Chapter 18

ANOVA Diagnostics and Remedial Measures

In any statistical analysis, there is always three steps.

1. *Diagnostics.*
   Choose which of a number of different statistical models best fits the data; in particular, decide which factors[1] to keep in or eliminate from the model. Check to see if the statistical model (single–factor ANOVA, in this case) is appropriate. That is, check to see if the assumptions required in applying the statistical model are satisfied by the data.

2. *Remedial Measures.*
   If the data does not satisfy the assumptions of the chosen statistical model, then investigate whether or not it is possible to alter the statistical model in some way (transformation of the variables or adding and eliminating variables) so as to make the model fit the data better. Sometimes the chosen statistical model is such a poor choice to analyze the given data set, it is necessary to try a second entirely different statistical model.

3. *Inference.*
   Analyze the data with the (appropriately modified, if necessary) statistical model, to perform tests and to calculate confidence intervals.

18.1   Residual Analysis

SAS program: att2-18-1-cotton-residual

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[1]Later, we look at ANOVA models which have more factors than just one, as is the case in the present single–factor ANOVA model.

One key quantity used in diagnosis, of checking to see if the assumptions required in using the single–factor ANOVA statistical model are satisfied by the data, is the *residual*. The residual, e_{ij}, is the difference between an observed data point, Y_{ij}, and the associated *expected* (predicted, fitted) point determined by the model, $\hat{Y}_{ij}$,

$$e_{ij} = Y_{ij} - \hat{Y}_{ij} = Y_{ij} - \bar{Y}_{i\cdot}.$$

There are different types of residuals, including

- *Studentized Residuals.*

$$r_{ij} = \frac{e_{ij}}{s\{e_{ij}\}}, \quad s\{e_{ij}\} = \sqrt{\frac{MSE(n_i - 1)}{n_i}}$$

  These residuals indicate, in standard deviation units, how far any particular residual is from the mean. Most residuals should fall within three standard deviations of the mean; often, a residual further than two (2) standard deviations from the mean is considered unusual.

- *Studentized Deleted Residuals.*

$$t_{ij} = e_{ij} \left[\frac{n_T - r - 1}{SSE\left(1 - \frac{1}{n_i}\right) - e_{ij}^2} \right]^{1/2}$$

  These standardized residuals account for when one observation is missing. If a studentized *deleted* residual is (roughly) equal it the associated studentized residual, then this indicates the one missing observation is not influential in the analysis.

- *Semistudentized Residuals.*

$$e_{ij}^* = \frac{e_{ij}}{\sqrt{MSE}}$$

  These residuals act "almost" like standardized residuals where, notice, $\sqrt{\frac{MSE(n_i-1)}{n_i}} \approx \sqrt{MSE}$ for large enough n. Semistudentized residuals are sometimes used instead of studentized residuals if the assumptions of the model require them; mostly, though, studentized residuals are used in residual analyses.

Diagnosis of departures of an ANOVA model from the data include,

- *Nonconsistancy of error (residual) variance*
  One of the assumptions required in a single–factor ANOVA statistical model is that the error variance is constant with respect to the different factor level means. This is checked using (standardized or not) residuals versus fitted plots

or dot plots. Residual plot (a) in the figure below indicates constant error variance; other possible nonconstant error variance patterns, with suggested model corrections, are given in (b), (c), (d) and (e).
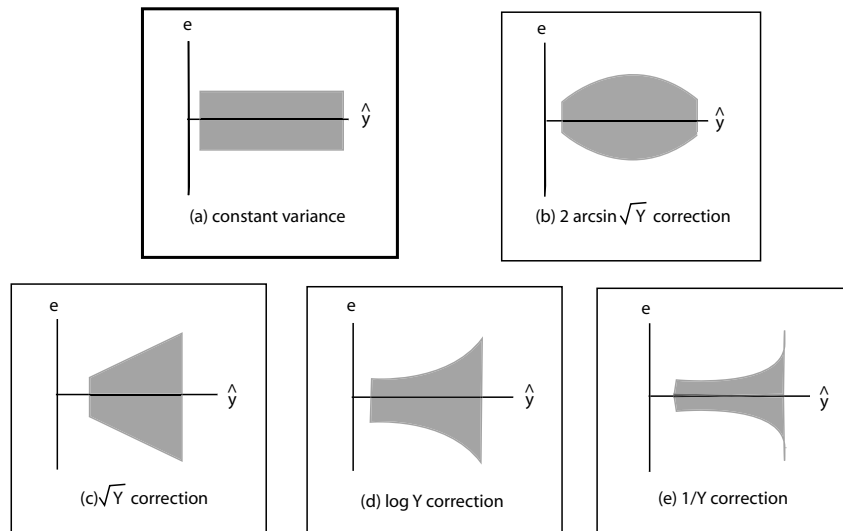

Figure 18.1 (Check for constant error variance.)

