# Chapter 3

# Inferences About Process Quality

We now look at some basic results in simple statistical inference, that will be used throughout the course.

## 3.1 Statistics and Sampling Distributions

A *statistic* is a function of a sample that does not contain unknown parameters. The probability distribution of a statistic is called a *sampling distribution.* Three sampling distributions are investigated: $t$, $\chi^2$ and $F$.

**Exercise 3.1 (Probabilities and Percentiles For the $t$ Distribution: Westville Temperatures)** In Westville, in February, the temperature, $T$, follows a $t$ distribution, where

$$t = \frac{\sqrt{n}(\overline{X} - \mu)}{S}.$$

Assume $n = 4$ temperatures are randomly sampled.

1. *Percentages.* Consider the following figure with four $t$ distributions, each with different shaded areas (probabilities).

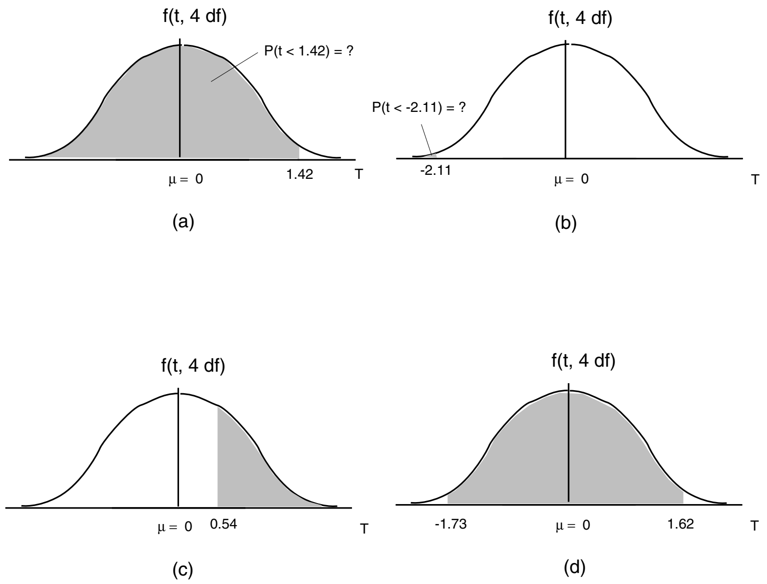



Figure 3.1 (Calculating Probabilities For the $t_4$ Distribution)

(For example, use WINDOW -3 3 1 -0.2 0.5 0.1, then Y = 2nd DISTR 4:$t$pdf( X , 4 ) GRAPH. To shade between -1.75 and 1.62, 2nd DISTR DRAW 2:Shade_t(-1.75,1.62,4) ENTER.)

(a) There are many different $t$ distributions, indexed by the *degrees of freedom* (df), where df $= n - 1$. If a sample of size $n = 5$ is taken, then the particular $t$ distribution under consideration is one with
(circle one) **4** / **19** / **20** degrees of freedom (df) and is denoted $t_4$.

(b) The $t$–distribution with 4 df, in diagram (a) of the figure above, say, is
(circle one) **skewed right** / **symmetric** / **skewed left**.

(c) The total area (probability) under this curve is
(circle one) **50%** / **75%** / **100%** / **150%**.

(d) The shape of this distribution is
(circle one) **triangular** / **bell–shaped** / **rectangular**.

(e) This distribution is centered at (circle one) $\boldsymbol{\mu = 0^o}$ / $\boldsymbol{\mu = 1.42^o}$.

(f) Since this distribution is symmetric,
(circle one) **25%** / **50%** / **75%** of the temperatures are above (to the right) of $0^o$.

(g) The probability of the temperature being less than $1.42^o$ is
(circle one) **greater than** / **about the same as** / **smaller than** 0.50. Use (a) in the figure above.

(h) The probability the temperature is less than $1.42^o$,
$P(t_4 < 1.42) =$ (circle one) **0.786** / **0.834** / **0.886** / **0.905**
(Use your calculator: 2nd DISTR 5:tcdf( − 2nd EE 99 , 1.42 , 4 ) ENTER.)

(i) $P(t_4 < -2.11) =$ (circle one) **0.023** / **0.051** / **0.124** / **0.243**. Use your calculator and (b) in the figure above with 4 df.

(j) $P(t_4 > 0.54) =$ (circle one) **0.309** / **0.356** / **0.435** / **0.470**. Use your calculator and (c) in the figure above with 4 df.

(k) $P(-1.73 < t_4 < 1.62) =$ (circle one) **0.647** / **0.734** / **0.801** / **0.830**. Use your calculator and (d) in the figure above with 4 df

(l) **True** / **False** The probability the temperature is *exactly* $1.42^o$, say, is *zero*.

(m) **True** / **False** $P(Z < 1.42^o) = P(t_4 \leq 1.42^o)$ with 4 df, where, recall, "Z" stands for the "standard normal".

(n) **True** / **False** $P(Z < 1.42^o) \approx P(t(60) \leq 1.42^o)$.

2. *Percentiles.* Consider the following figure with four $t$ distributions with 4 df, each with different *percentiles.*

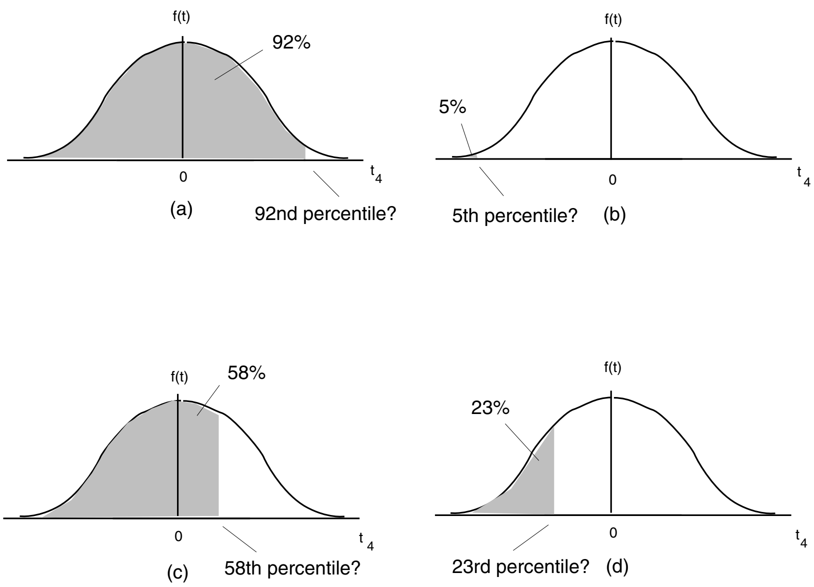



Figure 3.2 (Percentiles of a $t_4$ Distribution)

(a) **True** / **False** The 50th percentile is that temperature such that there is a 50% chance of temperatures being below (and also above) this temperature.

(b) Since the $t$ distribution with 4 df is symmetric, centered at $0^o$, and contains "100%" of the probability, the 50th percentile must be (circle one) **below** $0^o$ / **equal to** $0^o$ / **above** $0^o$.

(c) **True** / **False** The 75th percentile is that temperature such that there is a 75% chance of the temperatures being below this temperature and so a 25% chance of the temperatures being above this temperature.

(d) The 75th percentile must be
(circle one) **below** $0^o$ / **equal to** $0^o$ / **above** $0^o$.

(e) The third quartile must be
(circle one) **below** $0^o$ / **equal to** $0^o$ / **above** $0^o$.

(f) The 92nd percentile must be
(circle one) **below** $0^o$ / **equal to** $0^o$ / **above** $0^o$. Use (a) in the figure above with 4 df

(g) The 92nd percentile (92% left/below, 8% right/above) is
$t(\nu, \alpha) = t(4, 0.08) =$ (circle one) $\mathbf{0.95^o}$ / $\mathbf{1.23^o}$ / $\mathbf{1.72^o}$ / $\mathbf{2.21^o}$.
(Use your calculator: PRGM INVT ENTER 14 ENTER 0.92 ENTER.)

(h) The 5th percentile (5% left/below, 95% right/above) is $t_{\alpha,n-1} = t_{0.95,4} =$ (circle one) $\mathbf{-2.31^o}$ / $\mathbf{-2.13^o}$ / $\mathbf{-1.76^o}$ / $\mathbf{-0.76^o}$. Use your calculator and (b) in the figure above with 4 df

(i) The 58th percentile (58% left/below, 42% right/above) is $t_{\alpha,n-1} = t_{0.42,4} =$ (circle one) $\mathbf{0.22^o}$ / $\mathbf{0.97^o}$ / $\mathbf{1.21^o}$ / $\mathbf{1.35^o}$. Use your calculator and (c) in the figure above with 4 df

(j) The 23rd percentile (23% left/below, 77% right/above) is $t_{\alpha,n-1} = t_{0.77,4} =$ (circle one) $\mathbf{-1.58^o}$ / $\mathbf{-1.23^o}$ / $\mathbf{-0.82^o}$ / $\mathbf{-0.56^o}$. Use your calculator and (d) in the figure above with 4 df

(k) That temperature such that 77% of the temperatures are *above* this temperature is
(circle one) $\mathbf{-2.31^o}$ / $\mathbf{-1.54^o}$ / $\mathbf{-1.21^o}$ / $\mathbf{-0.82^o}$.

3. *Percentiles and Critical Values.*

(a) The 95th percentile for $t$ with 4 df, for example, is denoted $t_{0.05,4}$. The 5th percentile for $t$ with 4 df, on the other hand, is denoted (circle one)

i. $t_{0.95,4}$.
ii. $t_{0.05,4}$.

(b) The 99th percentile for $t$ with 11 df is (check none, one or more)

i. $t_{0.99,11}$.
ii. $t_{0.01,11}$.

(c) The amount of area between $t_{0.10,4}$ and $t_{0.90,4}$ is
(circle one) **0.80** / **0.09** / **0.88**.

(d) For the critical value at $\alpha = 0.10$ with 7 df, (circle one)

i. 10% of the area is *above* the critical value $t_{0.90,7}$
ii. 10% of the area is *below* the critical value $t_{0.90,7}$
iii. 10% of the area is *above* the critical value $t_{0.10,7}$

iv. 10% of the area is *below* the critical value $t_{0.10,7}$

(e) **True** / **False**. The $t$ distribution is a "flatter" version of the standard normal distribution. The larger the sample size, $n$, the less flat the $t$ distribution becomes, the more like the standard normal it becomes.

