Final for Mathematics 223
Introductory Analysis I - Spring 1999
Material Covered: Chapters 1–6 of Workbook and text
5:15–7:15pm, Wednesday, 5th May

This is a 2 hour final, worth 27% and marked out of 27 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): _____ . **ID Number:** _____.
last first

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1. The costs of manufacturing x oboes in one month was found to be a linear function of x. It cost \$32,000 to produce 550 oboes and \$82,500 to produce 1950 oboes.

(a) [1] The cost function is (circle closest one)

(i) $C(x) = 36.07x - 12,160.71$ (ii) $C(x) = 36.07x - 8,156.46$

(iii) $C(x) = 36.07x + 12,160.71$ (iv) $C(x) = 36.07x + 21,551.14$

(v) $C(x) = 36.07x + 32,661.99$

(b) [1] The cost per oboe is
(circle closest one) **23.75** / **27.67** / **30.88** / **33.00** / **36.07**

(c) [1] The fixed costs

are _____.

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2. [2] Consider the equation $(3 + y^2)^4 = 5x^3 + 2xy$. Then $\frac{dy}{dx} =$ (circle one)

(i) $\frac{15x^2+2y}{4y(3+y^2)^3-2x}$ (ii) $\frac{15x+2y}{8y(3+y^2)^3-2x}$ (iii) $\frac{15x^2+2y}{8y(3+y^2)^3-6x}$

(iv) $\frac{15x^2+2y}{8y(3+y^2)^3+2x}$ (v) $\frac{15x^2+2y}{8y(3+y^2)^3-2x}$

3. [3] Circle true or false.

(a) **True** / **False** If $\lim_{x\to\infty} f(x) = L$, then f has horizontal asymptotes.

(b) **True** / **False** Infinite limit and limit at infinity mean the same thing. In other words, they are just different ways of saying exactly the same thing.

(c) **True** / **False** A function f is said to be differentiable at a point c if

$$\lim_{x\to c} f(x) = f(c)$$

(d) **True** / **False** If a function is continuous at a point c, then it is differentiable at a point c.

(e) **True** / **False** If $(c, f(c))$ is an inflection point of the function f, then $f''(c) = 0$.

(f) **True** / **False** Let function f be defined on the finite closed interval I which contains point c. If $(c, f(c))$ is an extreme point, then c must either an endpoint of I or a point where the derivative is zero or a point where the derivative does not exist.

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4. Determine the derivatives of the following functions.

(a) [1] If $f(x) = e^{3x^2-5x}$, then $f'(x) =$ (circle one)

(i) $e^{3x^2-5x}(6x-5)(6)$ (ii) $(3x^2-5x)e^{3x^2-5x}$ (iii) $(6x-5)e^{3x^2-5x}$

(iv) $e^{3x^2-5x}(6x+5)$ (v) $e^{3x^2-5x}(3x^2-5)$

(b) [1] If $f(x) = \ln(3x^2-5x)$, then $f'(x) =$ (circle one)

(i) $\frac{(6x-5)(6)}{3x^2-5x}$ (ii) $\frac{1}{3x^2-5x}$ (iii) $\frac{3x^2-5}{3x^2-5x}$ (iv) $\frac{6x-5}{3x^2-5x}$ (v) $\frac{6x+5}{3x^2-5x}$

(c) [2] If $f(x) = \ln(3x^2-5x)e^{3x^2-5x}$, then $f'(x) =$ (circle one)

(i) $(6x-5)\ln(3x^2-5x)e^{3x^2-5x} - \frac{(6x-5)e^{3x^2-5x}}{3x^2-5x}$

(ii) $(6x-5)(6)\ln(3x^2-5x)e^{3x^2-5x} + \frac{(6x-5)(6)e^{3x^2-5x}}{3x^2-5x}$

(iii) $(6x-5)\ln(3x^2-5x)e^{3x^2-5x} + \frac{(6x-5)e^{3x^2-5x}}{3x^2-5x}$

(iv) $(6x-5)\ln(3x^2-5x)e^{3x^2+5x} + \frac{(6x-5)e^{3x^2-5x}}{3x^2-5x}$

(v) $(6x-5)\ln(3x^2-5x)e^{3x^2-5x} + \frac{(6x+5)e^{3x^2-5x}}{3x^2-5x}$

5. A psychologist has found that if a 16–year–old boy studies t hours on the piano, his performance score, S, can be predicted from the formula, $S(t) = 35t^2 - 3t^3$, where $0 \leq t \leq 7$.

(a) [1] The number of hours of study this boy needs to have the highest performance score is (circle closest one) **3.7** / **5.2** / **6.0** / **7.0** / **7.7**

(b) [1] The time when the boy's performance score is increasing most rapidly is (circle closest one) **1.4** / **2.2** / **3.9** / **5.0** / **6.3**

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6. After 5 years in the early 1990s, an initial population of 15 rap singers grows to 72. The music industry will only support 305 rap singers. It is assumed the growth in the number of rap singers follows the logistic formula, $f'(t) = \frac{dy}{dt} = ky(L - y)$.