- *Nonindependence of error terms*
  This type of error, because it often appears as a pattern in a time sequence, is checked using (standardized or not) residuals versus time (sequence) plots[2]. In general, residual plot (a) in the figure below indicates independent errors; other possible nonindependent error patterns are given in (b) and (c).

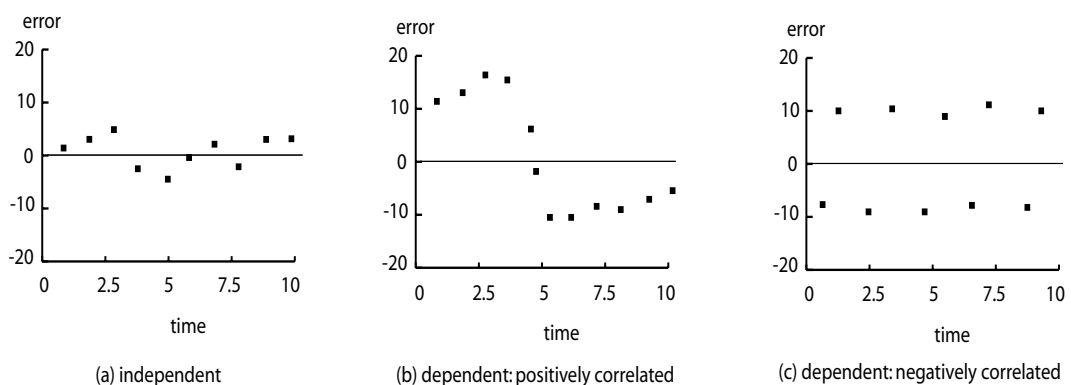Figure 18.2 (Check for nonindependence of error.)

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[2]Since type of error appears in statistical control theory or in time series analyses, not in the current design of experiments course, we will not spend much time investigating this type of error.

- *Outliers*
  Although not an assumption required in a single–factor ANOVA statistical model, outlying observations need to be investigated. They may arise as a consequence of data input error or they may reveal insight into the data. They are checked using (standardized or not) residuals versus fitted plots, and residual dot plots, box plots and stem–and–leaf plots. In general, standardized residual plot (a) in the figure below indicates no outliers, whereas residual plot (b) indicates there are outliers.

Figure 18.3 (Check for outliers.)

- *Omission of important explanatory variables*
  It is possible the single–factor ANOVA statistical model is missing some predictor variables. Omission of variables may appear as patterns in the residual plots, such as those that appear in the nonindependence patterns given above: time variables are missing in this case. Also, for example, residual plot (a) in the figure below indicates the variable gender does *not* need to be added to the model, whereas residual plot (b) indicates that the variable gender does need to be included in the model.
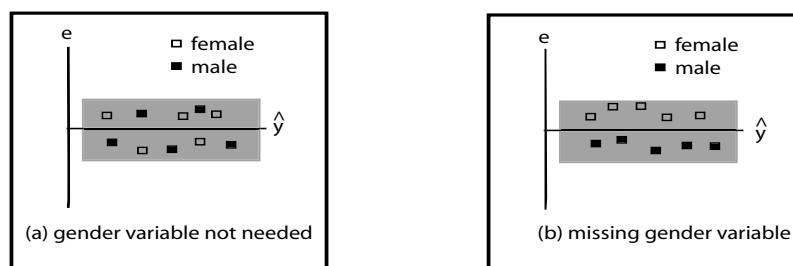

Figure 18.4 (Omission of important explanatory variables?)

