **2** (iii) **3**

(l) Slope of line through $(x, 3x)$ and $(x+h, 3(x+h))$ is

$$\frac{3(x+h)-(3x)}{(x+h)-x} = \frac{3x+3h-3x}{x+h-x} = \frac{3h}{h} =$$

(i) **1** (ii) **2** (iii) **3**

3. *Linear equation.*

(a) *Slope-intercept* form of linear equation is

$$y = mx + b$$

where, if slope $m = 0.4$ and $y$-intercept $b = 86.4$, then

i. $\boldsymbol{y = 0.2x + 86.4}$
ii. $\boldsymbol{y = 0.1x + 86.4}$
iii. $\boldsymbol{y = 0.4x + 86.4}$

(b) If slope $m = -0.2$ and y–intercept $b = 55$, then
i. $\boldsymbol{y = 0.4x + 55}$
ii. $\boldsymbol{y = 0.1x + 55}$
iii. $\boldsymbol{y = -0.2x + 55}$

(c) *Point-slope* form of linear equation is

$$(y - y_1) = m\,(x - x_1)$$

where, if $m = 0.4$ and $(x_1, y_1) = (4, 88)$, then
i. $\boldsymbol{(y - 88) = 0.4\,(x - 4)}$
ii. $\boldsymbol{(y - 0.4) = 88\,(x - 4)}$
iii. $\boldsymbol{(y - 88) = 0.4\,(x + 4)}$

(d) **True** / **False** The two equations, $(y-88) = 0.4(x-4)$ and $y = 0.4x+86.4$ are identical to one another, only written in slightly different ways because

$$\begin{aligned} y - 88 &= 0.4(x - 4) \\ y - 88 &= 0.4x - 0.16 \\ y &= 0.4x - 0.16 + 88 \\ y &= 0.4x + 86.4 \end{aligned}$$

(e) If $m = -0.2$ and $(x_1, y_1) = (4, 70)$, then
i. $\boldsymbol{(y - 88) = 0.4\,(x - 4)}$
ii. $\boldsymbol{(y - 0.4) = 88\,(x - 4)}$
iii. $\boldsymbol{(y - 70) = -0.2\,(x - 4)}$

which is equivalent to
i. $\boldsymbol{y = 0.2x + 70.8}$
ii. $\boldsymbol{y = -0.2x + 70.8}$
iii. $\boldsymbol{y = -0.2x - 70.8}$

(f) If $m = -1$ and $(x_1, y_1) = (1, 3)$, then
i. $\boldsymbol{(y + 1) = -\,(x - 3)}$
ii. $\boldsymbol{(y - 3) = -\,(x - 3)}$
iii. $\boldsymbol{(y - 1) = (x + 3)}$

which is equivalent to
i. $\boldsymbol{y = -x}$

ii. $\boldsymbol{y = x + 1}$
iii. $\boldsymbol{y = x}$

(g) If line is *horizontal* and $(x_1, y_1) = (-4, 70)$, then

i. $\boldsymbol{(y - (-4)) = 0\,(x - 70)}$
ii. $\boldsymbol{(y - 4) = 0\,(x - 70)}$
iii. $\boldsymbol{(y - (-4)) = 70\,(x - 70)}$

which is

i. $\boldsymbol{x = -4}$
ii. $\boldsymbol{y = -4}$
iii. **undefined**

(h) If line is *vertical* and $(x_1, y_1) = (-4, 70)$, then

i. $\boldsymbol{y = 70}$
ii. $\boldsymbol{x = 70}$
iii. **undefined**

(i) Line that passes through the two points (4,88) and (9,90) has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 88}{9 - 4} =$$

(i) **0.2** (ii) **0.3** (iii) **0.4**
and so

i. $\boldsymbol{(y - 88) = 0.4\,(x - 4)}$
ii. $\boldsymbol{(y - 88) = 0.2\,(x - 4)}$
iii. $\boldsymbol{(y - 88) = 0.3\,(x - 4)}$

which is equivalent to

i. $\boldsymbol{y = 0.2x + 86.4}$
ii. $\boldsymbol{y = 0.4x + 86.4}$
iii. $\boldsymbol{y = 0.1x + 86.4}$

(j) Line that passes through the two points (-3,4) and (7,10) has slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{7 - (-3)} =$$

