

Part II

Basic Methods of Statistical Process Control and Capability Analysis

Chapter 4

Methods and Philosophy of Statistical Process Control

4.1 Introduction

We look at the basic statistical process control (SPC) problem solving tools (the “magnificent seven”) which together work to stabilize and reduce variability in a process:

- histogram (stem and leaf plot)
- check sheet
- cause and effect diagram
- defect concentration diagram
- scatter diagram
- control chart

Most time, though, is spent discussing control charts.

4.2 Chance and Assignable Causes of Quality Variation

A process is either *in statistical control* or *out of control*. It is in statistical control if it is operating with only chance causes of variation; it is out of control, if it is operating in the presence of assignable causes. The goal of statistical process control (SPC) is to reduce or eliminate the variability in the process by identifying the assignable causes.

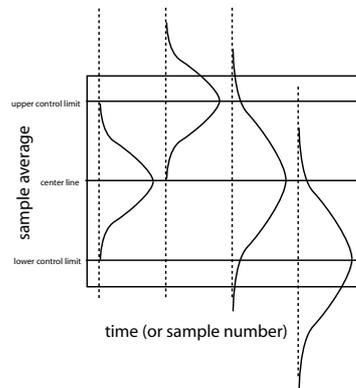
Exercise 4.1 (Chance and Assignable Causes of Quality Variation)

Figure 4.1 (In and out of control)

1. True / False

The in-control value for the mean and standard deviation are μ_0 and σ_0 respectively. More than this, the in-control mean is at the center line of the control chart and the in-control standard deviation is at the upper/lower control limits.

2. At time 1, the process is in-control because (choose one)

- (a) $\mu = \mu_0, \sigma = \sigma_0$
- (b) $\mu > \mu_0, \sigma = \sigma_0$
- (c) $\mu = \mu_0, \sigma > \sigma_0$
- (d) $\mu > \mu_0, \sigma > \sigma_0$

3. At time 2, the process is out of control because (choose one)

- (a) $\mu = \mu_0, \sigma = \sigma_0$
- (b) $\mu > \mu_0, \sigma = \sigma_0$
- (c) $\mu = \mu_0, \sigma > \sigma_0$
- (d) $\mu > \mu_0, \sigma > \sigma_0$

4. At time 3, the process is out of control because (choose one)

- (a) $\mu = \mu_0, \sigma = \sigma_0$
- (b) $\mu > \mu_0, \sigma = \sigma_0$
- (c) $\mu = \mu_0, \sigma > \sigma_0$
- (d) $\mu > \mu_0, \sigma > \sigma_0$

5. At time 4, the process is out of control because (choose one)

- (a) $\mu = \mu_0, \sigma = \sigma_0$
- (b) $\mu < \mu_0, \sigma = \sigma_0$
- (c) $\mu = \mu_0, \sigma > \sigma_0$
- (d) $\mu < \mu_0, \sigma > \sigma_0$

4.3 Statistical Basis of the Control Chart

Exercise 4.2 (Statistical Basis of the Control Chart)

1. Out of Control Situations

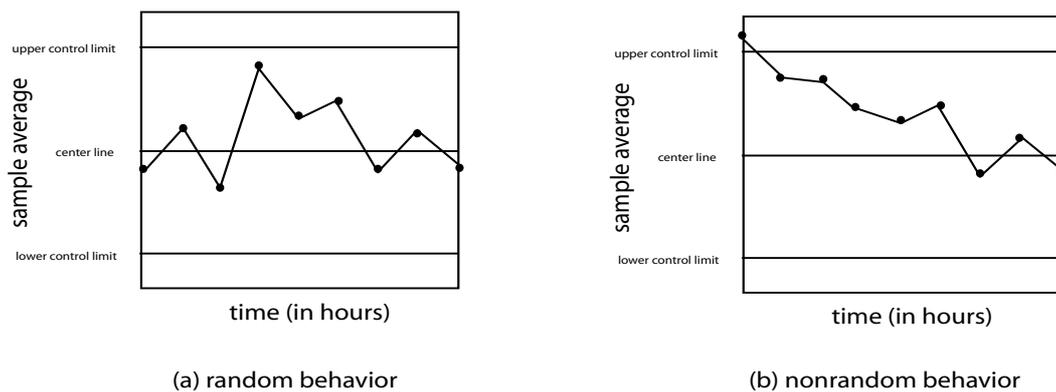


Figure 4.2 (Out of Control Situations)

A process is out of control if (choose none, one or more)

- (a) at least one point is beyond the control limits
- (b) the points are behaving in a systematic, nonrandom way.