- *Nonnormality of error terms*
  Another of the assumptions required in a single–factor ANOVA statistical model is that the errors are normally distributed. This assumption is checked using (standardized or not) residuals versus fitted plots, and residual dot plots, box

plots and normal probability plots. In general, residual plot (a) in the figure below indicates normally distributed errors; other possible nonnormal error patterns are given in (b), (c), (d) and (e).
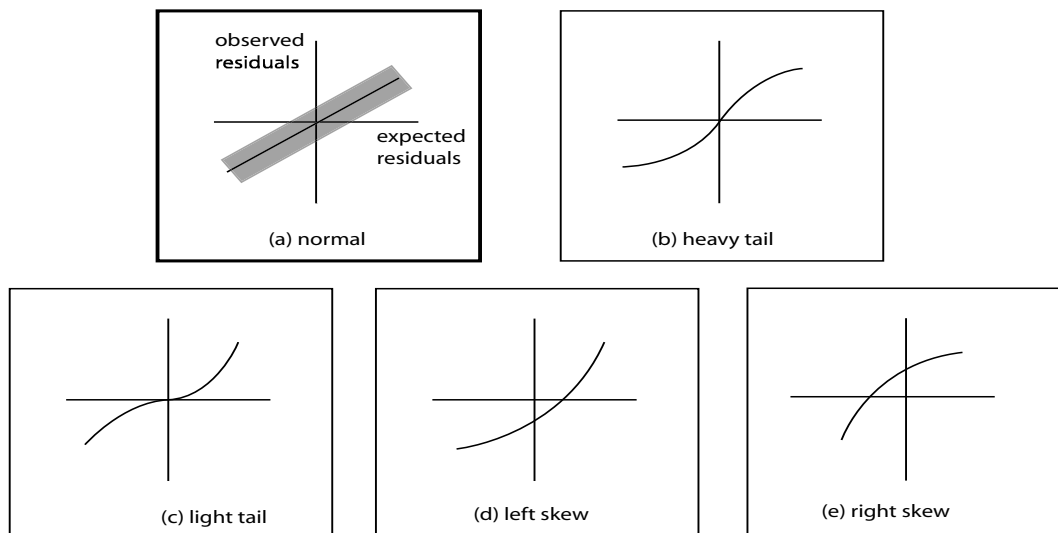

Figure 18.5 (Check for nonnormality of error.)

Exercise 18.1 (Single–Factor Analysis of Variance)

1. *Residual analysis diagnosis: cotton strength.*
   The tensile strength of fibers with varying amounts of cotton content is investigated.

5% cotton	7	7	15	11	9	10	$\bar{Y}_{1.} \approx 9.8$
10% cotton	12	17	12	18	18	16	$\bar{Y}_{2.} \approx 15.5$
15% cotton	14	18	18	19	19	17	$\bar{Y}_{3.} \approx 17.5$
20% cotton	19	25	22	19	23	24	$\bar{Y}_{4.} \approx 22$
25% cotton	7	10	11	15	11	14	$\bar{Y}_{5.} \approx 11.3$

   Use various residual analyses to check to see if the assumptions required in using the single–factor ANOVA statistical model are satisfied by this data.

   (a) *Constant error (residual) variance?*
       The residual plot for the drug data indicates that error variance (choose one) **is** / **is not** constant with respect to either the (qualitative) cotton types (1, 2, 3, 4 and 5) or the (quantitative) predicted ($\hat{Y} = \bar{Y}$) variable. The predicted (mean) values are (choose none, one or more)

       i. 9.833

      ii. 15.5

     iii. 17.5

     iv. 22

      v. 11.333

(b) *Nonindependence of error?*
There does appear to a curved pattern in the residual plots, so the assumption of independence appears to be (choose one) **satisfied / not satisfied**.

(c) *Outliers?*
There (choose one) **appears to be / does not appear to be** outliers because not only are none of the residuals in the studentized plot are further than two (2) standard deviations from the mean, but also the studentized *deleted* residual plot is very similar to the (ordinary) studentized residual plot.

(d) *Omission of important explanatory variables?*
There does appear to a curved pattern in the residual plots, so it may be possible that one or more explanatory variables (choose one) **are / are not** missing from the model.

(e) *Nonnormality of error terms?*
Since the normal probability plot is more or less a line, this indicates the residuals are (choose one) **normal / nonnormal**.