(i) **0.6** (ii) **0.7** (iii) **0.8**
and so

i. $\boldsymbol{(y - 0.4) = 0.7\,(x - 4)}$
ii. $\boldsymbol{(y - 4) = 0.6\,(x + 3)}$

iii. $\boldsymbol{(y - 70) = 0.8\,(x - 4)}$

which is

i. $\boldsymbol{y = 0.6x + 5.8}$
ii. $\boldsymbol{y = -0.2x + 55}$
iii. $\boldsymbol{y = 0.1x + 55}$

4. *More linear equations.*

(a) *General form* of $y = 0.4x + 86.4$ is

i. $ax + by = c$, where $a = 0.4$, $b = 1$ and $c = 86.4$
ii. $ax + by = c$, where $a = -0.4$, $b = -1$ and $c = 86.4$
iii. $ax + by = c$, where $a = -0.4$, $b = 1$ and $c = 86.4$

(b) *General form* of $y = \frac{3}{4}x - 2$ is

i. $ax + by = c$, where $a = \frac{3}{4}$, $b = 1$ and $c = 2$
ii. $ax + by = c$, where $a = -\frac{3}{4}$, $b = -1$ and $c = 2$
iii. $ax + by = c$, where $a = -\frac{3}{4}$, $b = 1$ and $c = -2$

(c) Equation with $x$-intercept -6 and $y$-intercept -2 is equivalent to line that passes through the two points

i. $\boldsymbol{(0, -6), (-2, 0)}$
ii. $\boldsymbol{(-6, 0), (-2, 0)}$
iii. $\boldsymbol{(-6, 0), (0, -2)}$

where these two points have slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{0 - (-6)} =$$

(i) $\boldsymbol{-\frac{1}{3}}$ (ii) $\boldsymbol{\frac{1}{3}}$ (iii) $\boldsymbol{\frac{2}{3}}$
and so

i. $\boldsymbol{(y - (-6)) = \frac{1}{3}\,(x - 0)}$
ii. $\boldsymbol{(y - 0) = -\frac{1}{3}\,(x - (-6))}$
iii. $\boldsymbol{(y - 0) = \frac{2}{3}\,(x - 0)}$

which is

i. $\boldsymbol{\frac{1}{3}x + y = -2}$
ii. $\boldsymbol{\frac{1}{3}x - y = -2}$
iii. $\boldsymbol{\frac{1}{3}x + y = 2}$

(d) Equation $3x - y = 10$ or $y = 3x - 10$ has slope (i) **3** (ii) **1** (iii) $\boldsymbol{-3}$
and is *parallel* to

i. $3x - y = 12$
ii. $x - y = 10$
iii. $3x - 3y = 10$

(e) Equation $3x - 3y = 10$ or $y = x - \frac{10}{3}$ has slope (i) $\mathbf{-3}$ (ii) $\mathbf{0}$ (iii) $\mathbf{1}$ and is *parallel* to

i. $5x - 5y = 10$
ii. $x + y = 10$
iii. $3x - y = 10$

(f) Equation $3x - y = 10$ or $y = 3x - 10$ has slope (i) $\mathbf{-3}$ (ii) $\mathbf{3}$ (iii) $\mathbf{6}$ and so a slope of a line *perpendicular* to this line is (i) $\mathbf{\frac{1}{3}}$ (ii) $\mathbf{1}$ (iii) $\mathbf{-\frac{1}{3}}$ and so the *perpendicular* line which passes through the point (1,5) is

i. $\boldsymbol{(y - (-6)) = \frac{1}{3}(x - 0)}$
ii. $\boldsymbol{(y - 0) = \frac{2}{3}(x - 0)}$
iii. $\boldsymbol{(y - 5) = -\frac{1}{3}(x - 1)}$

which is

i. $\frac{1}{3}x - y = 5$
ii. $\frac{1}{3}x + y = \frac{16}{3}$
iii. $3x - 3y = 10$

## 1.2 Linear Functions and Applications

We consider linear *functions* in this section, such as $f(x) = 2x + 4$, where $y = f(x)$ is the *dependent* variable and $x$ is the *independent* variable. All linear *equations* are linear *functions* except equations of the form $x = k$ where $k$ is a constant[1]. We look at how to solve two linear functions, to find their intersection, and, furthermore, give applications of solving linear functions. In particular, we look at economic supply, demand and equilibrium examples and also business cost and break-even analyses, as well as a finite mathematics feasible regions example. We also notice there are only three possible ways two lines can intersect: at one point, no point (inconsistent solution) or infinite points (dependent, identity solution).

**Exercise 1.2 (Linear Functions and Applications)**

1. *Supply, demand and equilibrium: vacuum cleaners.*

[1]Since $x = k$ is vertical, the slope is undefined and so this equation cannot also be a function.

