Final for Statistics 213
Probability and Decision Theory - Spring 1999
Material Covered: Chapters 1–4, 6–9 of Workbook and text
10:15am–12:15pm, Wednesday, 5th May

This is a 2 hour final, worth 27% and marked out of 27 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): ______________________ . **ID Number:** __________.
last first

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1. Consider the following linear programming problem.

$$\begin{array}{lcccccc} \text{Minimize} & 4x & + & 8y & & \\ \text{subject to} & 4x & + & y & \geq & 16 \\ & x & + & 5y & \geq & 20 \\ & 5x & + & 4y & \geq & 40 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(a) [1] The intersection of the half–planes defined by the constraints is (circle none, one or more)

(i) an optimal set.
(ii) an unbounded region.
(iii) a feasible set.
(iv) an inconsistent set.
(v) the solution set.

(b) [1] One vertex of the feasible region is (approximately) $(x, y) = (3.16, 3.37)$. The other vertex of the feasible region is (circle closest one)
$\mathbf{(7.27, 2.18)}$ / $\mathbf{(2.86, 5.71)}$ / $\mathbf{(5.71, 7.27)}$ / $\mathbf{(5.71, 2.86)}$ / $\mathbf{(2.18, 7.27)}$

(c) [1] The value of the objective function at the optimal solution is
(circle closest one) **39.6** / **46.5** / **57.1** / **66.9** / **81.0**

2. Consider the following supply and demand equations,

$$4x + 3p - 59 = 0, \qquad 4x - 6p - 14 = 0$$

where x is either quantity demanded or supplied and p is the unit price, in dollars.

(a) [1] The supply equation is (circle one) $\mathbf{4x + 3p - 59 = 0}$ / $\mathbf{4x - 6p - 14 = 0}$ because the slope is (circle one) **positive** / **zero** / **negative**.

(b) [1] The market equilibrium quantity is (circle closest one) **8** / **9** / **10** / **11** / **12**

(c) [1] The market equilibrium price is (circle closest one) **5** / **6** / **7** / **8** / **9**

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3. Consider the following system of equations

$$\begin{aligned} 2x - 5y &= 10 \\ 6x - 15y &= 30 \end{aligned}$$

(a) [1] This system has a (circle one)
unique / **inconsistent** / **dependent** / **hyperplane** / **homogeneous**
solution.

(b) [1] The two lines (circle one)

(i) intersect at an infinity of points.
(ii) intersect at one point.
(iii) do not intersect.
(iv) intersect at the origin.
(v) intersect at two points.

(c) [1] The solution of this system is (circle one)

(i) $(2.5t, t)$ (ii) $(t, 2.5t + 5)$ (iii) (t, t) (iv) $(2.5t, t + 5)$ (v) $(2.5t + 5, t)$

4. Consider the following simplex tableau.

x	y	z	w_1	w_2	w_3	w_4	P	
1	0	0	$\frac{2}{5}$	0	-$\frac{6}{5}$	-$\frac{8}{5}$	0	4
0	0	0	-$\frac{2}{5}$	1	$\frac{6}{5}$	$\frac{8}{5}$	0	5
0	1	0	0	0	1	0	0	12
0	0	1	0	0	0	1	0	6
0	0	0	72	0	-16	12	1	4920

(a) [1] There are (circle one) **two** / **three** / **four** / **five** / **six** slack variables in this simplex tableau.

(b) [1] The pivot is (circle one) $\boldsymbol{-\frac{8}{5}}$ / $\boldsymbol{-\frac{6}{5}}$ / $\mathbf{1}$ / $\boldsymbol{\frac{6}{5}}$ / $\boldsymbol{\frac{8}{5}}$.

(c) [1] The row operations necessary to pivot to the next simplex tableau are (circle one)

(i) $R_1 + \frac{5}{6}R_3, \quad R_2 - \frac{6}{5}R_3, \quad R_5 - 16R_3$

(ii) $-\frac{5}{6}R_1, \quad R_2 - \frac{6}{5}R_1, \quad R_3 - R_1, \quad R_5 + 16R_1$

(iii) $R_1 + \frac{6}{5}R_3, \quad R_2 - \frac{6}{5}R_3, \quad R_5 + 16R_3$

(iv) $\frac{5}{6}R_2, \quad R_1 + \frac{6}{5}R_2, \quad R_3 - R_2, \quad R_5 + 16R_2$

(v) $\frac{6}{5}R_2, \quad R_1 + \frac{6}{5}R_2, \quad R_3 - R_2, \quad R_5 - 16R_2$

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5. Permutations of the word "whoopee".