2. *Residual analysis diagnosis: pest–free apples.*
The number of pest–free apples (out of 100 per tree) produced by apple plants in five differently applied pesticide plots of land is tabulated below.

pesticide 1	12	13	15	15	15	16	16	17	18	22
pesticide 2	28	30	31	32	33	33	37	37	40	41
pesticide 3	42	48	48	54	54	55	56	60	63	64
pesticide 4	70	71	72	73	75	75	77	79	79	80
pesticide 5	92	94	95	96	96	97	97	98	98	99

(a) *Constant error (residual) variance?*
The residual plot for the drug data indicates that error variance (choose one) **is / is not** constant with respect to either the (qualitative) pesticide types (1, 2, 3, 4 and 5) or the (quantitative) predicted ($\hat{Y} = \bar{Y}$) variable. The predicted (mean) number of pest–free apple values are (choose none, one or more)

      i. 16

     ii. 34.2

     iii. 54.4

      iv. 75.1

      v. 96.2

(b) *Nonindependence of error?*
There does appear to a bowed–in–the–middle pattern in the residual plots, so the assumption of independence appears to be (choose one) **satisfied / not satisfied**.

(c) *Outliers?*
There (choose one) **appears to be / does not appear to be** outliers because not only are none of the residuals in the studentized plot are further than two (2) standard deviations from the mean, but also the studentized *deleted* residual plot is very similar to the (ordinary) studentized residual plot.

(d) *Omission of important explanatory variables?*
There does appear to a bowed–in–the–middle pattern in the residual plots, so it may be possible that one or more explanatory variables (choose one) **are / are not** missing from the model.

(e) *Nonnormality of error terms?*
Since the normal probability plot is more or less a line, this indicates the residuals are (choose one) **normal / nonnormal**.

18.2   Tests for Constancy of Error Variance

SAS program: att2-18-2-cotton-hartley,levene

In addition to the graphical residual analysis used to check to see if the assumptions required in using the single–factor ANOVA statistical model are satisfied by the data, analytical (numerical) procedures can also be used in this regard. Two tests are discussed in this section, the Hartley test and the modified Levene test[3], both of which are used to test for the constancy of error variance.

Exercise 18.2 (Tests for Constancy of Error Variance)

1. *Tests for Constancy: cotton strength.*
The tensile strength of fibers with varying amounts of cotton content is investigated.

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[3]The Neter text calculates the F test statistic relative to the *median*, whereas SAS calculates the test statistic relative to the *mean*. Consequently, the text and SAS will have (slightly) different results for the modified Levene test. We will use the SAS version of the modified Levene test in this course.

5% cotton	7	7	15	11	9	10	$\bar{Y}_{1.} \approx 9.8$
10% cotton	12	17	12	18	18	16	$\bar{Y}_{2.} \approx 15.5$
15% cotton	14	18	18	19	19	17	$\bar{Y}_{3.} \approx 17.5$
20% cotton	19	25	22	19	23	24	$\bar{Y}_{4.} \approx 22$
25% cotton	7	10	11	15	11	14	$\bar{Y}_{5.} \approx 11.3$

Use both the modified Levene test and the Hartley test at $\alpha = 0.05$ to check to see if the assumption of constant variance required when using the single–factor ANOVA statistical model is satisfied by this data.

(a) *Modified Levene test*

    i. *Statement.*
    Choose none, one or more.

        A. H_0 : error variance constant over cotton types *versus*
           H_1 : not constant for different cotton types

        B. $H_0 : \sigma_1^2 = \cdots = \sigma_5^2$ *versus*
           H_1 : not all σ_i^2 are the same

        C. H_0 : error variance constant over cotton types *versus*
           H_1 : increase for different cotton types

    ii. *Test.*
    From SAS, the p–value is (choose one) **< 0.0001 / 0.532 / 0.7915**
    The level of significance is $\alpha = 0.05$

    iii. *Conclusion.*
    Since the p–value is (choose one) **smaller / larger** than the level of significance we (choose one) **accept / reject** the null hypothesis that the error variance is constant.