**Exercise 3.2 (Probabilities and Percentiles For the Chi–Square, $\chi^2$, Distribution: Waiting Times)** The waiting time to pounce (in minutes) for a leopard, $\chi^2$, follows a *chi–square*, $\chi^2$, distribution, where

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}.$$

1. Consider the following figure with two $\chi^2$ distributions, each with different shaded areas (probabilities).

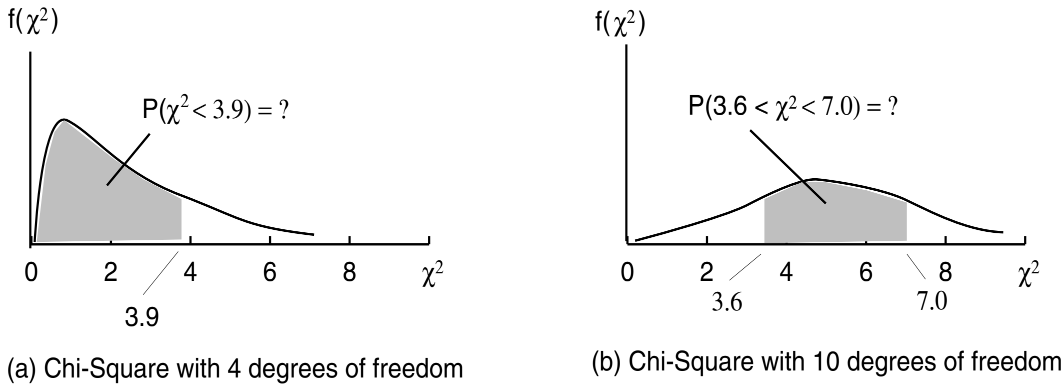



Figure 3.3 (Calculating Probabilities For the $\chi^2$ Distribution)

(For example, use WINDOW 0 15 1 -0.1 0.3 0.1, then Y = 2nd DISTR 6:$\chi^2$pdf( X , 4 ) GRAPH. To shade between 0 and 3.9, 2nd DISTR DRAW 2:Shade$\chi^2$(0,3.9,4) ENTER.)

(a) Like the $t$ distribution, different $\chi^2$ distributions are indexed using degrees of freedom (df), which equal df $= n - 1$. If a sample of size $n = 10$ had been taken, then the particular $\chi^2$ distribution considered would have (circle one) **9** / **10** / **11** degrees of freedom.

(b) The $\chi^2$ with 4 df distribution, $\chi^2_4$, in diagram (a) of the figure above, say, is
(circle one) **skewed right** / **symmetric** / **skewed left**.

(c) The total area (probability) under this curve is
(circle one) **50%** / **75%** / **100%** / **150%**.

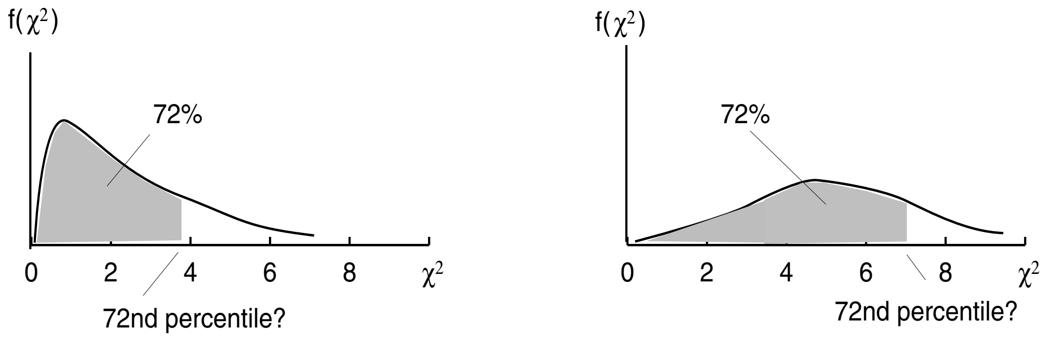
(d) The probability ($\chi^2$ with 4 df) of waiting less than 3.9 minutes is $P(\chi^2_4 < 3.9) =$ (circle one) **0.35 / 0.45 / 0.58 / 0.66**
(Use 2nd DISTR 7:$\chi^2$cdf(-2ndEE99,3.9,4).).

(e) For a $\chi^2_{10}$,
$P(3.6 < \chi^2_{10} < 7.0) =$ (circle one) **0.24 / 0.34 / 0.42 / 0.56**.

(f) **True / False** The probability the waiting time is *exactly* 3 minutes, say, is *zero*.

(g) **True / False** For a $\chi^2_{10}$, $P(Z < 3) = P(\chi^2_{10} \leq 3)$, where, recall, "Z" stands for the "standard normal".

2. Consider the following figure with two $\chi^2$ distributions, each with the 72nd percentile.




(a) Chi-Square with 4 degrees of freedom (b) Chi-Square with 10 degrees of freedom

Figure 3.4 (Percentiles For the $\chi^2$ Distribution)

(a) **True / False** The 50th percentile is that waiting time such that there is a 50% chance of the waiting times being below (and also above) this waiting time.

(b) The 72nd percentile (72% left/below, 28% right/above) for a $\chi^2$ with 4 df, is $\chi^2_{\alpha,n-1} = \chi^2_{0.28,4} =$ (circle one) **3.1 / 5.1 / 8.3 / 9.1**.
(Use PRGM INVCHI2 ENTER 4 ENTER 0.72 ENTER)

(c) The 72nd percentile (72% left/below, 28% right/above) for a $\chi^2$ with 10 df, is $\chi^2_{\alpha,n-1} = \chi^2_{0.28,10} =$ (circle one) **2.5 / 10.5 / 12.1 / 20.4**.

(d) The 32nd percentile (32% left/below, 68% right/above) for a $\chi^2$ with 18 df, is $\chi^2_{\alpha,n-1} = \chi^2_{0.68,18} =$ (circle one) **2.5 / 10.5 / 14.7 / 20.4**.

(e) The lower critical value, $\chi^2_{0.95,n-1}$, is equal to the
(circle one) ***5th* / *95th* / *97.5th*** percentile.

**Exercise 3.3(Probabilities and Percentiles For The F Distribution: Waiting**

**Time)** The waiting time to pounce (in minutes) for a leopard, $F$, follows an, $F$–distribution where

$$F = \frac{S_1^2}{S_2^2}.$$

1. *Probabilities.* Consider the following figure with two $F$ distributions, each with different shaded areas (probabilities).

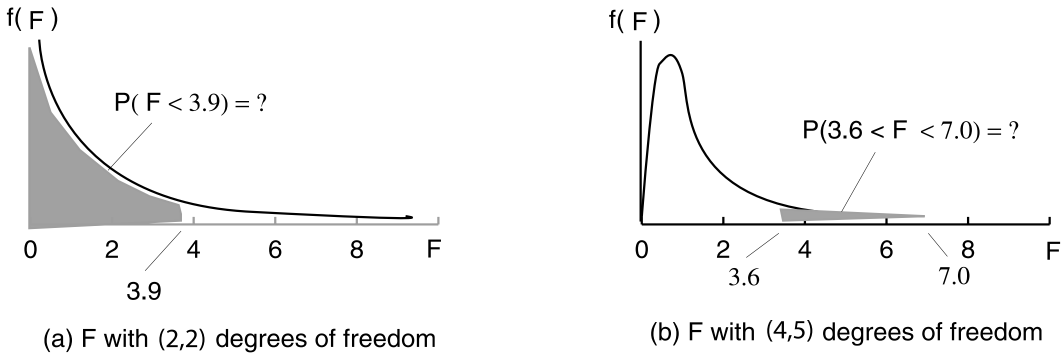



Figure 3.5 (Probabilities For the $F$ Distribution)

(Use WINDOW 0 8 1 -0.3 0.7 0.1, then Y = 2nd DISTR 8:Fpdf( X , 4 , 5 ) GRAPH. To shade between 3.6 and 7, 2nd DISTR DRAW 2:ShadeF(3.6,7,4,5) ENTER.)

(a) The $F$ distribution is indexed by *two* degrees of freedom ($df_1, df_2$), which are equal to $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$. If *two* samples of size $n_1 = 10$ and $n_2 = 11$ had been taken, then the particular $F$ distribution considered would had
(circle one) $\mathbf{(9, 10)}$ / $\mathbf{(10, 11)}$ / $\mathbf{(11, 12)}$ degrees of freedom.

(b) The $F$ distribution with (2,2) degrees of freedom, in diagram (a) of the figure above, say, is
(circle one) **skewed right** / **symmetric** / **skewed left**.

(c) The $F$ distribution with (4,5) degrees of freedom, in diagram (b) of the figure above, say, is
(circle one) **skewed right** / **symmetric** / **skewed left**.

(d) The total area (probability) under this curve is
(circle one) **50%** / **75%** / **100%** / **150%**.

(e) The probability of waiting less than 3.9 minutes for an $F$ with (2,2) degrees of freedom, is
$P(F_{2,2} < 3.9) =$ (circle one) **0.35** / **0.45** / **0.80** / **0.92**.
(Use 2nd DISTR 9:Fcdf(-2ndEE99,3.9,2,2).)

(f) For an $F$ with (4,5) degrees of freedom,
$P(3.6 < F_{4,5} < 7.0) =$ (circle one) **0.03** / **0.07** / **0.09** / **0.11**.

2. *Percentiles.* Consider the following figure with two $F$ distributions, each with the 72nd percentile.

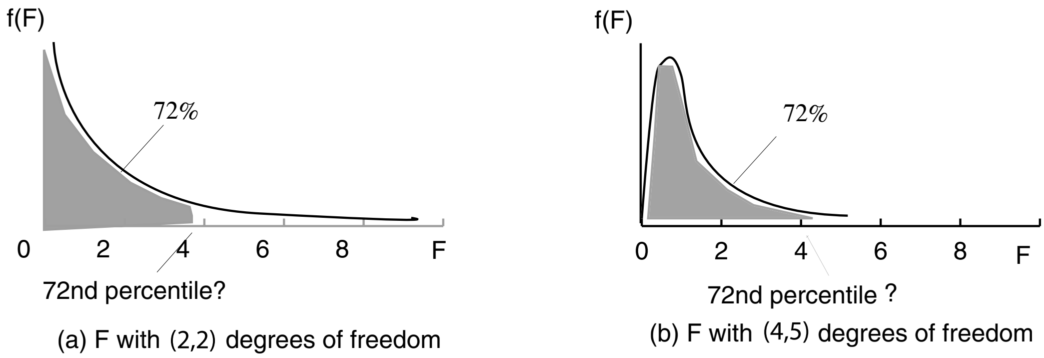



Figure 3.6 (Percentiles For the $F$ Distribution)

(a) The 72nd percentile (72% left/below, 28% right/above) for $F$ with (2,2) degrees of freedom is
upper $F_{\alpha,n_1-1,n_2-1} = F_{0.28,2,2} =$ (circle one) **0.5** / **1.7** / **2.6** / **3.1**.
(Use PRGM 6:INVF ENTER 2 ENTER 2 ENTER 0.72 ENTER)

(b) The 22nd percentile (22% left/below, 78% right/above) for $F$ with (3,8) degrees of freedom, is
lower $F_{\alpha,n_1-1,n_2-1} = F_{0.78,3,8} =$ (circle one) **0.22** / **0.37** / **2.61** / **3.12**.

(c) The 72nd percentile (72% left/below, 28% right/above) for $F$ with (4,5) degrees of freedom, is
upper $F_{\alpha,n_1-1,n_2-1} = F_{0.28,4,5} =$ (circle one) **0.5** / **1.7** / **2.6** / **3.1**.

(d) The critical value $F_{0.05,n_1-1,n_2-1}$ is equal to the
(circle one) ***5th*** / ***95th*** / ***97.5th*** percentile.

(e) **True** / **False**

$$F_{\alpha,n_1-1,n_2-1} = \frac{1}{F_{1-\alpha,n_1-1,n_2-1}}$$

(Use your calculators to try, for example, $n_1 - 1 = 3$, $n_2 - 1 = 4$ and $\alpha = 0.07$.)