(a) [1] The L in $f'(t) = \frac{dy}{dt} = ky(L - y)$ describes the (circle one)

(i) rate of change in the number

(ii) number

(iii) initial number

(iv) maximum number

(v) starting year

of rap singers and is given by (circle one) **5** / **15** / **72** / **305** / **1990**.

(b) [1] In this case, $y = \frac{L}{1+Be^{-Lkt}} =$ (circle closest one)

(i) $\frac{305}{1+17.4e^{-305kt}}$ (ii) $\frac{305}{1+18.7e^{-305kt}}$ (iii) $\frac{305}{1+19.3e^{-305kt}}$

(iv) $\frac{305}{1+20.4e^{-305kt}}$ (v) $\frac{305}{1+21.3e^{-305kt}}$

(c) [2] How many rap singers are there after 9 years? Circle closest one.

(i) 162 (ii) 172 (iii) 182 (iv) 190 (v) 200

7. Consider the function $f(x) = 3x^3 + 5x^2 - 170x$.

(a) [1] Function f is increasing on the interval(s) (circle closest one)

(i) $(-\infty, -4.94) \cup (-4.94, 3.83) \cup (3.83, \infty)$ (ii) $(-4.94, 3.83)$

(iii) $(-\infty, -4.94) \cup (3.83, \infty)$ (iv) $(-\infty, \infty)$ (v) $(-4.94, 3.83) \cup (3.83, \infty)$

(b) [1] The function f is concave down

on the interval(s) _____.

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8. Consider the function, $y = f(x) = 3x^5 - \sqrt{x}$.

(a) [1] $\frac{dy}{dx} =$ (circle one)

(i) $15x^4 + \frac{1}{2x^2}$ (ii) $15x^4 - \frac{x^2}{2}$ (iii) $15x^4 - \frac{2}{\sqrt{x}}$

(iv) $15x^4 - \frac{3}{2\sqrt{x}}$ (v) $15x^4 - \frac{1}{2\sqrt{x}}$

(b) [1] $f''(x) =$ (circle one)

(i) $60x^3 - \frac{1}{4}x^{-\frac{3}{2}}$ (ii) $60x^3 + \frac{1}{4}x^{-\frac{3}{2}}$ (iii) $60x^3 + \frac{1}{4}x^{\frac{3}{2}}$

(iv) $60x^3 + \frac{1}{2}x^{-\frac{3}{2}}$ (v) $60x^3 + \frac{1}{8}x^{-\frac{3}{2}}$

(c) [1] And so

$\frac{d^3y}{dx^3} =$ _____.

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9. [2] Consider the equation $y = f(x) = (x^2 + 4x)^{5x}$. Then $\frac{dy}{dx} =$ (circle one)

(i) $(x^2 + 4x)^{5x} \left[\frac{10x+20}{x+4} + 5\ln(x^2 + 4x)\right]$

(ii) $(x^2 - 4x)^{5x} \left[\frac{10x+20}{x+4} + 5\ln(x^2 + 4x)\right]$

(iii) $(x^2 + 4x)^{5x} \left[\frac{10x-20}{x+4} + 5\ln(x^2 + 4x)\right]$

(iv) $(x^2 + 4x)^{5x} \left[\frac{10x+20}{x+4} + 5\ln(x^2 + 8x)\right]$

(v) $(x^2 + 4x)^{5x} \left[\frac{10x+20}{x+4} - 5\ln(x^2 + 4x)\right]$

10. Try the following problems.

(a) [1] If \$655 is invested at a 11% yearly interest, which is compounded monthly, what will be its value after 10 years? Circle closest one.

(i) 1937.69 (ii) 1947.71 (iii) 1957.89 (iv) 2056.65 (v) 2114.50

(a) [1] If \$655 is invested at a 11% yearly interest, which is compounded yearly, and the inflation rate is 4% per year, the value of this money, 6 years from now is (circle closest one)

(i) 968.23 (ii) 1005.25 (iii) 1114.99 (iv) 1187.24 (v) 1200.01

1. (a) iii; (b) 36.07; (c) 12,161.50
2. v
3. (a) T; (b) F; (c) F; (d) F; (e) T; (f) T
4. (a) iii; (b) iv; (c) iii
5. (a) 7.0; (b) 3.9
6. (a) iv, 305; (b) iii; (c) ii
7. (a) iii; (b) $(-\infty, -0.56)$
8. (a) v; (b) ii; (c) $180x^2 - \frac{3}{8}x^{-\frac{5}{2}}$
9. i
10. (a) iii; (b) i