(b) *Hartley test*

    i. *Statement.*
    Choose none, one or more

        A. H_0 : error variance constant over cotton types *versus*
           H_1 : not constant for different cotton types

        B. $H_0 : \sigma_1^2 = \cdots = \sigma_5^2$ *versus*
           H_1 : not all σ_i^2 are the same

        C. H_0 : error variance constant over cotton types *versus*
           H_1 : increase for different cotton types

    ii. *Test.*
    From SAS, the test statistic is

$$H = \frac{max(s_i^2)}{min(s_i^2)} = \frac{2.994}{1.871}$$

which equals **1.3 / 1.5 / 1.6**

Also, from table B.6, page 1355, the number of levels is $r = 5$ and $df = 6 - 1 = 5$, the critical value is (choose one) **13.7 / 16.3 / 18.7**

iii. *Conclusion.*

Since the test statistic is (choose one) **smaller / larger** than the critical value we (choose one) **accept / reject** the null hypothesis that the error variance is constant.

2. *Tests for Constancy: pest–free apples.*

The number of pest–free apples (out of 100 per tree) produced by apple plants in five differently applied pesticide plots of land is tabulated below.

pesticide 1	12	13	15	15	15	16	16	17	18	22
pesticide 2	28	30	31	32	33	33	37	37	40	41
pesticide 3	42	48	48	54	54	55	56	60	63	64
pesticide 4	70	71	72	73	75	75	77	79	79	80
pesticide 5	92	94	95	96	96	97	97	98	98	99

Use both the modified Levene test and the Hartley test at $\alpha = 0.05$ to check to see if the assumption of constant variance required when using the single–factor ANOVA statistical model is satisfied by this data.

(a) *Modified Levene test*

i. *Statement.*

Choose none, one or more.

A. H_0 : error variance constant over cotton types *versus*
   H_1 : not constant for different cotton types

B. $H_0 : \sigma_1^2 = \cdots = \sigma_5^2$ *versus*
   H_1 : not all σ_i^2 are the same

C. H_0 : error variance constant over cotton types *versus*
   H_1 : increase for different cotton types

ii. *Test.*

From SAS, the p–value is (choose one) **0.0153 / 0.532 / 0.7915**

The level of significance is $\alpha = 0.05$

iii. *Conclusion.*

Since the p–value is (choose one) **smaller / larger** than the level of significance we (choose one) **accept / reject** the null hypothesis that the error variance is constant.

(b) *Hartley test*

i. *Statement.*

Choose none, one or more

A. H_0 : error variance constant over cotton types *versus*
   H_1 : not constant for different cotton types

B. $H_0 : \sigma_1^2 = \cdots = \sigma_5^2$ *versus*
   H_1 : not all σ_i^2 are the same

C. H_0 : error variance constant over cotton types *versus*
   H_1 : increase for different cotton types

ii. *Test.*
    From SAS, the test statistic is

$$H = \frac{\mathrm{m}ax(s_i^2)}{\mathrm{m}in(s_i^2)} = \frac{6.963}{2.098}$$

which equals **3.3 / 3.5 / 3.6**
Also, from table B.6, page 1355, the number of levels is $r = 5$ and
$df = 10 - 1 = 9$, the critical value is (choose one) **6.72 / 7.11 / 8.74**

iii. *Conclusion.*
     Since the test statistic is (choose one) **smaller / larger** than the
     critical value we (choose one) **accept / reject** the null hypothesis
     that the error variance is constant.

(c) *A final point*
    The Hartley test and modified Levene tests (choose one) **always / some-
    times** agree with one another.

18.3   Overview of Remedial Measures

If the data does not satisfy the assumptions of the chosen statistical model, it is
sometimes possible to take remedial action; in other words, to alter the statistical
model in some way so as to make the data fit the model better. We look at two
remedial methods for correcting for two common departures from the ANOVA model.

1. The weighted least squares method is used to make the nonconstant error vari-
   ances more constant.

2. Transformations (both simple and according to the Box–Cox procedure) of the
   response variable are used to make nonconstant error variance more constant
   and also to make nonnormal data more normal.

18.4   Weighted Least Squares

SAS program: att2-18-4-soil-weightLS

The weighted least squares method for single–factor ANOVA models is used in this section to attempt to make *non*constant error variances constant.