## 3.2 Point Estimation of Process Parameters

One important aspect of statistics is to do with the idea of taking a statistic from a sample to be used to guess the value of an unknown parameter of the population distribution from which this sample is taken. We look at criteria used to decide when statistics "closely" estimate their corresponding parameters.

- *Unbiased.* The statistic $\hat{\theta}$ is an *unbiased estimator* of the parameter $\theta$ if

$$E[\hat{\theta}] = \theta$$

  Roughly speaking, a statistic $\hat{\theta}$ is considered "close" to a parameter $\theta$ if the expected value of the statistic (the weighted average of all the possible values of the statistic) is equal to the parameter.

- *Minimum Variance.* The statistic $\hat{\theta}$ is a *minimum variance* estimator of the parameter if, for any other estimator, $\hat{\theta}^*$,

$$\sigma^2(\hat{\theta}) \leq \sigma^2(\hat{\theta}^*), \quad \text{for all } \hat{\theta}^*$$

  Roughly speaking, a statistic $\hat{\theta}$ is considered "close" to a parameter $\theta$ if the spread in all of the possible values of the statistic $\hat{\theta}$ is smaller than the spread in all of the possible values of any other statistic $\hat{\theta}^*$; in other words, the statistic is more precise than any other statistic. Of course, the statistic may be more precisely *wrong* than the other statistics and so this criterion is often coupled with the unbiased criteria above to account for this problem.

**Exercise 3.4 (Statistical Estimation)**

1. *A First Look: Darts.*
   Where our darts land on the board will represent the different observed values of a statistic. The goal in our game of darts is to try and hit the bulls eye. When our darts land on or very near the bulls–eye, this is equivalent to our observed values being equal to or near equal to the actual unknown parameter value. Identify whether or not the standard deviation (variance) is small or large (whether the chance error is small or large) and whether or not the bias is small or large for the four different dart patterns below. Use this to identify the best dart hit pattern.

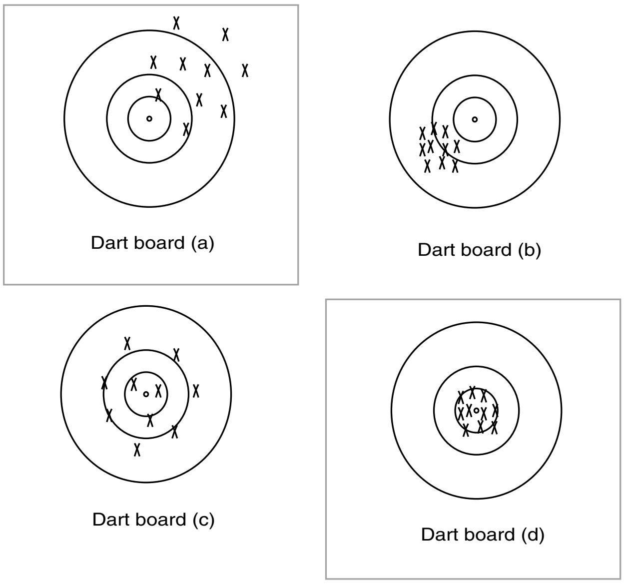



Figure 3.7 (Bias and Variance in Darts and Dart–Board)

(a) **True** / **False** The dart hits on board (a) have a **large SD** because the they are spread out and, more than this, have a **large bias** because the average of the dart hits (the "center" of the cloud of dart hits) is up and to the right of the bulls eye.

(b) Dart hits on board (b):
**small SD** / **large SD** and **small bias** / **large bias**

(c) Dart hits on board (c):
**small SD** / **large SD** and **small bias** / **large bias**

(d) Dart hits on board (d):
**small SD** / **large SD** and **small bias** / **large bias**

(e) **True** / **False** The best dart hit pattern occurs on dart board (d) because it has both small SD and small bias.

(f) (Big Picture: Sample and Statistic, Population and Parameter.) *If* the best dart hit pattern (d) does occur (where 30 hits are recorded, say), then match the following terms with the darts example.

| terms | darts example |
|---|---|
| (i) statistic | (i) bulls eye |
| (ii) parameter | (ii) "center" of dart hit pattern |
| (iii) population | (iii) *all possible* dart hits |
| (iv) sample | (iv) 30 dart hits |

| terms | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|
| darts example | | | | |

(g) The center of the dart hit pattern for the first group of 30 darts thrown is probably (circle one) **different from** / **the same as** the center of the dart hit pattern for the second group of the 30 darts thrown. In general, the value of the *statistic* (*estimator*) (circle one) **changes** / **remains the same** from one random sample to the next.

(h) The location of the bull's eye for the first group of 30 darts thrown is (circle one) **different from** / **the same as** the location of the bull's eye for the second group of the 30 darts thrown. In general, the value of the *parameter* (circle one) **changes** / **remains the same** from one random sample to the next.

(i) **True** / **False** The center of the dart hit pattern (statistic) could be thought of as a "point estimate" of the location of the bull's eye (parameter). In the same way, the center of the dart hit pattern "plus or minus" the SD in the dart hit pattern could be thought of as an "confidence interval estimate" of the location of the bull's eye.

2. *Mean, $\overline{X}$, Is Unbiased: Waiting To Catch A Fish.*
The distribution of the number of minutes waiting to catch a fish, $Y$, is given below.

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | 0.1 | 0.2 | 0.7 |

Demonstrate if $\overline{X}$ is a biased or unbiased estimator of $\mu$.

(a) The mean waiting time is
$\mu = 1(0.1) + 2(0.2) + 3(0.7) =$ (circle one) **0.3** / **1.8** / **2.6**.
(Use your calculator: STAT ENTER; type 1, 2 and 3, into $L_1$ and 0.1, 0.2 and 0.7 into $L_2$; then define $L_3 = L_1 \times L_2$; then STAT CALC ENTER 2nd $L_3$ ENTER; then read $\sum x$.)

(b) Complete the following table of joint distribution probabilities *as well as* the average times spent waiting to catch two fish.

| | $P(x_1, x_2)$<br>$\bar{x}$ | <br>1 | $x_1$<br>2 | <br>3 |
|---|---|---|---|---|
| | 1 | 0.01<br>1 | 0.02<br>$\frac{3}{2}$ | 0.07<br>2 |
| $x_2$ | 2 | 0.02<br>______ | 0.04<br>2 | 0.14<br>$\frac{5}{2}$ |
| | 3 | 0.07<br>______ | 0.14<br>$\frac{5}{2}$ | 0.49<br>3 |

(c) **True** / **False** Another way of presenting the joint distribution information given in the previous question is

| $(x_1, x_2)$ | $Pr(x_1, x_2)$ | $\bar{x}$ |
|---|---|---|
| (1,1) | 0.01 | 1 |
| (1,2) | 0.02 | 3/2 |
| (1,3) | 0.07 | 2 |
| (2,1) | 0.02 | 3/2 |
| (2,2) | 0.04 | 2 |
| (2,3) | 0.14 | 5/2 |
| (3,1) | 0.07 | 2 |
| (3,2) | 0.14 | 5/2 |
| (3,3) | 0.49 | 3 |

(d) Complete the *sampling distribution* of the average waiting time, *statistic* $\overline{X}$,

| $\bar{x}$ | 1 | $\frac{3}{2}$ | 2 | $\frac{5}{2}$ | 3 |
|---|---|---|---|---|---|
| $P(\overline{X} = \bar{x})$ | ______ | 0.04 | 0.18 | ______ | 0.49 |

(e) The mean waiting time is
$\mu_{\overline{X}} = 1(0.01) + \frac{3}{2}(0.04) + 2(0.18) + \frac{5}{2}(0.28) + 3(0.49) =$
(circle one) **0.3** / **1.8** / **2.6**.

(f) Since, for $n = 2$, $\mu_{\overline{X}} = \mu$, this indicates the statistic $\overline{X}$ is a/n (circle one) **biased** / **unbiased** estimator of parameter $\mu$.

3. *Variance, $S^2$, Is Unbiased But Statistic, $S_n^2$, Is Biased: Waiting To Catch A Fish.*
The distribution of the number of minutes waiting to catch a fish, $Y$, is given below.

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | 0.1 | 0.2 | 0.7 |

Demonstrate if either $S^2 = \frac{1}{n-1} \sum (X - \mu)^2$ or $S_n^2 = \frac{1}{n} \sum (X - \mu)^2$ is a biased or unbiased estimator of $\sigma^2$.

(a) Since $\mu = 2.6$ from above, the variance in waiting time is
$\sigma^2 = (1 - 2.6)^2(0.1) + (2 - 2.6)^2(0.2) + (3 - 2.6)^2(0.7) =$
(circle one) **0.33** / **0.44** / **0.69**.
(Use your calculator: STAT ENTER; type 1, 2 and 3, into $L_1$ and 0.1, 0.2 and 0.7 into $L_2$; then define $L_3 = (L_1 - 2.6)^2 \times L_2$; then STAT CALC ENTER 2nd $L_3$ ENTER; then read $\sum x$.)

(b) Complete the following table for the variance.

| $(x_1, x_2)$ | $Pr(x_1, x_2)$ | $s^2 = \frac{1}{n-1}\sum(x-\mu)^2$ | $s_n^2 = \frac{1}{n}\sum(x-\mu)^2$ |
|---|---|---|---|
| (1,1) | 0.01 | 0 | 0 |
| (1,2) | 0.02 | 0.5 | 0.25 |
| (1,3) | 0.07 | 2 | 1 |
| (2,1) | 0.02 | ______ | 0.25 |
| (2,2) | 0.04 | 0 | ______ |
| (2,3) | 0.14 | 0.5 | 0.25 |
| (3,1) | 0.07 | 2 | 1 |
| (3,2) | 0.14 | ______ | 0.25 |
| (3,3) | 0.49 | 0 | ______ |

(To find the variance, $s^2$, of (1,2), say, type 2nd LIST MATH 8:variance({1,2}) ENTER; do not forget the curly brackets! To find $s_n^2$, of (1,2), say, type STAT EDIT and then type 1 and 2 into $L_1$, then STAT CALC, read $\sigma_X$, then square this number.)

(c) Complete the *sampling distribution* of the variance in the waiting time, *statistic* $S^2$,

| $s^2$ | 0 | 0.5 | 2 |
|---|---|---|---|
| $P(S^2 = s^2)$ | ______ | 0.32 | 0.14 |

(d) The mean of $S^2$ is
$\mu_{S^2} = 0(0.54) + 0.5(0.32) + 2(0.14) =$ (circle one) **0.33** / **0.44** / **0.69**.

(e) Since, for $n = 2$, $\mu_{S^2} = \sigma^2$, this indicates the statistic $S^2$ is a/n (circle one) **biased** / **unbiased** estimator of parameter $\sigma$.

(f) Complete the *sampling distribution* of $S_n^2$,

| $s_n^2$ | 0 | 0.25 | 1 |
|---|---|---|---|
| $P(S_n^2 = s_n^2)$ | ______ | 0.32 | 0.14 |

(g) The mean of $S_n^2$ is
$\mu_{S_n^2} = 0(0.54) + 0.25(0.32) + 1(0.14) =$ (circle one) **0.22** / **0.44** / **0.69**.