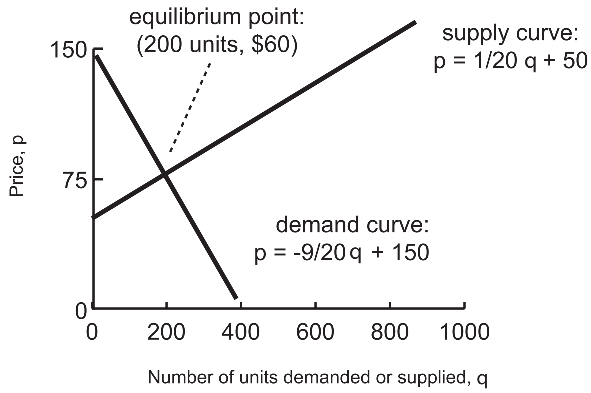


Figure 1.4 (Supply, demand and equilibrium for vacuum cleaners)

Determine equilibrium point, where supply and demand equal one another.

(a) *Using algebra to find equilibrium point.*

i. *Supply* function for vacuum cleaners is
(i) $\boldsymbol{p = S(q) = \left(\frac{1}{20}\right) q + 50}$
(ii) $\boldsymbol{p = D(q) = -\left(\frac{9}{20}\right) q + 150}$
sellers increase *supply*, produce more quantity $q$, if price $p$ increases

ii. *Demand* function for vacuum cleaners is
(i) $\boldsymbol{p = S(q) = \left(\frac{1}{20}\right) q + 50}$
(ii) $\boldsymbol{p = D(q) = -\left(\frac{9}{20}\right) q + 150}$
buyers *de*crease *demand*, buy less quantity $q$, if price $p$ increases

iii. Equilibrium occurs at intersection of supply and demand

$$\left(\frac{1}{20}\right) q + 50 = -\left(\frac{9}{20}\right) q + 150,$$

so $\left(\frac{10}{20}\right) q = 100$ and $q = \frac{100}{0.5} =$ (i) **100** (ii) **200** (iii) **300** units
where $p = \left(\frac{1}{20}\right)(200) + 50 =$ (i) **\$50** (ii) **\$60**
so equilibrium is (i) **(200, \$50)** (ii) **(200, \$60)**

(b) *Using TI-84+ to geometrically find equilibrium point.*
Quantity, price where supply equals demand is
$(q, p) =$ (i) **(\$60, 200)** (ii) **(200, \$60)** (iii) **(260, \$60)**

First clear previous plots.

Enter $\left(\frac{1}{20}\right) x + 50$ beside $Y_1 =$ and $-\left(\frac{9}{20}\right) x + 150$ beside $Y_2 =$.

Enter domain: Press WINDOW, set 0, 1000, 1, 0, 150, 1, 1.

Graph: Press Graph.

Determine intersection: 2nd CALC, intersect, ENTER to First curve? and ENTER to Second curve?, arrow close to intersection, ENTER, and intersection is X = 200, Y = 60.

2. *Another supply, demand and equilibrium example.* Determine equilibrium point.

$$p = S(q) = 1.4q - 0.4$$
$$p = D(q) = -2.1q + 3.1$$

Equilibrium occurs at intersection of supply and demand

$$1.4q - 0.4 = -2.1q + 3.1,$$

so $3.5q = 3.5$ and $q =$ (i) **1** (ii) **2** (iii) **3** units
where $p = 1.4(1) - 0.4 =$ (i) **1** (ii) **2** (iii) **3**
so equilibrium is $(q, p) =$ (i) $\mathbf{(1, 1)}$ (ii) $\mathbf{(-1, 1)}$ (iii) $\mathbf{(-1, -1)}$

3. *Cost analysis: machine usage costs.* Consider *linear cost function*:

$$C(x) = mx + b$$

where $m$ is *marginal cost* (or *cost per item*) and $b$ is *fixed cost*.