Exercise 18.3 (Weighted Least Squares) Soil–water fluxes were measured for different grades of soil.

grade 1 soil	365.04	372.41	368.71	361.41	357.81
grade 2 soil	249.64	247.15	244.69	239.85	242.26
grade 3 soil	149.90	148.41	146.94	145.47	151.41

1. *Weighted Least Squares.*

   (a) *Statement.*
   Choose none, one or more.

      i. H_0 : error variance constant over cotton types *versus*
         H_1 : not constant for different cotton types
      ii. $H_0 : \mu_1 = \mu_2 = \mu_3$ *versus*
         $H_1 : \mu_i \neq \mu_j, i \neq j$
      iii. H_0 : error variance constant over cotton types *versus*
         H_1 : increase for different cotton types

   (b) *Test.*
   From SAS, the test statistic is

   $$F_w^* = \frac{SSE_w(F) - SSE_w(R)}{r - 1} \div \frac{SSE_w(F)}{n_T - r}$$
   $$= \frac{7149.70963 - 12}{3 - 1} \div \frac{12}{15 - 3}$$

   (circle one) **35.76 / 357.69 / 3568.85**
   And so the p–value, with $r - 1 = 3 - 1 = 2$ and $n_T - r = 15 - 3 = 12$ degrees of freedom, is given by

   $$\text{p–value} = P(F \geq 3568.85)$$

   which equals (circle one) **0.00 / 0.35 / 0.43**.
   The level of significance is 0.05.

   (c) *Conclusion.*
   Since the p–value, 0.00, is smaller than the level of significance, 0.05, we (circle one) **accept / reject** the null hypothesis that mean water–soil fluxes are the same.

2. *Understanding Weighted Least Squares.*

(a) *Error (residual) variance constant?*
From SAS, the residual plot indicates that error variance (choose one) **is** / **is not** constant with respect to the predicted $(\hat{Y} = \bar{Y})$ variable and so we will use weighted least squares to correct for this problem.

(b) *Comparing weighted test statistic with unweighted test statistic.*
From SAS, the weighted test statistic is

$$
\begin{aligned}
F_w^* &= \frac{SSE_w(F) - SSE_w(R)}{r - 1} \div \frac{SSE_w(F)}{n_T - r} \\
&= \frac{7149.70963 - 12}{3 - 1} \div \frac{12}{15 - 3} \\
&= 3568.85
\end{aligned}
$$

whereas, from SAS[4], the *un*weighted test statistic is

$$
\begin{aligned}
F_w^* &= \frac{SSE_w(F) - SSE_w(R)}{r - 1} \div \frac{SSE_w(F)}{n_T - r} \\
&= \frac{118041 - 215.13312}{3 - 1} \div \frac{215.13312}{15 - 3} \\
&= 3286.12
\end{aligned}
$$

the weighted test statistic is (choose one) **more** / **less** significant than the unweighted test statistic.

(c) *Comparing weighted residuals with unweighted residuals.*
The weighted least squares analysis weights the *larger* residuals (choose one) **more than** / **the same as** / **less than** the smaller residuals. In particular, the weights assigned to each residual is

$$
w_{ij} = \frac{1}{s_i^2}
$$

and so the three different weights used for this data are

i	$\bar{Y}$	s^i	$w_{ij} = \frac{1}{s_i^2}$
grade 1 soil	365.076	5.77125	0.03002
grade 2 soil	244.718	3.86913	0.06680
grade 3 soil	148.426	2.34645	0.18163

(d) *ANOVA Model As A Multiple Linear Regression.*
True / False
The single–factor ANOVA model

$$
Y_{ij} = \mu_i + \varepsilon_{ij}
$$

---

[4]This test statistic can be obtained in two different ways: either from the single–factor ANOVA or as the ratio of two appropriately defined least square regressions.

can be rewritten as a multiple linear regression model,

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

where

$$\underset{15\times1}{\mathbf{Y}} = \begin{bmatrix} Y_{11} \\ Y_{12} \\ \vdots \\ Y_{35} \end{bmatrix} = \begin{bmatrix} 365.04 \\ 372.41 \\ \vdots \\ 151.41 \end{bmatrix}, \quad \underset{3\times1}{\beta} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix}, \quad \underset{15\times1}{\varepsilon} = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{12} \\ \vdots \\ \varepsilon_{35} \end{bmatrix}$$

and

$$\underset{15\times3}{\mathbf{X}} = \begin{bmatrix} X_{1,1} & X_{1,2} & X_{1,3} \\ \vdots & \dots & \vdots \\ X_{15,1} & X_{15,2} & X_{15,3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