(h) Since, for $n = 2$, $\mu_{S^2} \neq \sigma^2$, this implies the statistic $S_n^2$ is a/n (circle one) **biased** / **unbiased** estimator of parameter $\sigma$.

4. *Unbiased Sample Standard Deviations.*

(a) $\hat{\sigma} = \frac{S}{c_4}$
Let

$$\hat{\sigma} = \frac{S}{c_4}$$

where

$$E(S) = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma(n/2)}{\Gamma[(n-1)/2]}\sigma = c_4\sigma$$

then $\hat{\sigma}$ is a (choose one) **biased** / **unbiased** estimator of $\sigma$.
Hint: Is $E(\hat{\sigma}) = \sigma$?

(b) $\hat{\sigma} = \frac{R}{d_2}$
Let

$$\hat{\sigma} = \frac{R}{d_4}$$

where $R$ is the range, $R = x_{\max} - x_{\min}$, and where

$$E(R) = d_4\sigma$$

then $\hat{\sigma}$ is a (choose one) **biased** / **unbiased** estimator of $\sigma$.
Hint: Is $E(\hat{\sigma}) = \sigma$?

5. *Minimum Variance.*
The distribution of the number of minutes waiting to catch a fish, $Y$, is given below.

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | 0.1 | 0.2 | 0.7 |

Recall, $\sigma^2 = 0.44$. Compare $\sigma^2(S^2) = \mu_{(S^2-\sigma^2)^2}$ with $\sigma^2(S_n^2) = \mu_{(S_n^2-\sigma^2)^2}$.

(a) Complete the following table.

| $(x_1, x_2)$ | $Pr(x_1, x_2)$ | $(s^2 - \sigma^2)^2$ | $(s_n^2 - \sigma^2)^2$ |
|---|---|---|---|
| (1,1) | 0.01 | $(0 - 0.44)^2 = 0.1936$ | 0.1936 |
| (1,2) | 0.02 | $(0.5 - 0.44)^2 = 0.0036$ | $(0.25 - 0.44)^2 = 0.0361$ |
| (1,3) | 0.07 | 2.4336 | 0.3136 |
| (2,1) | 0.02 | ______ | 0.0361 |
| (2,2) | 0.04 | 0.1936 | ______ |
| (2,3) | 0.14 | 0.0036 | 0.0361 |
| (3,1) | 0.07 | 2.4336 | 0.3136 |
| (3,2) | 0.14 | ______ | 0.0361 |
| (3,3) | 0.49 | 0.1936 | ______ |

(Type STAT EDIT and enter the nine $s^2$ values into $L_1$ and then define $L_2 = (L_1 - 0.44)^2$; do the same for the nine $s_n^2$ values.)

(b) $\sigma^2(S^2) = \mu_{(S^2-\sigma^2)^2} = 0.01(0.1936) + 0.02(0.0036) + \cdots + 0.49(0.1936) =$ (circle one) **0.2232** / **0.4464** / **0.6911**. (Type STAT EDIT and enter the nine $(s^2 - \sigma^2)^2$ values into $L_1$ and the nine $P(x_1, x_2)$ values into $L_2$ and then define $L_3 = L_1 \times L_2$. Then STAT CALC $L_3$ and read off $\sum x$.)

(c) $\sigma^2(S_n^2) = \mu_{(S_n^2-\sigma^2)^2} = 0.01(0.1936) + 0.02(0.0361) + \cdots + 0.49(0.1936) =$ (circle one) (circle one) **0.16** / **0.44** / **0.69**.

(d) Since $\sigma^2(S^2) = 0.4464$ is larger than $\sigma^2(S_n^2) = 0.16$, $S^2$ is (circle one) **not as spread out** / **more spread out** from $\sigma$ than is $S_n^2$. So, unbiased $S^2$ is "more spread out" than biased $S_n^2$ from the parameter $\sigma^2$.

## 3.3 Statistical Inference for a Single Sample

In this section, we will look at a tests and confidence intervals related to a single sample.

**Exercise 3.5 (Statistical Inference for a Single Sample)**

1. *Hypothesis Test Of Population Mean–Normal Population: Hourly Wage.*
The average hourly wage in the US is assumed to be \$10.05 in 1985. Big business in the midwest, however, claimed the average hourly wage to be larger than this. A random sample of size $n = 15$ of workers in the midwest determine their average hourly wage to be $\bar{x} = \$10.83$ and the standard deviation in their wages to be $s = 3.25$. Does this data support big business's claim at $\alpha = 0.05$?

   (a) *P–Value Versus Level of Significance, Standardized.*

      i. *Statement.* The statement of the test, in this case, is (circle one)
         A. $H_0 : \mu = \$10.05$ versus $H_1 : \mu < \$10.05$
         B. $H_0 : \mu = \$10.05$ versus $H_1 : \mu \neq \$10.05$
         C. $H_0 : \mu = \$10.05$ versus $H_1 : \mu > \$10.05$

      ii. *Test.* Since the standardized test statistic of $\bar{x} = \$10.83$ is

$$\text{t test statistic } = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{10.83 - 10.05}{3.25/\sqrt{15}} =$$

(circle one) **0.42** / **0.93** / **1.52**,
the p–value is given by

$$\text{p–value} = P(T \geq 0.93)$$

which equals (circle one) **0.18** / **0.20** / **0.23**.
(Use 2nd DISTR 5:tcdf(0.93,E99,14).)

      iii. *Conclusion.* Since the p–value, 0.18, is greater than the level of significance, $\alpha = 0.05$, we (circle one) **accept** / **reject** the null guess of \$10.05. Use the figure below to help you answer this question.

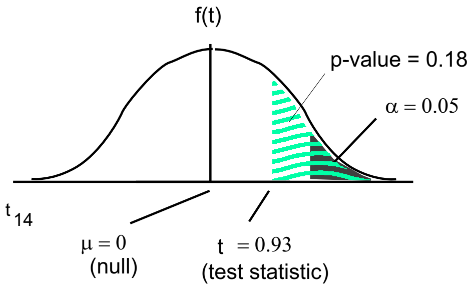



Figure 3.8 (Testing $\mu$: P–Value Versus Level of Significance, Small Sample, Right Sided, Wages)

(b) *Test Statistic Versus Critical Value, Standardized.*

i. *Statement.* The statement of the test, in this case, is (circle one)

A. $H_0 : \mu = \$10.05$ versus $H_1 : \mu < \$10.05$

B. $H_0 : \mu = \$10.05$ versus $H_1 : \mu \neq \$10.05$

C. $H_0 : \mu = \$10.05$ versus $H_1 : \mu > \$10.05$

ii. *Test.* The standardized test statistic of $\bar{x} = \$10.83$ is

$$\text{t test statistic } = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{10.83 - 10.05}{3.25/\sqrt{15}} =$$

(circle one) **0.42** / **0.93** / **1.52**.
The standardized critical value at $\alpha = 0.05$ is
(circle one) **1.18** / **1.76** / **2.32**
(Use PRGM ENTER INVT ENTER 14 ENTER 0.95 ENTER)

iii. *Conclusion.* Since the test statistic, 0.93, is smaller than the critical value, 1.76, we (circle one) **accept** / **reject** the null guess of \$10.05.

(c) *More Questions.*

i. Since the value of the $t$–test statistic, 0.93, is (circle one)

A. smaller than the critical value, 1.76, we will *reject* that the average hourly wage is \$10.05.

B. larger than the level of significance, $\alpha = 0.05$, we will *reject* that the average hourly wage is \$10.05.

C. smaller than the critical value, 1.76, we will *accept* that the average hourly wage is \$10.05.

D. larger than the p–value, 0.18, we will accept the average hourly wage is \$10.05

ii. Since the value p–value, 0.18, is (circle one)

A. smaller than the critical value, 1.76, we will *accept* that the average hourly wage is \$10.05.

B. smaller than the critical value, 1.76, we will *reject* that the average hourly wage is \$10.05.

C. larger than the level of significance, $\alpha = 0.05$, we will *accept* that the average hourly wage is \$10.05.

D. smaller than the observed $t$ test statistic, we will accept the average hourly wage is \$10.05.

iii. Match the statistical items with the appropriate parts of this wage example.

| terms | wage example |
|---|---|
| **(a)** population | **(a)** all US wages |
| **(b)** sample | **(b)** wages of 15 workers |
| **(c)** statistic | **(c)** observed average wage of 15 workers |
| **(d)** parameter | **(d)** average wage of US workers, $\mu$ |

| terms | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| wage example | | | | |

iv. **True** / **False**. There is *no non*standard version of the $t$–test.

v. The $t$–test is used instead of the $z$–test, if the sample size is (circle one) **small** / **large**.

2. *Confidence Interval Of Population Mean: PNC Student Height.*
A random sample of size thirty ($n = 30$) is taken from PNC students and it is found that the average height of these students is $\bar{x} = 5.6$ feet tall and that the standard deviation in the height of these students is $s = 3$ feet.

(a) *Two–Sided.* An 80% *confidence interval* (CI) for $\mu$ is given by $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 5.6 \pm 1.28 \times \frac{3}{\sqrt{30}} \approx$
(circle none, one or more) $\mathbf{5.6 \pm 0.55}$ / $\mathbf{5.6 \pm 0.70}$ / $\mathbf{(4.19, 7.01)}$.
(STAT TESTS 7:ZInterval ENTER Stats 3 5.6 30 0.80 Calculate ENTER.)

(b) *Two–Sided.* An 60% CI for $\mu$ is given by $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} =$
(circle none, one or more)
$\mathbf{5.6 \pm 0.85 \times \frac{3}{\sqrt{30}}}$ / $\mathbf{(5.14, 6.06)}$ / $\mathbf{5.6 \pm 2.58 \times \frac{3}{\sqrt{30}}}$.
(STAT TESTS 7:ZInterval ENTER Stats 3 5.6 30 0.60 Calculate ENTER;
Also, $z_{\alpha/2} = z_{0.40/2} = z_{0.20}$ is the 80th percentile
and so use 2nd DISTR invNorm(0.80).)

(c) *Two–Sided.* An 99% CI for $\mu$ is given by $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 5.6 \pm 2.58 \times \frac{3}{\sqrt{30}} \approx$
(circle one) $\mathbf{(4.19, 7.01)}$ / $\mathbf{(4.03, 7.32)}$ / $\mathbf{(3.95, 7.45)}$.
(STAT TESTS 7:ZInterval ENTER Stats 3 5.6 30 0.99 Calculate ENTER.)

(d) *Lower–Sided.* The 95% lower confidence interval (LCI) for $\mu$ is given by $(\bar{x} - z_{\alpha}\frac{\sigma}{\sqrt{n}}, \infty) = (5.6 - 1.645 \times \frac{3}{\sqrt{30}}, \infty) \approx$
(circle one) $\mathbf{(4.6991, \infty)}$ / $\mathbf{(5.6691, \infty)}$ / $\mathbf{(6.5009, \infty)}$.

(STAT TESTS 7:ZInterval ENTER Stats 3 5.6 30 0.90 Calculate ENTER; notice it is 0.90, not 0.95!)