2. Control Charts and Confidence Intervals

Every hour, a random sample of size thirty ($n = 30$) CD disks is taken from an assembly line and it is found that the average width of the plastic protective coating on these disks is $\bar{x} = 5.6$ microns and that the standard deviation in the coating is $s = 3$ microns.

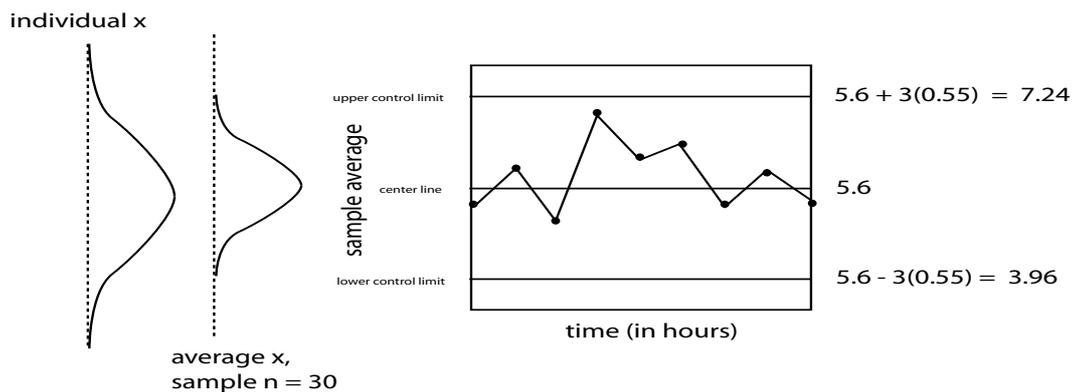


Figure 4.3 (Statistical Basis of the Control Chart)

An 99.73% (three-sigma, or 3σ) confidence interval for μ is given by

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 5.6 \pm 3 \times \frac{3}{\sqrt{30}} \approx$$

(circle none, one or more) 5.6 ± 0.55 / 5.6 ± 0.70 / **(3.96, 7.24)**.

(STAT TESTS 7:ZInterval ENTER Stats 3 5.6 30 0.9973 Calculate ENTER.)

A control chart is equivalent to using this confidence interval four times, at hours 1, 2, 3 and 4, to determine if the process is in or out of control. Control charts developed in this way are called *Shewhart control charts*.

3. Control Charts and Hypothesis Tests

The control limits are equivalent to testing

- (a) $H_0 : \mu = 5.6$ versus $H_1 : \mu < 5.6$
- (b) $H_0 : \mu = 5.6$ versus $H_1 : \mu > 5.6$
- (c) $H_0 : \mu = 5.6$ versus $H_1 : \mu \neq 5.6$

4. Control and Warning Limits

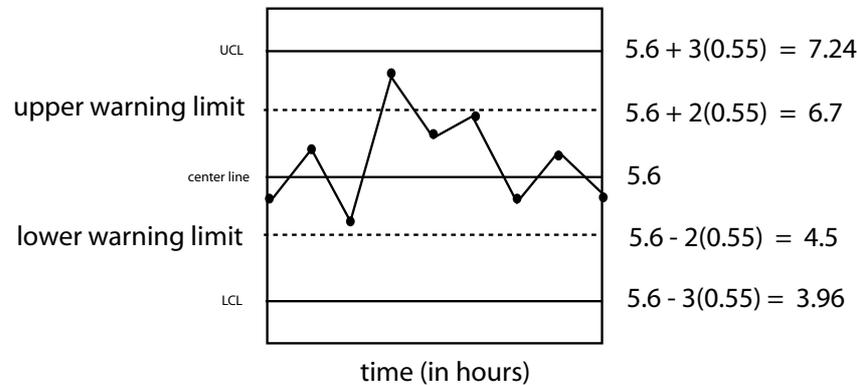


Figure 4.4 (Control and Warning Limits)

True / False The general model of a control chart is

- $UCL = \mu_W + L\sigma_W$
- Center Line = μ_W
- $LCL = \mu_W - L\sigma_W$

where L is the distance of the control limit from the center line, μ_W and σ_W are the mean and standard deviation of the sample statistic W . The UCL (upper control limit) and LCL (lower control limit) are called *action limits* and generally at 3-sigma. *Warning limits*, often placed at 2-sigma, increase the sensitivity of the chart, but also increase the number of false alarms.

5. Uses of Control Charts

Control charts are used to (choose none, one or more)

- (a) detect assignable causes
- (b) detect and eliminate assignable causes
- (c) estimate process parameters such as mean, standard deviation, fraction nonconforming and so on to determine the *capability* of the process to produce acceptable products.