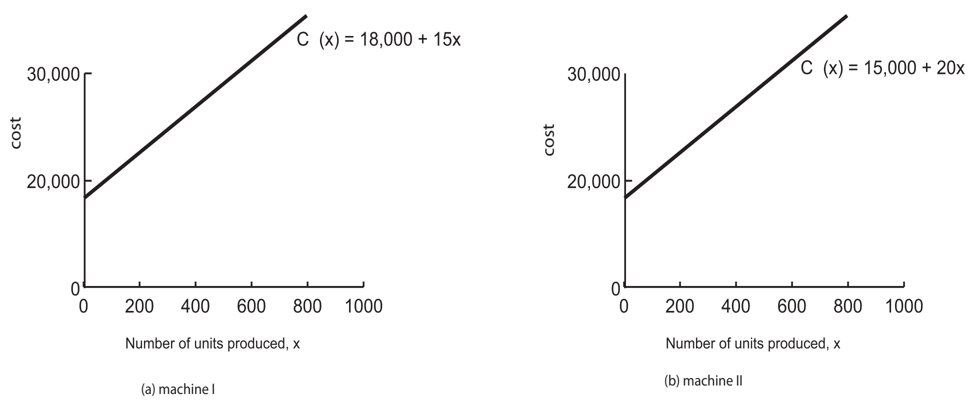


Figure 1.5 (Two cost functions)

(a) *Machine costs I.* Monthly fixed cost of using machine I is \$18,000. Marginal cost of manufacturing one widget using machine I is \$15.

i. Linear cost function in terms of $x$ widgets is
   A. $C(x) = 15x + 15000$
   B. $C(x) = 18000x + 15$
   C. $C(x) = 15x + 18000$

ii. Total cost of 300 widgets is
   $C(300) = 15(300) + 18000 =$ (i) $\mathbf{22,000}$ (ii) $\mathbf{22,500}$ (iii) $\mathbf{23,000}$

iii. Additional cost of making 301st widget: (i) **\$15** (ii) **\$16** (iii) **\$17**

(b) *More machine costs II.* Monthly fixed costs of using machine II are \$15,000. Marginal costs of manufacturing one widget using machine II is \$20.

i. Linear cost function in terms of $x$ widgets is
   A. $C(x) = 20x + 15000$
   B. $C(x) = 18000x + 15$
   C. $C(x) = 15x + 18000$

ii. Total cost of 300 widgets is
   $C(300) = 20(300) + 15000 =$ (i) **21,000** (ii) **22,500** (iii) **23,000**

iii. Additional cost of making 301st widget: (i) **\$15** (ii) **\$17** (iii) **\$20**

4. *Break-even cost analysis for machine I.*

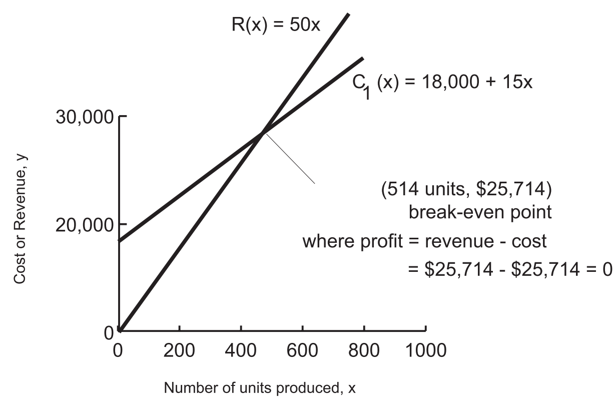


Figure 1.6 (Cost and revenue function for machine)

Determine break-even point if monthly fixed cost is \$18,000 and cost per item is \$15 and also each unit of product sells for \$50.

(a) *Using algebra to find equilibrium point.*

i. Cost and revenue functions for machine are
   (i) $\boldsymbol{C(x) = 18000 + 15x, R(x) = 20x}$
   (ii) $\boldsymbol{C(x) = 15000 + 20x, R(x) = 15x}$
   (iii) $\boldsymbol{C(x) = 18000 + 15x, R(x) = 50x}$

ii. Break-even occurs at intersection of cost and revenue

$$C(x) = R(x)$$

or,

$$18000 + 15x = 50x,$$

so $35x = 18000$ and $x = \frac{18000}{35} \approx$ (i) **500** (ii) **514.3** (iii) **525.4**
where $C(514.3) = 18000 + 15(514.3) \approx$ (i) **\$24,714** (ii) **\$25,714**
so break-even is (i) **(514, \$25,714)** (ii) **(515, \$25,714)**

(b) *Using TI-84+ to geometrically find break-even point.*
Quantity of units and corresponding cost/revenue where revenue equals costs is (quantity, cost/revenue) = $(x, y) \approx$
(i) **(514, \$25, 714)** (ii) **(515, \$25, 714)** (iii) **(516, \$25, 714)**

First clear previous plots.

Enter $18000 + 15x$ beside $Y_1 =$ and $50X$ beside $Y_2 =$.