(e) *Upper–Sided.* The 90% upper confidence interval, UCI for $\mu$ is given by $(-\infty, \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}}) = (-\infty, 5.6 + 1.28 \times \frac{3}{\sqrt{30}}) \approx$
(circle one) $(\boldsymbol{\infty}, \mathbf{4.8981})$ / $(\boldsymbol{\infty}, \mathbf{5.8981})$ / $(\boldsymbol{\infty}, \mathbf{6.3019})$.
(STAT TESTS 7:ZInterval ENTER Stats 3 5.6 30 0.80 Calculate ENTER; notice it is 0.80, not 0.90!)

3. *Hypothesis Test Of Population Variance: Car Door and Jamb.*
In a random sample of 28 cars, the variance in the distance between door and jamb was found to be $s^2 = 0.7$. Test if the variance is *greater* than 0.4 at $\alpha = 0.05$.

(a) *Test Statistic Versus Critical Value, Standardized.*

i. *Statement.* The statement of the test is (circle one)
A. $H_0 : \sigma^2 = 0.4$ versus $H_1 : \sigma^2 > 0.4$
B. $H_0 : \sigma^2 = 0.4$ versus $H_1 : \sigma^2 < 0.4$
C. $H_0 : \sigma^2 = 0.4$ versus $H_1 : \sigma^2 \neq 0.4$

ii. *Test.*
The standardized test statistic of $s^2 = 0.7$ is

$$\chi^2 \text{ test statistic } = \frac{(n-1)s^2}{\sigma^2} = \frac{(28-1)(0.7)}{0.4} =$$

(circle one) **30.2** / **47.3** / **82.7**.
The standardized upper critical value at $\alpha = 0.05$ is
(circle one) **39.2** / **40.1** / **43.2**
(Use PRGM INVCHI2 ENTER 27 ENTER 0.95 ENTER)

iii. *Conclusion.*
Since the test statistic, 47.3, is larger than the critical value, 40.1, we (circle one) **accept** / **reject** the null hypothesis that $\sigma^2 = 0.4$.

(b) *P–Value Versus Level of Significance, Standardized.*

i. *Statement.*
The statement of the test is (circle one)
A. $H_0 : \sigma^2 = 0.4$ versus $H_1 : \sigma^2 > 0.4$
B. $H_0 : \sigma^2 = 0.4$ versus $H_1 : \sigma^2 < 0.4$
C. $H_0 : \sigma^2 = 0.4$ versus $H_1 : \sigma^2 \neq 0.4$

ii. *Test.*
Since the standardized test statistic is $\chi^2 = 47.3$, the p–value is given by

$$\text{p–value} = P(\chi^2_{27} \geq 47.3)$$

which equals (circle one) **0.01** / **0.05** / **0.10**.
(Use 2nd DISTR 7:$\chi^2$cdf(47.3,E99,27).)
The level of significance is 0.05.

iii. *Conclusion.*
Since the p–value, 0.01, is smaller than the level of significance, 0.05, we (circle one) **accept** / **reject** the null hypothesis that $\sigma^2 = 0.4$.

(c) *Related Questions.*

i. Since the $\chi^2$ test statistic is (circle one)
   A. larger than the critical value, $\chi^2_{0.05}$, we will accept that the variance between door and jamb exceeds 0.4 mm.
   B. larger than the level of significance, $\alpha = 0.05$, we will accept that the variance between door and jamb exceeds 0.4 mm.
   C. smaller than the critical value, $\chi^2_{0.05}$, we will accept that the variance between door and jamb exceeds 0.4 mm.
   D. larger than the p–value we will accept that the variance between door and jamb exceeds 0.4 mm.

ii. Since the p–value is (circle one)
   A. larger than the critical value, $\chi^2_{0.05}$, we will accept that the variance between door and jamb exceeds 0.4 mm.
   B. smaller than the level of significance, $\alpha = 0.05$, we will accept that the variance between door and jamb exceeds 0.4 mm.
   C. smaller than the critical value, $\chi^2_{0.05}$, we will accept that the variance between door and jamb exceeds 0.4 mm.
   D. larger than the test statistic we will reject that the variance between door and jamb is equal to 0.4 mm.

iii. *Population, Parameter, Sample and Statistic.*
Match the statistical items with the appropriate parts of this jamb example.

| terms | jamb example |
|---|---|
| **(a)** population | **(a)** variance in jamb–door distance, of 28 cars, $s^2$ |
| **(b)** sample | **(b)** variance in jamb–door distance, of all cars, $\sigma^2$ |
| **(c)** statistic | **(c)** jamb–door distances, of all cars |
| **(d)** parameter | **(d)** jamb–door distances, of 28 cars |

| terms | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| jamb example | | | | |

4. *Confidence Interval Of Population Variance: Car Door and Jamb.*
In a random sample of 28 cars, the variance in the distance between door and

jamb was found to be $s^2 = 0.7$ and assume $\sigma^2 = 0.4$. Calculate a 95% CI using the formula,

$$\left( \frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}} \right)$$

(a) The upper critical value is $\chi^2_{\alpha/2,n-1} = \chi^2_{0.05/2,27} =$
(circle one) **8.7 / 40.1 / 43.2**
(Use PRGM INVCHI2 ENTER 27 ENTER 0.975 ENTER)

(b) The lower critical value is $\chi^2_{1-\alpha/2,n-1} = \chi^2_{1-0.05/2,27} =$
(circle one) **14.6 / 40.1 / 43.2**
(Use PRGM INVCHI2 ENTER 27 ENTER 0.025 ENTER)

(c) So, the CI is
$\left( \frac{(n-1)s^2}{\chi^2_{0.05/2,27}}, \frac{(n-1)s^2}{\chi^2_{1-0.05/2,27}} \right) = \left( \frac{(28-1)0.7}{43.2}, \frac{(28-1)0.7}{14.6} \right) =$
(circle one) $\mathbf{(0.6, 1.6)}$ / $\mathbf{(0.5, 1.2)}$ / $\mathbf{(0.4, 1.3)}$.

(d) Since the 95% CI (0.4, 1.3) does *not* include 0.4, this indicates that the variance in the distance between door and jamb (circle one) **is / is not** 0.4 mm.

5. *Test of Proportion*[1]*: Greenhouse Sprinkling System.*
In a greenhouse, 8% of all plants are assumed to be under–watered. Technical trouble with the greenhouse sprinkling system has raised the concern that the percent under–watered has increased in the past few weeks. Of $n = 60$ plants chosen at random, $\frac{7}{60}$ths ($\frac{7}{60} \approx 0.117$) of them are found to be under–watered. Does this data support the concern about under–watering at $\alpha = 0.05$?

   (a) *P–Value Versus Level of Significance, Standardized.*

      i. The statement of the test, in this case, is (circle one)
         A. $H_0 : p = 0.08$ versus $H_1 : p < 0.08$
         B. $H_0 : p \leq 0.08$ versus $H_1 : p > 0.08$
         C. $H_0 : p = 0.08$ versus $H_1 : p > 0.08$

      ii. *Test.* The standardized observed proportion is

$$\text{z test statistic} = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} = \frac{0.117 - 0.08}{\sqrt{\frac{0.117(0.883)}{60}}}$$

which equals (circle one) **0.89 / 1.06 / 1.55**. The p–value, the chance the standardized observed proportion is 0.89 or more, guessing the

[1]The test given here is not exactly the same as the one given in the Montgomery text. The one here is *not* corrected by 0.5, as is done in the text. Also, the one here uses $\hat{p}$, rather than $p$ in calculating the standard error.

population proportion is 0.08, is given by

$$\text{p–value} = P(Z \geq 0.89)$$

which equals (circle one) **0.04** / **0.15** / **0.19**.
(Use 2nd DISTR 2:normalcdf(0.89,E99).)
The level of significance is given by $\alpha = 0.05$.

iii. *Conclusion.* Since the p–value, 0.19, is *greater* than the level of significance, $\alpha = 0.05$, we (circle one) **accept** / **reject** the null guess of 0.08.

(b) *Test Statistic Versus Critical Value, Standardized.*

i. The statement of the test, in this case, is (circle one)

A. $H_0 : p = 0.08$ versus $H_1 : p < 0.08$
B. $H_0 : p \leq 0.08$ versus $H_1 : p > 0.08$
C. $H_0 : p = 0.08$ versus $H_1 : p > 0.08$

ii. *Test.* The standardized observed test statistic is

$$\text{z test statistic} = \frac{\hat{p} - p}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}} = \frac{0.117 - 0.08}{\sqrt{\frac{0.117(0.883)}{60}}} \approx 0.89$$

The critical value is (circle one) **0.04** / **0.15** / **1.65**.
(Use 2nd DISTR 3:invNorm(0.95).)

iii. *Conclusion.* Since the test statistic, 0.89, is smaller than the critical value, 1.65, we (circle one) **accept** / **reject** the null guess of 0.08.

(c) *Population, Sample, Statistic, Parameter.*
Match the statistical items with the appropriate parts of this plant example.

| terms | plant example |
|---|---|
| **(i)** population | **(i)** all (watered or under–watered) plants |
| **(ii)** sample | **(ii)** proportion under–watered, of all plants, $p$ |
| **(iii)** statistic | **(iii)** 60 (watered or under–watered) plants |
| **(iv)** parameter | **(iv)** proportion under–watered, of 60 plants, $\hat{p}$ |

| terms | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|
| example | | | | |

6. *Confidence Interval: Greenhouse Sprinkling System.*
In a random sample of 60 plants, $\frac{7}{60}$ths ($\frac{7}{60} \approx 0.117$) of them are found to be under–watered.

(a) A (two–sided) 95% CI is given by
$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.117 \pm z_{0.05/2}\sqrt{\frac{0.117(0.883)}{60}} =$
(circle one) **(0.036, 0.198)** / **(0.046, 0.208)** / **(0.056, 0.218)**.
(To determine z(0.025), type 2nd DISTR 3:invNorm(0.975) ENTER)

(b) A (two–sided) 99% CI is given by
$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.117 \pm z_{0.01/2}\sqrt{\frac{0.117(0.883)}{60}} =$
(circle one) **(0.010, 0.224)** / **(0.020, 0.234)** / **(0.030, 0.244)**.

7. *Normal Probability Plot: Checking For Normality of Error Terms.*
We *assume* the chance of the size of the error follows a bell–shaped normal distribution and no other probability distribution such as a $\chi^2$ or $F$ distribution, say[2]. A *normal probability plot* of residuals versus expected residuals,

$$e \quad \text{versus} \quad \sigma\left[z\left(\frac{j-0.5}{n}\right)\right],$$

which appears to be *linear* provides some evidence that the error is, in fact, normally distributed. Consider the following five normal probability plots.

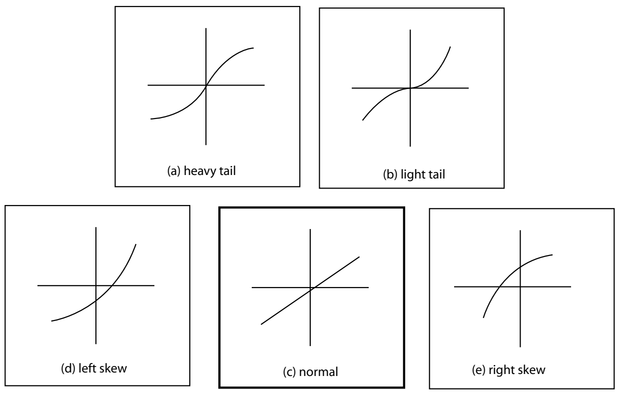



Figure 3.9 (Checking For Normality of Error Terms)

Plot (c) indicates
**heavy tail** / **light tail** / **normality**
**left skew** / **right skew** / **none of these**

## 3.4 Statistical Inference for Two Samples

### Exercise 3.6 (Statistical Inference for Two Samples)

[2]There are probability plots for these other distributions where, again, linearity in the plot indicates the presence of the required underlying distribution and non–linearity indicates the required underlying distribution is not present.