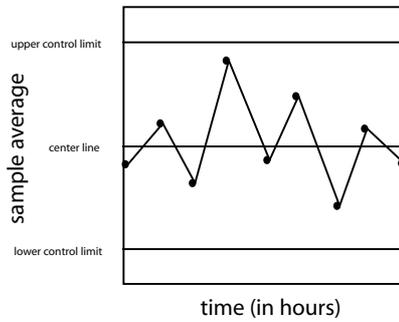
6. Types of Control Charts

The types of control charts include (choose none, one or more)

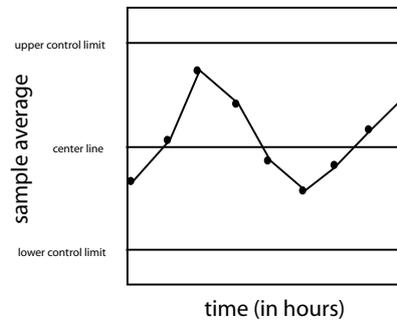
- (a) variable control charts, which are applied to continuous data, to measurement data.
- (b) attribute control charts, which are applied to discrete data, to the number or percentage conforming or nonconforming.

The control chart in the CD plastic coating example here is an example of a (choose one) **variable** / **attributes** control chart.

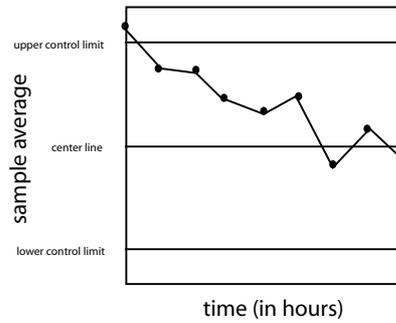
7. Types of Process Variability



(a) stationary behavior, uncorrelated data



(b) stationary behavior, autocorrelated data



(c) nonstationary behavior

Figure 4.5 (Types of Process Variability)

The types of process variability include (choose none, one or more)

- (a) stationary behavior, uncorrelated data
- (b) stationary behavior, autocorrelated data
- (c) nonstationary behavior

8. Sample Size and Sampling Frequency

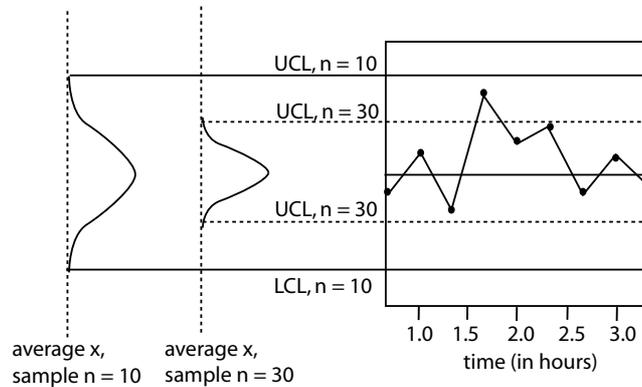


Figure 4.6 (Types of Process Variability)

It is most easy to detect changes in a process for (choose one)

- (a) small sample size ($n = 10$), slow sampling frequency (every 1 hour)
- (b) small sample size ($n = 10$), frequent sampling (every 1/2 hour)
- (c) large sample size ($n = 30$), slow sampling frequency (every 1 hour)
- (d) large sample size ($n = 30$), frequent sampling (every 1/2 hour)

9. Average Run Length (ARL) and Average Time to Signal (ATS)

A $3\text{-}\sigma$ control limit problem, where there is a $1 - 0.9973 = 0.0027$ probability the process (assuming it is stationary and uncorrelated) falls beyond the control limits, has an *average run length* (ARL) of

$$ARL = \frac{1}{0.0027} =$$

(choose one) **250 / 370 / 450** samples. In other words, an out-of-control signal will be generated every 370 samples, on average. In the samples are fixed $h = 1$ hour apart, then an out-of-control signal will be generated every

$$ATS = ARL \times h = 370 \times 1 =$$

(choose one) **250 / 370 / 450** hours.

10. More Average Run Length (ARL) and Average Time to Signal (ATS)

A $2\text{-}\sigma$ control limit problem, where there is a $1 - 0.9545 = 0.0455$ probability the process falls beyond the control limits, has an ARL of

$$ARL = \frac{1}{0.0455} = \text{(choose one) } \mathbf{11 / 37 / 45} \text{ samples}$$

and if $h = 0.5$, an ATS of

$$ATS = ARL \times h = \text{(choose one) } \mathbf{22 / 44 / 45} \text{ hours}$$

11. Rational Subgroups

To be “rational”, to detect assignable causes effectively, we would like to *maximize between* sample subgroup variation and *minimize within* sample subgroup variation.

