**Final for Mathematics 223**
**Introductory Analysis I - Spring 1998**
**Material Covered: Chapters 1–6 of Workbook and text**

This is a 2 hour final, worth 27% and marked out of 27 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

**Name (please print):** _____ . **ID Number:** _____.
last first

---

1. Suppose 280 tons of corn were harvested in 5 days and 940 tons in 20 days. If the relationship between tons of corn harvested and days, $d$, to harvest $T(d)$ tons of corn is linear, express $T$ as a function of $d$.

   A. $T(d) = 5d + 280$ B. $T(d) = -44d + 500$ C. $T(d) = 44d + 60$ D. $T(d) = 60d + 44$ E. None of these.

2. When 30 orange trees are planted per acre, each tree yields 150 oranges. For each additional tree planted per acre, the yield per tree is decreased by 3 oranges. Express the total yield of oranges per acre as a function, $Y$, of the number of trees planted per acre, $x$, if $x \geq 30$.

   A. $Y(x) = 4500+60x-3x^2$ B. $Y(x) = \dfrac{1}{x} + 180$ C. $Y(x) = 150x-3x^2$
   D. $Y(x) = 240x - 3x^2$ E. None of these.

3. A furniture manufacturer can sell dining-room tables for \$70 each. The manufacturer's total cost consists of a fixed overhead of \$8,000 plus production costs of \$30 per table. How many tables must the manufacturer sell to break-even?

   A. 80 B. 267 C. 200 D. 20 E. None of these.

4. If $f(u) = \sqrt{1+u}$ and $g(x) = \begin{cases} 2x-3 & \text{if } x \geq 1 \\ 7+x^2 & \text{if } x < 1 \end{cases}$, then $f(g(-1)) =$

   A. 0 B. 3 C. $\sqrt{7}$ D. 7 E. None of these.

5. If $f(x) = \dfrac{2}{x}$, then $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} =$

   A. $\dfrac{-2}{x^2}$ B. $\dfrac{2}{x+\Delta x} - \dfrac{2}{x}$ C. $\dfrac{2}{x(x+\Delta x)}$ D. $\dfrac{-2}{x(x+\Delta x)}$ E. None of these.

6. The domain of the function $f$ given by $f(x) = \dfrac{1}{\sqrt[3]{x-1}}$ is all real numbers $x$ such that

A. $x < 1$ and $x > 1$ B. $x > 1$ C. $x > 0$ D. $x > 2$ E. None of these.

7. $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x^2 - 4} =$

A. 1.25 B. 1 C. $\infty$ D. 0 E. None of these.

8. If $g(x) = \dfrac{x^2 + 2}{x}$, then $g\left(\dfrac{2}{x}\right) =$

A. $\dfrac{4x^2 + 8}{x^3}$ B. $\dfrac{x^2 + 2}{x}$ C. $\dfrac{2x}{x^2 + 4}$ D. $\dfrac{x}{2x^2 + 4}$ E. None of these.

9. If $2\sqrt{y} - y = x$, and this equation defines $y$ as a differentiable function of $x$, then $\dfrac{dy}{dx} =$

A. $\dfrac{1}{\sqrt{y} - 1}$ B. $\sqrt{y} - 1$ C. $\dfrac{\sqrt{y}}{1 - \sqrt{y}}$ D. $\dfrac{x}{\sqrt{y} - 1}$ E. None of these.

10. The derivative of the function $f$ given by $f(x) = \dfrac{x^2 + 1}{x + 5}$ is

A. $\dfrac{2x(x+5) - (x^2+1)}{(x+5)^2}$ B. $2x$ C. $\dfrac{(x^2+1) + 2x(x+5)}{(x+5)^2}$ D. $\dfrac{(x^2+1) - 2x(x+5)}{(x+5)^2}$ E. None of these.

11. If $y = (3 - x^2)^3$, then $\dfrac{d^2y}{dx^2} =$

A. $-6x(3 - x^2)^2$ B. $6(3 - x^2)(5x^2 - 3)$ C. $6(3 - x^2)$ D. $24x^2(3 - x^2)$
E. None of these.

12. After $t$ years (yr) the population of a certain town is given by $p(t) = 50 + 5t$ thousand people. A population of $p$ thousand people has an associated $CO_2$ level given by $c(p) = \dfrac{\sqrt{p^2 + 1}}{2}$ parts per million (ppm). Two years from now ($t = 0$), the rate of change of the $CO_2$ level with respect to $t$ will be

A. $\dfrac{5}{2\sqrt{5}}$ ppm/yr B. $\dfrac{150}{\sqrt{3601}}$ ppm/yr C. $30\sqrt{3601}$ ppm/yr D. $\dfrac{30}{\sqrt{3601}}$ ppm/yr

E. None of these.

13. If $f(x) = \frac{1}{3}x^3 - 9x + 2$, then on the closed interval $0 \le x \le 4$ the function $f$ has

A. an absolute maximum at $x = 3$ and as an absolute minimum at $x = 0$.
B. an absolute maximum at $x = 4$ and as an absolute minimum at $x = 3$.
C. an absolute maximum at $x = 0$ and as an absolute minimum at $x = 4$.
D. an absolute maximum at $x = 0$ and as an absolute minimum at $x = 3$.
E. None of these.