1. *Hypothesis Test For Difference in Two Means: Progesterone.*
A study is conducted to determine the cellular response to progesterone in females. Blood cells from four females are injected with progesterone; blood cells from four *different* females are, for comparison purposes, left untreated. Test if the average progesterone response is *greater* than the average control response at 5%.

   (a) *Test Statistic Versus Critical Value, Standardized.*

      i. *Statement.* If the average progesterone response, $\mu_1$, is *greater* than the average control response, $\mu_2$, then the statement of the test is (circle one)

      A. $H_0 : \mu_1 - \mu_2 = 0$ versus $H_1 : \mu_1 - \mu_2 < 0$
      B. $H_0 : \mu_1 - \mu_2 \leq 0$ versus $H_1 : \mu_1 - \mu_2 > 0$
      C. $H_0 : \mu_1 - \mu_2 = 0$ versus $H_1 : \mu_1 - \mu_2 > 0$

      ii. *Test.*
      First, calculate the average progesterone amount and the average control amount, by completing the table below.

| female | progesterone (1) | female | control (2) |
|---|---|---|---|
| 1 | 5.85 | 5 | 5.23 |
| 2 | 2.28 | 6 | 1.21 |
| 3 | 1.51 | 7 | 1.40 |
| 4 | 2.12 | 8 | 1.38 |
| average | $\bar{x}_1 = \frac{5.85+2.28+1.51+2.12}{4} = 2.94$ | | $\bar{x}_2 =$ ______ |

The difference of the averages is $\bar{x}_1 - \bar{x}_2 = 2.94 - 2.305 = 0.635$ and the *pooled* standard deviation of the two samples is

$$s = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(4-1)1.97^2 + (4-1)s_2^2}{4+4-2}}$$

which is equal to (circle one) **1.97** / **2.93** / **3.52**,
(To find $s_2^2$, fill in $L_1$ and $L_2$, STAT CALC 1–Var Stats $L_1$ and $L_2$, then find $\bar{x}_1 - \bar{x}_2$, $s_1$, $s_2$ and $s$.)
The standardized test statistic of $\bar{x}_1 - \bar{x}_2 = 0.635$ is then

$$t \text{ test statistic} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{0.635 - 0}{1.96\sqrt{\frac{1}{4} + \frac{1}{4}}} =$$

(circle one) **0.46** / **2.93** / **4.56**.
The standardized upper critical value at $\alpha = 0.05$ with $n_1 + n_2 - 2 = 4 + 4 - 2 = 6$ df is (circle one) **1.18** / **1.94** / **2.35**
(Use PRGM ENTER INVT ENTER 6 ENTER 0.95 ENTER)

iii. *Conclusion.*
Since the test statistic, 0.46, is smaller than the critical value, 1.94, we (circle one) **accept** / **reject** the null hypothesis that $\mu_1 - \mu_2 = 0$.

(b) *P–Value Versus Level of Significance, Standardized.*

i. *Statement.*
The statement of the test is (circle one)
A. $H_0 : \mu_1 - \mu_2 = 0$ versus $H_1 : \mu_1 - \mu_2 < 0$
B. $H_0 : \mu_1 - \mu_2 \leq 0$ versus $H_1 : \mu_1 - \mu_2 > 0$
C. $H_0 : \mu_1 - \mu_2 = 0$ versus $H_1 : \mu_1 - \mu_2 > 0$

ii. *Test.*
Since the standardized test statistic is $t = 0.46$, the p–value is given by

$$\text{p–value} = P(T \geq 0.46)$$

which equals (circle one) **0.002** / **0.25** / **0.33**.
(Use 2nd DISTR 5:tcdf(0.46,E99,6).)
The level of significance is 0.05.

iii. *Conclusion.*
Since the p–value, 0.33, is larger than the level of significance, 0.05, we (circle one) **accept** / **reject** the null hypothesis that $\mu_1 - \mu_2 = 0$.

(c) *Related Questions.*

i. The control blood samples (circle one) **depend on** / **are independent of** the four samples of progesterone–infected blood.

ii. A *paired* two sample test is probably a (circle one) **better** / **worse** test than the present two sample independent test in the sense it controls for differences due to different females, whereas the present test does not do this.

iii. The average in the differences calculated in the paired test is always (circle one) **smaller than** / **equal to** / **larger than** the difference of the averages calculated in the present independent two sample test.

iv. **True** / **False**. We must assume the $n$ data are sampled at random from both the populations, which are both normally and independently distributed. Also, the population standard deviations are equal.

v. If the sample size is large enough, we could, because of the CLT, use the (circle one) **normal** / **$t$–distribution**.

vi. *Population, Sample, Statistic, Parameter.*
Match the statistical items with the appropriate parts of the infection example.

| terms | infection example |
|---|---|
| **(a)** populations | **(a)** cellular response, 8 females |
| **(b)** samples | **(b)** $\bar{x}_1 - \bar{x}_2$ |
| **(c)** statistic | **(c)** cellular response, all females |
| **(d)** parameter | **(d)** $\mu_1 - \mu_2$ |

| terms | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| infection example | | | | |

2. *Confidence Intervals For Differences In Means: Plasma Levels.*
The plasma levels for a random sample of nine 17–year–old males and six 17–year–old females are given by:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| males (1) | 3.06 | 2.78 | 2.87 | 3.52 | 3.81 | 3.60 | 3.30 | 2.77 | 3.62 |
| females (2) | 1.31 | 1.17 | 1.72 | 1.20 | 1.55 | 1.53 | | | |

Verify that the average, standard deviation and number of the plasma levels for males and females are:

| | males (1) | females (2) |
|---|---|---|
| $\bar{x}$ | 3.259 | 1.413 |
| $s$ | 0.400 | 0.220 |
| $n$ | 9 | 6 |

We are interested in knowing whether the (population) average plasma level for the males, $\mu_1$, is the same or different than the (population) average plasma level for the females, $\mu_2$. We will consider two cases, when we do not or do pool the standard deviations.

(a) *Unpooled Standard Deviation Case.*
Since the standard deviations in plasma levels for the males and females are *very* different from another, $s_1 = 0.400 \neq s_2 = 0.220$, we will *not pool* these two standard deviations into *one* standard deviation.

i. We will take the *difference* of the point estimates for these two averages, $\bar{X}_1 - \bar{X}_2$. If this difference is close to zero, we will decide that the population averages are the (circle one) **same** / **different**; otherwise, we will decide the averages are different.

ii. The degrees of freedom for the $t$ distribution are given by
df $= n_1 + n_2 - 2 = 9 + 6 - 2 =$ (circle one) **13** / **14** / **15**.

iii. The *difference* in the average plasma levels for the males and females is given by
$\bar{x}_1 - \bar{x}_2 = 3.259 - 1.413 =$ (circle one) **−1.846** / **1.846** / **2.345**.

iv. The standard *error* for the confidence interval is given by
$\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = \sqrt{\frac{0.400^2}{9}+\frac{0.220^2}{6}} =$ (circle one) **0.09 / 0.16 / 0.35**.

v. For a 95% CI of $\mu_1 - \mu_2$ and 13 df,
$t_c =$ (circle one) **1.77 / 2.16 / 2.58**.
(Remember to use the "INVT" program on your calculator and to determine the 97.5th percentile for $t_c$: PRGM INVT ENTER 13 ENTER 0.975 ENTER.)

vi. Thus, the 95% CI for $\mu_1 - \mu_2$ is
$(\bar{x}_1 - \bar{x}_2) \pm t_c\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 1.846 \pm 2.16(0.16) =$ (circle one)
**(1.23, 3.56) / (1.50, 2.19) / (0.94, 4.56)**.
(Use your calculator: STAT TESTS 0:2–SampTInt... Stats 3.259 0.4 9 1.413 0.22 6 0.95 No ENTER Calculate ENTER.)

vii. Since this confidence interval does not include zero, this indicates that the average plasma level for males is (circle one) the **same / different** than the average plasma level for females.

viii. Match the statistical items with the appropriate parts of this plasma example.

| terms | plasma example |
|---|---|
| **(a)** population | **(a)** nine male, six female plasma levels |
| **(b)** sample | **(b)** $\bar{x}_1 - \bar{x}_2$ |
| **(c)** statistic | **(c)** all 17–year–olds plasma levels |
| **(d)** parameter | **(d)** $\mu_1 - \mu_2$ |

| terms | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| plasma example | | | | |

ix. The sample sizes of the males, $n_1 = 9$, and the females, $n_2 = 6$, are both (circle one) **small / large**.

x. Since the sample sizes are small, we (circle one) **can / cannot** use the central limit theorem (CLT) and, thus, we cannot use the normal distribution for this exercise. We must assume that the plasma levels for both the males and females follow a normal distribution and so be able to use the $t$ distribution.

(b) *Pooled Standard Deviation Case.*
Even though the standard deviations in plasma levels for the males and females are *very* different from another, $s_1 = 0.400 \neq s_2 = 0.220$, we will (wrongly) *pool* these two standard deviations into *one* standard deviation and then calculate a CI for $\mu_1 - \mu_2$.

i. The pooled variance, $s_c^2$, is given by,
$s_c^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = \frac{(9-1)0.400^2+(6-1)0.220^2}{9+6-2} =$ (circle one)
**0.045 / 0.117 / 0.342**

ii. Thus, the 95% CI for $\mu_1 - \mu_2$ is
$(\bar{x}_1 - \bar{x}_2) \pm t_c\sqrt{\frac{s_c^2}{n_1} + \frac{s_c^2}{n_2}} = 1.846 \pm 2.16(0.117) =$ (circle one)
$\mathbf{(1.23, 3.56)}$ / $\mathbf{(1.50, 2.19)}$ / $\mathbf{(1.45, 2.09)}$.

iii. The 95% CI for $\mu_1 - \mu_2$ in the pooled case, here, is (circle one) **wider** / **narrower** than the 95% CI for $\mu_1 - \mu_2$ in the unpooled case, above.

iv. Thus, the 87% CI for $\mu_1 - \mu_2$ is
$(\bar{x}_1 - \bar{x}_2) \pm t_c\sqrt{\frac{s_c^2}{n_1} + \frac{s_c^2}{n_2}} = 1.846 \pm t_c(0.117) =$ (circle one)
$\mathbf{(1.66, 2.04)}$ / $\mathbf{(1.76, 2.14)}$ / $\mathbf{(1.86, 2.24)}$.
(Remember to use the "INVT" program on your calculator and to determine the 93.5th percentile for $t_c$ with 13 df.)

3. *Hypothesis Test For Paired Difference in Two Means: Progesterone.*
A study is conducted to determine the cellular response to progesterone in females. Blood cells from female 1 are broken into two groups. One group of these blood cells are injected with progesterone; the other group, called the *control*, is, for comparison purposes, left untreated. The blood cells of females 2, 3 and 4 are handled in the same way. Test if the average progesterone response is *greater* than the average control response at 5%.