Enter domain: Press WINDOW, set 0, 1000, 1, 0, 50000, 1, 1.

Graph: Press ZOOM, ZoomFit.

Determine intersection: 2nd CALC, intersect, ENTER to First curve? and ENTER to Second curve?, arrow close to intersection, ENTER, and intersection is X = 514.28.., Y = 25714.28...

5. *Break-even cost analysis for machine II.* Determine break-even point if monthly fixed cost is \$15,000 and cost per item is \$20 and each unit of product sells for \$50.

(a) Cost and revenue functions for machine are
(i) $\boldsymbol{C(x) = 18000 + 15x, R(x) = 20x}$
(ii) $\boldsymbol{C(x) = 15000 + 20x, R(x) = 50x}$
(iii) $\boldsymbol{C(x) = 18000 + 15x, R(x) = 50x}$

(b) Break-even occurs at

$$15000 + 20x = 50x,$$

so $30x = 15000$ and $x = \frac{15000}{30} \approx$ (i) **500** (ii) **514.3** (iii) **525.4** units
where $C(500) = 15000 + 20(500) \approx$ (i) **\$25, 000** (ii) **\$26, 000**
so break-even is (i) **(500, \$25, 000)** (ii) **(515, \$25, 714)**

(c) Profit from 600 units
$R(x) - C(x) = 50x - (1500 + 20x) = 30x - 15000 = 30(600) - 15000 =$
(i) **\$1000** (ii) **\$2000** (iii) **\$3000**

(d) Number of units which give a profit of \$12000:
since $R(x) - C(x) = 30x - 15000 = 12000$, $30x = 27000$ so
$x =$ (i) **800** (ii) **900** (iii) **1000**

6. *Equations and corner points of shaded regions.*

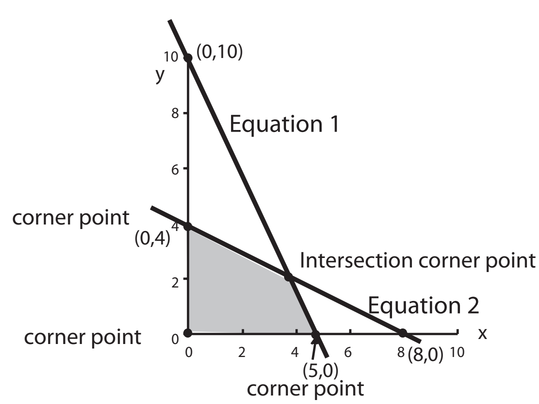

Figure 1.7 (Equations and corner points of shaded region)

(a) Equation 1 passes through y-intercept $(x, y) = (0, 10)$ and x-intercept $(x, y) = (5, 0)$ and so has slope $m = \frac{10-0}{0-5} = -2$ and so $y - y_1 = m(x - x_1)$ or $y - 10 = -2(x - 0)$ or

   i. $\boldsymbol{2x + y = 10}$

   ii. $\boldsymbol{-2x + y = 10}$

   iii. $\boldsymbol{2x + y = -10}$

(b) Equation 2 passes through y-intercept $(x, y) = (0, 4)$ and x-intercept $(x, y) = (8, 0)$ and so has slope $m = \frac{4-0}{0-8} = -0.5$ and so $y - y_1 = m(x - x_1)$ or $y - 4 = -0.5(x - 0)$ or

   i. $\boldsymbol{2x + y = 8}$

   ii. $\boldsymbol{x + 2y = 8}$

   iii. $\boldsymbol{2x + y = -8}$

(c) Corner point intersection of two equations,

$$\begin{aligned} 2x + y &= 10 \\ x + 2y &= 8 \end{aligned}$$

is, since $y = 10 - 2x$ and $2y = 8 - x$ or $y = 4 - 0.5x$, so

$$10 - 2x = 4 - 0.5x,$$

so $1.5x = 6$ and $x = \frac{6}{1.5} =$ (i) **2** (ii) **3** (iii) **4**
where $y = 10 - 2x = 10 - 2(4) =$ (i) **0** (ii) **2**
so $(x, y) =$ (i) $\mathbf{(0, 1)}$ (ii) $\mathbf{(2, 2)}$ (iii) $\mathbf{(4, 2)}$