18.5   Transformations of Response Variable

SAS program: att2-18-5-soil-trans,boxcox

Transformations, using both simple rules and according to the Box–Cox procedure, of the response variable, Y_{ij}, are used to make nonconstant error variance more constant and also to make nonnormal data more normal. The simple rules are

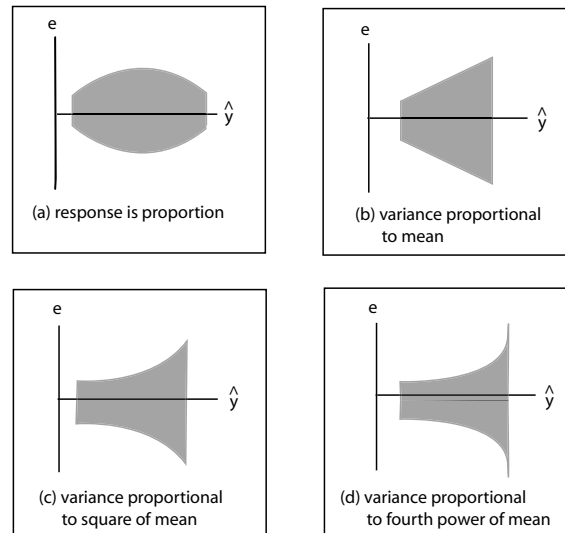| problem with variance	mathematical description	remedy transformation
(a) proportional to the mean	$\sigma^2 \ \alpha \ k\|\mu\|$ or $\sigma \ \alpha \ k\|\mu^{1/2}\|$	$Y' = \sqrt{Y}$ or $Y' = \sqrt{Y} + \sqrt{Y+1}$
(b) proportional to the mean^2	$\sigma^2 \ \alpha \ k\|\mu\|^2$ or $\sigma \ \alpha \ k\|\mu\|$	$Y' = \ln Y$
(c) proportional to the mean^4	$\sigma^2 \ \alpha \ k\|\mu\|^4$ or $\sigma \ \alpha \ k\|\mu\|^2$	$Y' = \frac{1}{Y}$
(d) proportional is a proportion	$\sigma^2\{p_{ij}\} \ \alpha \ \pi_i(1-\pi_i)$	$Y' = 2\arcsin\sqrt{Y}$

Figure 18.6 (Simple Rule Transformations.)

The Box–Cox transformation is given by

$$Y' = Y^\lambda$$

where the "best" λ is the one that maximizes[5] the SSE of all of the models for different λ. The Box–Cox transformation procedure is not discussed at great length.

Exercise 18.4 (Transformations of Response Variable) Soil–water fluxes were measured for different grades of soil.

grade 1 soil	365.04	372.41	368.71	361.41	357.81
grade 2 soil	249.64	247.15	244.69	239.85	242.26
grade 3 soil	149.90	148.41	146.94	145.47	151.41

1. *Residual plot and simple rule transformations.*
   From SAS, the residual plot seems to indicate the problem with the variance is proportional to the (choose one)

   (a) mean

   (b) mean^2

   (c) mean^4

   (d) none of these; the response is a proportion

   and so the transformation best suited to make the variance constant appears to be (choose one)

   ---

   [5]This λ is the maximum likelihood estimate for the statistical model.

   (a) $Y' = \sqrt{Y}$ or $Y' = \sqrt{Y} + \sqrt{Y+1}$

   (b) $Y' = \ln Y$

   (c) $Y' = \frac{1}{Y}$

   (d) $Y' = 2 \arcsin \sqrt{Y}$

2. *Variance–mean ratios and simple rule transformations.*
   Consider the following table of statistics, where s_i^2 and $\bar{Y}_i$ have both been taken from the SAS output,

i	$s_i^2/\bar{Y}_i$	$s_i^2/\bar{Y}_i^2$	$s_i^2/\bar{Y}_i^4$
grade 1 soil	$5.77125^2/365.076 = 0.091234$	0.000249904	1.87×10^{-9}
grade 2 soil	0.061173	0.000249974	4.17×10^{-9}
grade 3 soil	0.037095	0.000249921	1.13×10^{-8}

Clearly, the statistic, $s_i^2/\bar{Y}^2$, is the most constant of the three statistics; in other words, the variance is proportional to the (choose one)

   (a) mean

   (b) mean^2

   (c) mean^4

   (d) none of these; the response is a proportion

Consequently, this indicates the transformation best suited to make the variance constant is (choose one)

   (a) $Y' = \sqrt{Y}$ or $Y' = \sqrt{Y} + \sqrt{Y+1}$

   (b) $Y' = \ln Y$

   (c) $Y' = \frac{1}{Y}$

   (d) $Y' = 2 \arcsin \sqrt{Y}$

3. *Box–cox transformation*
   From the SAS output, the various *SSE* values for given box–cox λ transformations, are given below.