(a) *Test Statistic Versus Critical Value, Standardized.*

i. *Statement.*
If the average progesterone response, $\mu_1$, is *greater* than the average control response, $\mu_2$, then the difference in responses, $\mu_d = \mu_1 - \mu_2$, must be greater than zero and so the statement of the test is (circle one)

A. $H_0 : \mu_d = 0$ versus $H_1 : \mu_d < 0$
B. $H_0 : \mu_d \leq 0$ versus $H_1 : \mu_d > 0$
C. $H_0 : \mu_d = 0$ versus $H_1 : \mu_d > 0$

ii. *Test.*
First, calculate the *differences* between progesterone and control, by completing the table below.

| female | progesterone (1) | control (2) | differences, $d_i$ |
|---|---|---|---|
| 1 | 5.85 | 5.23 | ____________ |
| 2 | 2.28 | 1.21 | ____________ |
| 3 | 1.51 | 1.40 | $d_3 = 1.51 - 1.40 = 0.11$ |
| 4 | 2.12 | 1.38 | ____________ |

Since the average of the differences is $\bar{d} = \frac{0.62+1.07+0.11+0.74}{4} = 0.635$ and the standard deviation of the differences, $s_d$, is
(circle one) **0.398** / **0.931** / **1.522**,

(Fill in $L_1$ and $L_2$; $L_3 = L_1 - L_2$; STAT CALC 1–Var Stats $L_3$.)
The standardized test statistic of $\bar{d} = 0.635$ is then

$$\text{t test statistic } = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{0.635 - 0}{0.398/\sqrt{4}} =$$

(circle one) **1.42** / **2.93** / **3.19**.
The standardized upper critical value at $\alpha = 0.05$ with $n-1 = 4-1 = 3$ df is (circle one) **1.18** / **1.76** / **2.35**
(Use PRGM ENTER INVT ENTER 3 ENTER 0.95 ENTER)

iii. *Conclusion.*
Since the test statistic, 3.19, is larger than the critical value, 2.35, we (circle one) **accept** / **reject** the null hypothesis that $\mu_d = 0$.

(b) *P–Value Versus Level of Significance, Standardized.*

i. *Statement.*
The statement of the test is (circle one)
A. $H_0 : \mu_d = 0$ versus $H_1 : \mu_d < 0$
B. $H_0 : \mu_d \leq 0$ versus $H_1 : \mu_d > 0$
C. $H_0 : \mu_d = 0$ versus $H_1 : \mu_d > 0$

ii. *Test.*
Since the standardized test statistic is $t = 3.19$, the p–value is given by

$$\text{p–value} = P(T \geq 3.19)$$

which equals (circle one) **0.02** / **0.05** / **0.08**.
(Use 2nd DISTR 5:tcdf(3.19,E99,3).)
The level of significance is 0.05.

iii. *Conclusion.*
Since the p–value, 0.02, is smaller than the level of significance, 0.05, we (circle one) **accept** / **reject** the null hypothesis that $\mu_d = 0$.

(c) *Related Questions.*

i. The control blood samples (circle one) **depend on** / **are independent of** the four samples of progesterone–infected blood. These dependent samples are said to have been *paired* (or, more generally, *blocked*) across females.

ii. **True** / **False**. We must assume the $n$ data pairs are sampled at random, and these differences are normally distributed.

iii. If the sample size is large enough, we could, because of the CLT, use the (circle one) **normal** / $t$**–distribution**.

4. *Hypothesis Test For Two Variances: Plasma Levels.*
   The average and standard deviation of the plasma levels for males and females is given below, where, notice, $s_1^2 > s_2^2$ (which, if not true, must be made true).

| | males (1) | females (2) |
|---|---|---|
| $\bar{x}$ | 3.259 | 1.413 |
| $s^2$ | 0.16 | 0.13 |
| $n$ | 9 | 6 |

Test if $\sigma_1^2 > \sigma_2^2$ at $\alpha = 0.05$.

(a) *Test Statistic Versus Critical Value.*

i. *Statement.*
The statement of the test is (circle one)
A. $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 > \sigma_2^2$
B. $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 < \sigma_2^2$
(Is this a sensible, considering that $s_1^2 > s_2^2$?)
C. $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$
where, notice, if $\sigma_1^2 = \sigma_2^2$, then
$\frac{\sigma_1^2}{\sigma_2^2} =$ (circle one) **−1 / 0 / 1**
and if $\sigma_1^2 > \sigma_2^2$, then
$\frac{\sigma_1^2}{\sigma_2^2} >$ (circle one) **−1 / 0 / 1**.

ii. *Test.*
The test statistic is

$$F \text{ test statistic } = \frac{s_1^2}{s_2^2} = \frac{0.16}{0.13} =$$

(circle one) **1.03 / 1.23 / 2.27**.
The upper critical value at $\alpha = 0.05$, with $n_1 - 1 = 9 - 1 = 8$ and $n_2 - 1 = 6 - 1 = 5$ degrees of freedom, is
(circle one) **3.22 / 4.15 / 4.82**
(Use PRGM INVF ENTER 8 ENTER 5 ENTER 0.95 ENTER)

iii. *Conclusion.*
Since the test statistic, 1.23, is less than the critical value, 4.82, we (circle one) **accept / reject** the null hypothesis that $\sigma_1^2 = \sigma_2^2$.

(b) *P–Value Versus Level of Significance.*

i. *Statement.*
The statement of the test is (circle one)
A. $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 > \sigma_2^2$

B. $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 < \sigma_2^2$
C. $H_0 : \sigma_1^2 = \sigma_2^2$ versus $H_1 : \sigma_1^2 \neq \sigma_2^2$

ii. *Test.*
Since the test statistic is $F = 1.23$, with $n_1 - 1 = 9 - 1 = 8$ and $n_2 - 1 = 6 - 1 = 5$ degrees of freedom, the p–value is given by

$$\text{p–value} = P(F \geq 1.23)$$

which equals (circle one) **0.14** / **0.35** / **0.43**.
(Use 2nd DISTR 9:$F$cdf(1.23,E99,8,5).)
The level of significance is 0.05.

iii. *Conclusion.*
Since the p–value, 0.43, is larger than the level of significance, 0.05, we (circle one) **accept** / **reject** the null hypothesis that $\sigma_1^2 = \sigma_2^2$.

(c) *Related Questions.*
Match the statistical terms with the appropriate parts of the plasma example.

| terms | plasma example |
|---|---|
| **(a)** population | **(a)** nine male, six female plasma levels |
| **(b)** sample | **(b)** $F = \frac{s_1^2}{s_2^2}$ |
| **(c)** statistic | **(c)** all 17–year–olds plasma levels |
| **(d)** parameter | **(d)** $F = \frac{\sigma_1^2}{\sigma_2^2}$ |

| terms | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| plasma example | | | | |

5. *Hypothesis Test For Two Proportions: Male Urban and Rural Pigeons.*
A comparison of the number of male pigeons in urban and rural areas was undertaken, with the following results, based on large random samples.

| | urban (1) | rural (2) | total |
|---|---|---|---|
| male pigeons | 358 | 6786 | 7144 |
| female pigeons | 49 | 577 | 626 |
| total | 407 | 7363 | 7770 |

Does this data support the claim there is a smaller proportion of male pigeons in the urban than in rural areas at $\alpha = 0.05$?

(a) *Test Statistic Versus Critical Value, Standardized.*

i. *Statement.*
If the proportion of male pigeons in the urban, $p_1$, is *less* than the proportion of male pigeons in rural life, $p_2$, then the statement of the test is (circle one)

A. $H_0 : p_1 - p_2 = 0$ versus $H_1 : p_1 - p_2 \neq 0$
B. $H_0 : p_1 - p_2 = 0$ versus $H_1 : p_1 - p_2 < 0$
C. $H_0 : p_1 - p_2 = 0$ versus $H_1 : p_1 - p_2 > 0$

ii. *Test.*
The standardized test statistic of

$$\hat{p}_1 - \hat{p}_2 = \frac{358}{407} - \frac{6786}{7363} = 0.8796 - 0.9216 =$$

(circle one) **−0.042** / **−0.076** / **−0.123**,
with standard error given by,

$$\hat{\sigma}_{\hat{p}} = \sqrt{\frac{0.8796(1-0.8796)}{407} + \frac{0.9216(1-0.9216)}{7363}} =$$

(circle one) **0.0164** / **0.0174** / **0.0184**,
is given by,

$$z \text{ test statistic } = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}} = \frac{-0.042 - 0}{0.0164} =$$

(circle one) **−1.23** / **−2.56** / **−4.56**.
The standardized *lower* critical value at $\alpha = 0.05$ is
(circle one) **−1.18** / **−1.65** / **−2.33**
(Use 2nd DISTR 3:invNorm(0.05))

iii. *Conclusion.*
Since the test statistic, -2.56, is smaller than the critical value, -1.65, we (circle one) **accept** / **reject** the null hypothesis that $p_1 - p_2 = 0$.

(b) *P–Value Versus Level of Significance, Standardized.*

i. *Statement.*
The statement of the test is (circle one)
A. $H_0 : p_1 - p_2 = 0$ versus $H_1 : p_1 - p_2 \neq 0$
B. $H_0 : p_1 - p_2 = 0$ versus $H_1 : p_1 - p_2 < 0$
C. $H_0 : p_1 - p_2 = 0$ versus $H_1 : p_1 - p_2 > 0$

ii. *Test.*
Since the standardized test statistic is $z = -2.56$, the p–value is given by

$$\text{p–value} = P(Z \leq -2.56)$$

which equals (circle one) **0.001** / **0.005** / **0.011**.
(Use 2nd DISTR 2:normalcdf(−E99,-2.56).)
The level of significance is 0.05.

iii. *Conclusion.*
Since the p–value, 0.005, is smaller than the level of significance, 0.05, we (circle one) **accept** / **reject** the null hypothesis that $p_1 - p_2 = 0$.

## 3.5 What If There Are More Than Two Populations?

The single–factor analysis of variance (ANOVA) procedure is demonstrated in this section. This ANOVA test uses the data to decide if two or more of the population factor level (treatment) means are the same or different. An important part of this procedure involves the calculation of the ANOVA table,

| Source | Degrees of Freedom, *df* | Sum Of Squares, *SS* | Mean Squares, *MS* |
|---|---|---|---|
| Treatment (between treatments) | $a-1$ | $SSTR = \sum n_i(\overline{y}_{i\cdot} - \overline{y}_{\cdot\cdot})^2$ | $MSTR = \frac{SSTR}{a-1}$ |
| Error (within treatments) | $a(n-1)$ | $SSE = \sum\sum(y_{ij} - \overline{y}_{i\cdot})^2$ | $MSE = \frac{SSE}{a(n-1)}$ |
| Total | $an-1$ | $SSTO = \sum\sum(y_{ij} - \overline{y}_{\cdot\cdot})^2$ | |

which leads to the calculation of the observed $F$,

$$F = \frac{MSTR}{MSE}$$

A large $F$ indicates that the population factor level means are different; a small $F$ indicates the means are the same.