7. *Intersection of lines: one, none or infinity of points.*

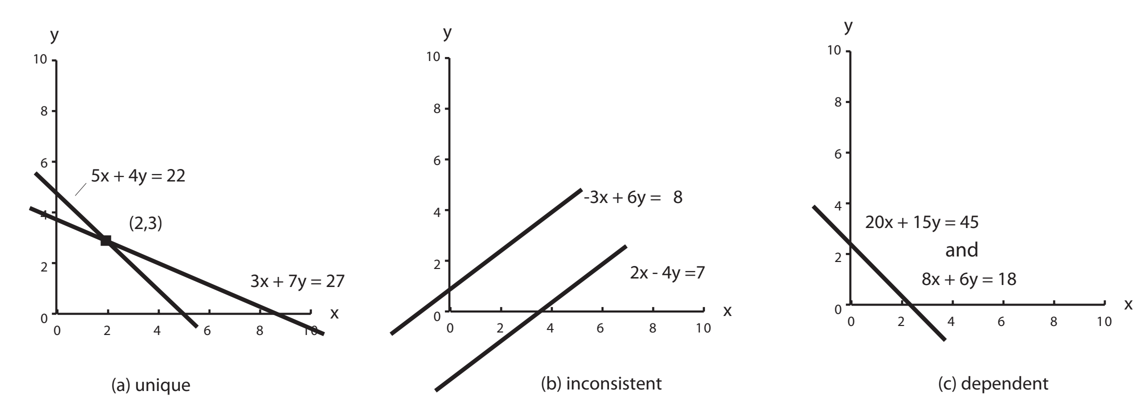


Figure 1.8 (Intersection of lines: one, none or infinity of points)

Two lines intersect

- at one point (they would *not* be parallel to one another)
- at no point (they are parallel and distinct)
- along a *line* (an infinity) of points (they are parallel and coincident)

(a) *Intersect at one point.*

$$\begin{aligned} 5x + 4y &= 22 \\ 3x + 7y &= 27 \end{aligned}$$

is, since $4y = 22 - 5x$, or $y = \frac{22}{4} - \frac{5}{4}x$ and $7y = 27 - 3x$ or $y = \frac{27}{7} - \frac{3}{7}x$, so

$$\frac{22}{4} - \frac{5}{4}x = \frac{27}{7} - \frac{3}{7}x,$$

so $\frac{23}{28}x = \frac{23}{14}$ and $x = \frac{28}{14} = \mathbf{2}$ / $\mathbf{3}$ / $\mathbf{4}$

Use calculator: $\frac{22}{4} - \frac{27}{7} = 1.64..$ then MATH ENTER ENTER for $\frac{23}{14}$; similar for $\frac{23}{28}$

where $y = \frac{22}{4} - \frac{5}{4}x = \frac{22}{4} - \frac{5}{4}(2) = \frac{27}{7} - \frac{3}{7}x = \frac{27}{7} - \frac{3}{7}(2) =$ (i) **1** (ii) **3**

so $(x, y) =$ (i) $\mathbf{(0, 1)}$ (ii) $\mathbf{(2, 2)}$ (iii) $\mathbf{(2, 3)}$

(b) *Intersect at no point.*

$$\begin{aligned} -3x + 6y &= 8 \\ 2x - 4y &= 7 \end{aligned}$$

is, since $6y = 8 + 3x$, or $y = \frac{8}{6} + \frac{3}{6}x$ and $4y = -7 + 2x$ or $y = -\frac{7}{4} + \frac{2}{4}x$, so

$$\frac{8}{6} + \frac{3}{6}x = -\frac{7}{4} + \frac{2}{4}x,$$

so $\frac{37}{12} = 0x$ and $x =$ (i) **0** (ii) **3** (iii) **4** / **huh?**

Use calculator: $\frac{8}{6} + \frac{7}{4} = 3.083...$ then MATH ENTER ENTER for $\frac{37}{12}$

so $(x, y) =$ (i) $\mathbf{\left(0, \frac{8}{6}\right)}$ (ii) $\mathbf{\left(0, -\frac{7}{4}\right)}$ (iii) **inconsistent (no solution)**

(c) *Intersect at infinity of points.*

$$\begin{aligned} 20x + 15y &= 45 \\ 8x + 6y &= 18 \end{aligned}$$

is, since $15y = 45 - 20x$, or $y = 3 - \frac{20}{15}x$ and $6y = 18 - 8x$ or $y = 3 - \frac{8}{6}x$, so

$$3 - \frac{20}{15}x = 3 - \frac{8}{6}x,$$

or $0x = 0$

so $(x, y) =$ (i) $\mathbf{(0, 1)}$ (ii) $\mathbf{(2, 2)}$ (iii) **identity (infinity of points)**

## 1.3 The Least Squares Line

Not covered