λ	2.7	2.8	2.9	3.0	3.1
SSE	171.59	171.42	171.37	171.44	171.62

In this case, the best (minimum *SSE*) box–cox transformation of the data is given by $Y' = Y^\lambda$ where (choose one)

   (i) $\lambda = 2.7$

(ii) $\lambda = 2.8$

(iii) $\lambda = 2.9$

(iv) $\lambda = 3.0$

(v) $\lambda = 3.1$

4. *Comparing three methods.*
   The residual plot, table and box–cox methods (choose one) differ on what transformation of Y to use to make the variance constant. Match the method used and the transformations suggested below.

method	suggested transformation
(a–1) residual plot	(A) $Y' = \sqrt{Y}$
(a–2) residual plot	(B) $Y' = \sqrt{Y + 1}$
(b) table	(C) $Y' = \ln Y$
(c) box–cox	(D) $Y' = Y^{2.9}$

method	(a–1)	(a–2)	(b)	(c)
transformation	(A)	(B)	(C)	(D)

5. *Using the suggested transformations in a single–factor ANOVA.*
   Use all of the suggested transformations to test if the mean fluxes for the three different soil types are the same or different at $\alpha = 0.05$.

   (a) *Statement.*
       Choose none, one or more.

       i. H_0 : error variance constant over soil grades *versus*
          H_1 : not constant for different soil grades

       ii. $H_0 : \mu_1 = \mu_2 = \mu_3$ *versus*
           $H_1 : \mu_i \neq \mu_j, i \neq j$

       iii. H_0 : error variance constant over soil grades *versus*
            H_1 : increase for different soil grades

   (b) *Test.*
       From SAS, the test statistic for
       i. $Y' = \sqrt{Y}$
          is (choose one) **3794.62 / 3793.33 / 4067.73 / 1495.68**
       ii. $Y' = \sqrt{Y + 1}$
           is (choose one) **3794.62 / 3793.33 / 4067.73 / 1495.68**
       iii. $Y' = \ln Y$
            is (choose one) **3794.62 / 3793.33 / 4067.73 / 1495.68**

  iv. $Y' = Y^{2.9}$
  is (choose one) **3794.62** / **3793.33** / **4067.73** / **1495.68**

where all associated p–values are essentially zero (0).
The level of significance is 0.05.

(c) *Conclusion.*
Since the p–value, 0.00, is smaller than the level of significance, 0.05, we (circle one) **accept** / **reject** the null hypothesis that mean water–soil fluxes are the same.

6. *How Is The Transformed Data Used In Inference Procedures?*
The transformed data can be used to (choose none, one or more)

(a) *test* the equality of factor level means since the single–factor ANOVA test is a *relative* one which does not depend on the absolute size of the factor level means, but only on whether these means are the same or different from one another in a relative sense.

(b) *estimate* the transformed factor level means but these estimates would then have to be transformed back into the original data units. For example, if the data is transformed using $Y' = \ln Y$, the transformation $Y'' = e^{Y'}$ would need to be used to give estimates in the original data units.

18.6 Effects of Departures from Model

What if, after attempting remedial measures, the data still does not conform to the assumptions of the single–factor ANOVA model? In fact, this model is robust to nonnormality and unequal error variances, but is sensitive to nonindependence of the error terms. In other words, as long as the data is not extremely nonnormal and does not have extremely unequal error variances, then it is still possible to perform reasonably accurate estimation and testing using the single–factor ANOVA model. This is not true if the error terms are nonindependent. Since it is difficult to correct for nonindependence after the data has been collected, it is very important to prevent this problem in the first place, to collect the data in a appropriate random manner.

18.7 Nonparametric Rank F Test

This material is not covered.

18.8 Case Example–Heart Transplant

This is an excellent example.