14. The total cost $C(q)$, in dollars, to manufacture $q$ units is given by $C(q) = 5q^2 - 20q + 80$. For what value of $q$ is the average cost a minimum?

A. 2 units B. 4 units C. 60 units D. 80 units E. None of these.

15. A manufacturer can produce cameras at a cost of 40 dollars each and estimates that if the cameras are sold at a price of $p$ dollars each, then consumers will buy approximately $D(p) = 800e^{-0.01p}$ cameras per week. At what price should the manufacturer sell the cameras to maximize the weekly profit? Give your answer to the nearest dollar.

A. 100 dollars B. 40 dollars C. 80 dollars D. 140 dollars E. None of these.

16. If $yx^2 + y^3 = x - y$ and the equation defines $y$ as a differentiable function of $x$, then the slope of the tangent line to the graph of the equation is given by

A. $y = 1 - 2xy - 3y^2$ B. $1 - 2xy - x^2 - 3y^2$ C. $\frac{1 - 2xy}{3y^2 + 1}$ D. $\frac{1 - 2xy}{x^2 + 3y^2 + 1}$
E. None of these.

17. The level of air pollution in a certain city is proportional to the square of the population. Use differentials to estimate the percentage by which the air-pollution level will increase if the population increases by 5%.

A. 5% B. 10% C. 15% D. 20% E. None of these.

18. If the concentration $C(t)$ of a certain drug remaining in the bloodstream $t$ minutes after it is injected is given by $C(t) = \frac{t}{5t^2 + 125}$, then the concentration is a maximum when $t =$

A. 25 min B. 15 min C. 5 min D. There is no maximum concentration.

E. None of these.

19. A display case is in the shape of a rectangular box with a square base. Suppose that the volume of the display case is 21 cubic feet and it costs one dollar per square foot to build the class top and fifty cents per square foot to build the sides and base. If $x$ is the length of one side of the base, for what value of $x$ is the cost to construct the display case a minimum? Round your answer to two decimal places.

A. 3.04 feet B. 2.41 feet C. 3.74 feet D. 2.24 feet E. None of these.

20. Each machine at a certain factory can produce 50 units per hour. The setup cost is 80 dollars per machine and the operation cost is 5 dollars per hour. How many machines should be used to produce 8000 units at least possible cost? Give your answer to the nearest integer.

A. 2 machines B. 3 machines C. 5 machines D. 6 machines E. None of these.

21. A cylindrical can (yes, like a "beer can") with no top has been made from $27\pi$ square inches of aluminum. Express the volume of the can as a function, $V$, of the radius of the can, $r$.

A. $V(r) = 27\pi r^2$ B. $V(r) = \pi r^2(27 - r^2 - 2r)$ C. $V(r) = \dfrac{4\pi}{3} r(27 - r^2)$

D. $V(r) = \dfrac{\pi}{2}(27r - r^3)$ E. None of these.

22. $\lim\limits_{x\to\infty} \dfrac{x^3 - 4x^2 + 1}{1 + x - 2x^2} =$

A. 0 B. $\infty$ C. $-\infty$ D. $-2$ E. None of the these.

23. If $y = \ln\left(\sqrt{1 - x^2}\right)$, then $\dfrac{dy}{dx} =$

A. $\dfrac{1}{\sqrt{1 - x^2}}$ B. $\dfrac{-2x}{\sqrt{1 - x^2}}$ C. $\dfrac{-x}{1 - x^2}$ D. $\dfrac{1}{2(1 - x^2)}$ E. None of these.

24. $\dfrac{d}{dx}\, e^{x^2} =$

A. $e^{x^2}$ B. $x^2 e^{x^2 - 1}$ C. $2xe^{x^2 - 1}$ D. $2xe^{x^2}$ E. None of these.

25. If $e^{s+t} = st + e$ and the equation defines $s$ as a differentiable function of $t$, then $\dfrac{ds}{dt}$ at the point $(0, 1)$ on the graph of the equation is

A. $\dfrac{1}{e}$ B. $\dfrac{1 - e}{e}$ C. $\dfrac{2 - e}{e}$ D. $-1$ E. None of these.

26. A particle moves along a coordinate line with position $s(t)$, in centimeters (cm), at time $t$, in seconds (sec), given by $s(t) = \frac{1}{3}t^3 - \frac{1}{2}t^2 - 2t$ for $t \geq 1$. What is the acceleration when the velocity is zero?

A. 3 cm/sec$^2$ and $-3$ cm/sec$^2$ B. 3 cm/sec$^2$ C. 1 cm/sec$^2$ D. 2 cm/sec$^2$
E. None of these.

27. Use calculus to find the intervals on which the graph of $y = x^3 - 6x^2 + 9x - 4$ is increasing.

A. $x < 1$ and $x > 3$ B. $1 < x < 3$ C. $x < 0$ and $x > 2$ D. $0 < x < 2$
E. None of these.

28. Use calculus to find the intervals on which the graph of $y = x^3 - 3x^2 + 5$ is concave down.

A. $x < 1$ B. $x > 1$ C. $x < 0$ and $x > 2$ D. $0 < x < 2$ E. None of these.

1. C 2. D 3. C 4. B 5. D 6. A 7. A 8. B 9. C 10. A 11. B 12. B 13. D 14. B 15. D 16. D 17. B 18. C 19. B 20. B 21. D 22. C 23. C 24. D 25. B 26. B 27. A 28. A