(a) [1] The number of different permutations that can be made from the letters in the word *whoopee* is (circle one) $\mathbf{7!}$ / $\boldsymbol{P(7,7)}$ / $\boldsymbol{\frac{7!}{2!2!}}$ / $\boldsymbol{P(7,2)}$ / $\boldsymbol{P(7,5)}$

(b) [1] The chance the letters in the word *whoopee* form a word beginning and ending in the letter e is (circle one) $\boldsymbol{\frac{7!}{P(7,7)}}$ / $\boldsymbol{\frac{2!2!}{P(7,7)}}$ / $\boldsymbol{\frac{2!2!}{5!}}$ / $\boldsymbol{\frac{2!5!}{7!}}$ / $\boldsymbol{\frac{P(7,5)}{P(7,7)}}$

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6. Let $P(G) = 0.6$, $P(H) = 0.3$, and $P(G \cap H) = 0.2$.

(a) [1] $P(G|H) =$ ____________________.

(b) [1] $P(G \cup H) =$ ____________________.

(c) [1] $P(G \cup H^c) =$ ____________________.

7. A lawyer, who presently represents 14 defendants, estimates she wins 27% of her cases. Assume this problem obeys the conditions of a binomial experiment.

(a) [1] The functional form of the binomial distribution in this case is given by (circle one)

(i) $C(14, x(0.27)^x(1-0.73)^{14-x} \quad x = 0, 1, 2, \ldots 14$

(ii) $C(14, x)(1-0.27)^x(0.73)^{14-x} \quad x = 0, 1, 2, \ldots 14$

(iii) $C(14, x)(0.27)^x(0.73)^{14-x} \quad x = 0, 1, 2, \ldots 14$

(iv) $C(14, x)(0.73)^x(0.27)^{14-x} \quad x = 0, 1, 2, \ldots 14$

(v) $C(14, x)(0.27)^x(0.27)^{14-x} \quad x = 0, 1, 2, \ldots 14$

(b) [1] The probability of winning 2 or 3 trials, using the binomial is (circle closest one) **0.377** / **0.402** / **0.411** / **0.423** / **0.443**.

(c) [1] The standard deviation in the number of trials she expects to win is (circle closest one) **1.32** / **1.45** / **1.50** / **1.55** / **1.66**.

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8. Consider the transition matrix given by,

$$A = \left[\begin{array}{c|c} I & O \\ \hline R & S \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0.2 & 0.1 & 0.2 & 0.5 \\ 0.3 & 0.2 & 0.2 & 0.3 \end{array}\right]$$

(a) [2] The steady state matrix is given by (fill in the blanks)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & 0 & 0 \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & 0 & 0 \end{bmatrix}$$

(b) [1] If the initial vector is $X_0 = (0.5, 0.2, 0.2, 0.1)$, the final steady state vector is (fill in the blanks)

$$\begin{bmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{bmatrix}$$

9. Consider a zero sum game with mixed strategies. where player R's mixed strategy is P, players C's mixed strategy is Q and the payoff matrix is A, all given below,

$$P = \begin{bmatrix} 0.3 & 0.7 \end{bmatrix}$$

and

$$A = \begin{bmatrix} 15 & 3 \\ -1 & 3 \end{bmatrix}$$

and

$$Q = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$$

(a) [1] The expected value of the game is
(circle closest one) **0.65** / **3.08** / **6.53** / **8.08** / **9.11**

(b) [1] The optimal mixed strategy

for player R is ____.

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10. [2] Kane Manufacturing has a division that produces two models of hibachis, model A and model B. To produce each model A hibachi requires 3.5 pounds of cast iron and 5 minutes of labor. To produce each model B hibachi requires 5 pounds of cast iron and 3 minutes of labor. The profit for each model A hibachi is \$2 and the profit for each model B hibachi is \$1.50. If 1050 pounds of cast iron and 19 hours of labor are available for the production of hibachis per day, Kane Manufacturing is interested in determining how many hibachis of each model each division should produce in order to maximize its profits. Give the linear programming (LP) problem for this situation. (Just write the LP problem; do *not* solve it.)

1. (a) ii,iii,v; (b) (2.18,7.27); (c) 39.6
2. (a) $4x - 6p - 14 = 0$,positive; (b) 11; (c) 5
3. (a) dependent; (b) i; (c) v
4. (a) four; (b) $\frac{6}{5}$; (c) iv
5. (a) $\frac{7!}{2!2!}$; (b) $\frac{2!5!}{7!}$
6. (a) 0.67; (b) 0.7; (c) 0.9
7. (a) iii; (b) 0.377; (c) 1.66
8. (a) 0.63,0.37,0.61,0.39; (b) 0.69,0.31,0,0
9. (a) 3.08; (b) $\frac{4}{16}$,$\frac{12}{16}$
10. max $2x + 1.5y$ st $3.5x + 5y \leq 1050$, $5x + 3y \leq 1140$, $x \geq 0$, $y \geq 0$