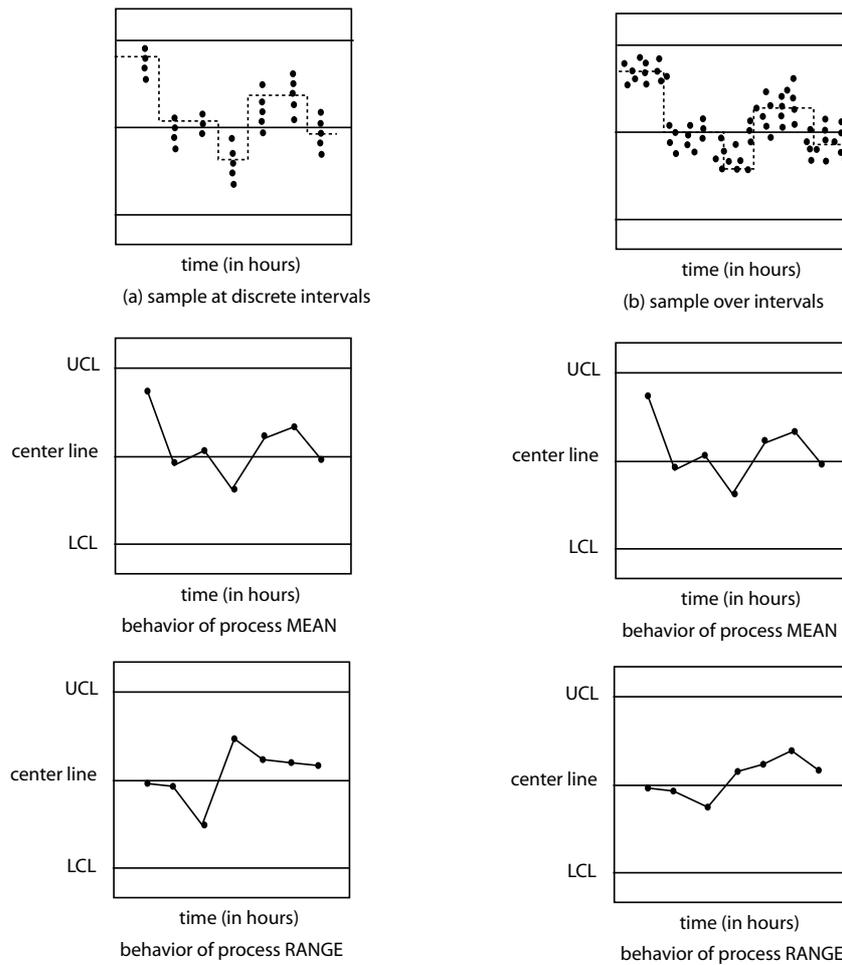


Figure 4.7 (Rational Subgroups)

The main advantage of a “snapshot” approach to rational subgroups of samples (figure (a)) is this approach is (choose one)

- (a) effective in detecting major shifts in the process¹
- (b) effective if the mean has quickly wandered in and out of control

The main advantage of a sample-over-the-interval approach to rational subgroups of samples (figure (b)) is this approach is (choose one)

- (a) effective in detecting major shifts in the process

¹This is because there are a lot of observations at a fixed point.

(b) effective if the mean has quickly wandered in and out of control²

12. *Analysis of Patterns on Control Charts*

Patterns indicate out-of-control processes and include (choose none, one or more)

- (a) a *run* of 8 or more observations; in other words, a sequence of observations all either above or below the center line
- (b) cyclical or trend patterns
- (c) two or three consecutive points beyond the $2\text{-}\sigma$ warning limits
- (d) four or five consecutive points beyond the $1\text{-}\sigma$ limits

4.4 The Rest of the “Magnificent Seven”

The “magnificent seven” which, together, work to stabilize and reduce variability in a process, to eliminate waste, to identify improvement opportunities, are

- histogram (stem and leaf plot)
- check sheet
- cause and effect diagram
- defect concentration diagram
- scatter diagram
- control chart

4.5 Implementing SPC

Statistical process control is only effective by using a management leadership and team approach.

4.6 An Application of SPC

An interesting example of statistical process control involving a copper plating operation is given in the text.

²This is because the observations are spread out over an interval of time and not at one point in time.

4.7 Nonmanufacturing Applications of Statistical Process Control

The SPC methods can be applied to nonmanufacturing processes such as finance, marketing, customer support, engineering development and design and so on.