**Exercise 3.7 (Single–Factor Analysis of Variance)**

1. *Hypothesis Test For More Than Two Populations: Drugs.*

| | | | | | | |
|---|---|---|---|---|---|---|
| drug 1 | 5.90 | 5.92 | 5.91 | 5.89 | 5.88 | $\overline{X}_{1\cdot} \approx 5.90$ |
| drug 2 | 5.50 | 5.50 | 5.50 | 5.49 | 5.50 | $\overline{X}_{2\cdot} = 5.50$ |
| drug 3 | 5.01 | 5.00 | 4.99 | 4.98 | 5.02 | $\overline{X}_{3\cdot} \approx 5.00$ |

Test if at least two of the three average patient responses to the drug are different at $\alpha = 0.05$.

(a) *Test Statistic Versus Critical Value.*

i. The statement of the test is (check none, one or more):

A. $H_0 : \mu_1 = \mu_2 = \mu_3$ versus $H_a :\ \mu_1 \neq \mu_2, \mu_1 = \mu_3$.

B. $H_0 : \mu_1 = \mu_2 = \mu_3$ versus $H_a :\ \mu_1 \neq \mu_3, \mu_1 \neq \mu_2$.

C. $H_0 : \mu_1 = \mu_2 = \mu_3$ versus
$H_a :$ at least one $\mu_i \neq \mu_j, i \neq j;\ i, j = 1, 2, 3$.

D. $H_0$ : means the same versus $H_a$ : means different

ii. *Test.* After some effort, the ANOVA table is given by,

| Source | Degrees of Freedom | Sum Of Squares | Mean Squares |
|---|---|---|---|
| Treatment (Drugs) | 2 | 2.033 | 1.0165 |
| Error | 12 | 0.0022 | 0.00018 |
| Total | 14 | 2.035 | |

(To use your TI–83, enter the data $L_1, L_2, L_3$ into STAT EDIT ENTER, then type perform the ANOVA by STAT TESTS ANOVA( ENTER $L_1, L_2, L_3$) ENTER.)
and so the test statistic is

$$F \text{ test statistic } = \frac{1.0165}{0.00018} =$$

(circle one) **1.02** / **123** / **5647.2**.
The upper critical value at $\alpha = 0.05$, with $t - 1 = 3 - 1 = 2$ and $N - t = 15 - 3 = 12$ degrees of freedom, is
(circle one) **3.22** / **3.89** / **4.82**
(Use PRGM INVF ENTER 2 ENTER 12 ENTER 0.95 ENTER)

iii. *Conclusion.* Since the test statistic, 5647.2, is larger than the critical value, 3.89, we (circle one) **accept** / **reject** the null hypothesis that the average patient responses to the three drugs are the same.

(b) *P–Value Versus Level of Significance.*

i. The statement of the test is (check none, one or more):

A. $H_0 : \mu_1 = \mu_2 = \mu_3$ versus $H_a : \mu_1 \neq \mu_2, \mu_1 = \mu_3$.

B. $H_0 : \mu_1 = \mu_2 = \mu_3$ versus $H_a : \mu_1 \neq \mu_3, \mu_1 \neq \mu_2$.

C. $H_0 : \mu_1 = \mu_2 = \mu_3$ versus
$H_a :$ at least one $\mu_i \neq \mu_j, i \neq j;\ i, j = 1, 2, 3$.

D. $H_0$ : means the same versus $H_a$ : means different

ii. *Test.* Since the test statistic is $F = 5647.2$, the p–value, with $t - 1 = 3 - 1 = 2$ and $N - t = 15 - 3 = 12$ degrees of freedom, is given by

$$\text{p–value} = P(F \geq 5647.2)$$

which equals (circle one) **0.00** / **0.35** / **0.43**.
(Use 2nd DISTR 9:$F$cdf(5647.2,E99,2,12).)
The level of significance is 0.05.

iii. *Conclusion.* Since the p–value, 0.00, is smaller than the level of significance, 0.05, we (circle one) **accept** / **reject** the null hypothesis that the average patient responses to the three drugs are the same.

(c) *ANOVA Assumptions.*
**True / False**
The ANOVA assumptions for a single–factor ANOVA model I are,

- populations have a normal distribution
- population variances are equal (are all equal to a constant)
- observed responses are independent random samples from $r$ factor levels

2. *Diagnosis: Drugs.*
Fifteen different patients are subjected to three drugs.

| | | | | | | |
|---|---|---|---|---|---|---|
| drug 1 | 5.90 | 5.92 | 5.91 | 5.89 | 5.88 | $\bar{Y}_{1\cdot} \approx 5.90$ |
| drug 2 | 5.51 | 5.50 | 5.50 | 5.49 | 5.50 | $\bar{Y}_{2\cdot} \approx 5.50$ |
| drug 3 | 5.01 | 5.00 | 4.99 | 4.98 | 5.02 | $\bar{Y}_{3\cdot} \approx 5.00$ |

Use various residual analyses to check to see if the assumptions required in using the single–factor ANOVA statistical model are satisfied by this data.

(a) *Constant error (residual) variance?* One of the assumptions required in a single–factor ANOVA statistical model is that the error variance is constant with respect to the different factor level means. In general (not for this drug data example!), residual plot (a) in the figure below indicates constant error variance; other possible nonconstant error variance patterns are given in (b), (c), (d) and (e).

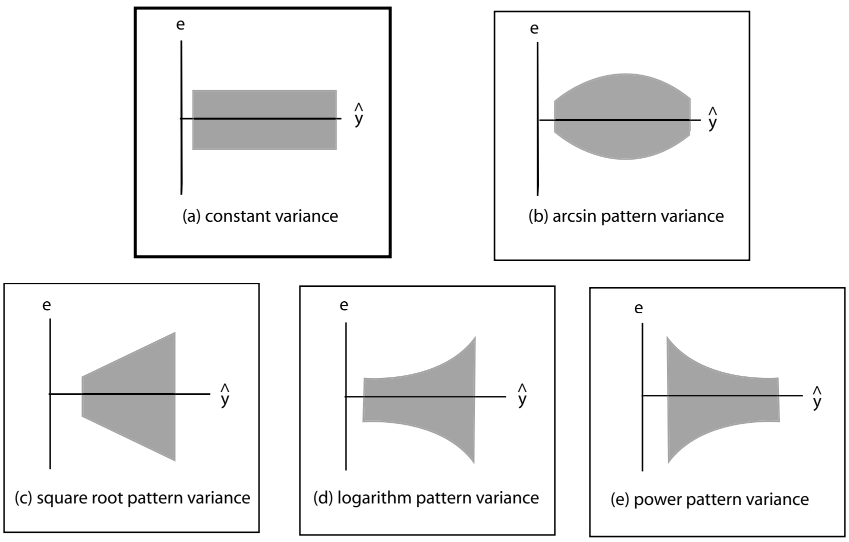



Figure 3.10 (Check for constant error variance.)

The residual plot for the drug data indicates that error variance (choose one) **is** / **is not** constant with respect to the predicted ($\hat{Y} = \bar{Y}$) variable. (TI–83 calculator: Type data in $L_1$, $L_2$, $L_3$ then PRGM EVPPLOT ENTER 3 ENTER)

(b) *Nonindependence of error?* Another of the assumptions required in a single–factor ANOVA statistical model is that the errors are independent of one another. In general, residual plot (a) in the figure below indicates independent errors; other possible nonindependent error patterns are given in (b), (c), (d) and (e).

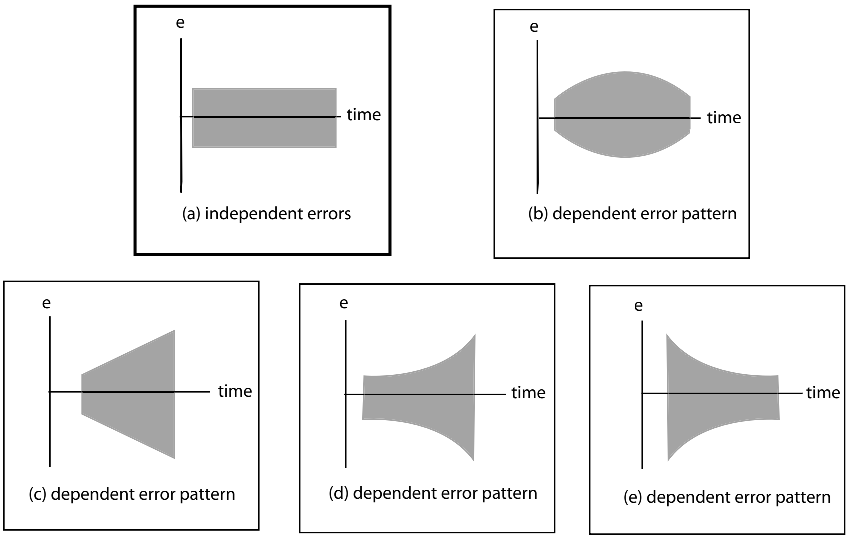



Figure 3.11 (Check for nonindependence of error.)

There does not appear to be a sequence (time) element in the drug data and so it (choose one) **is** / **is not** necessary to check the assumption of nonindependence. However, *if* the drug 1 responses were recorded on one day, the drug 2 responses were recorded on the next day and drug 3 responses were recorded on the third day, the residual plot for the drug data indicates that the errors (choose one) **are** / **are not** independent of one another.
(TI–83 calculator: PRGM EVPPLOT ENTER 3 ENTER)

(c) *Nonnormality of error terms?* Another of the assumptions required in a single–factor ANOVA statistical model is that the errors are normally distributed. In general, residual plot (a) in the figure below indicates normally distributed errors; other possible nonnormal error patterns are given in (b), (c), (d) and (e).

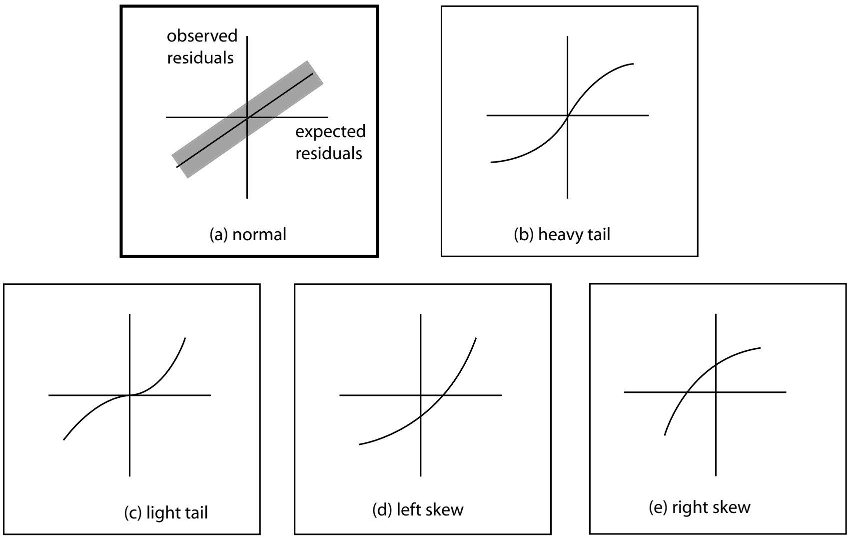



Figure 3.12 (Check for nonnormality of error.)

The residuals in the drug data appear to be (choose one) **normal** / **non-normal**.
(TI–83 calculator: PRGM QQPLTANV ENTER 3 ENTER)