Final for Statistics 213
Probability and Decision Theory - Spring 2000
Material Covered: Chapters 1–9 of Workbook and text
For: 3rd May

This is a 2 hour final, worth 28% and marked out of 28 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): ______________________ . **ID Number:** __________.
last first

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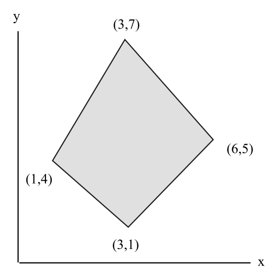
1. Consider the following feasible region.



(a) [1] Identify which, if any, of the following inequalities, are constraints for the feasible region.

(i) $y \leq \frac{3}{2}x + \frac{5}{2}$ (ii) $y \geq -\frac{3}{2}x + \frac{11}{2}$ (iii) $y \geq \frac{4}{3}x - 3$
(iv) $y \leq -\frac{2}{3}x + 9$ (v) $y \geq -\frac{2}{3}x + 9$

(b) [1] If the objective function is $Z = 3x + 2y$, then the minimum point(s) occur(s) at/along (circle one)

(i) (3,7) (ii) the edge connecting (3,7) and (6,5) (iii) (6,5)
(iv) the edge connecting (1,4) and (3,1) (v) (3,1)

(c) [1] If the objective function is $Z = 5x - 7y$, then the minimum point(s) occur(s) at/along (circle one)

(i) (3,7) (ii) the edge connecting (3,7) and (6,5) (iii) (6,5)
(iv) the edge connecting (1,4) and (3,1) (v) (3,1)

2. Consider the following augmented matrix.

$$A = \left[\begin{array}{ccc|c} 2 & 0 & 3 & -1 \\ 3 & -2 & 1 & 9 \\ 1 & 1 & 4 & 4 \end{array}\right]$$

(a) [1] This is a system of (circle none, one or more)

(i) four equations and three variables. (ii) three three–dimensional hyperplanes. (iii) three equations and four variables. (iv) three equations and three variables. (v) three planes.

(b) [1] The first step in the Gauss–Jordan method for solving this system of equations is (circle one)

(i) $R_2 - \frac{1}{2}R_1$ (ii) $\frac{1}{2}R_1 + 3R_2$ (iii) $\frac{1}{2}R_1 - 3R_2$ (iv) $\frac{1}{2}R_1$ (v) $R_3 - \frac{1}{2}R_1$

(c) [1] The solution to this system of equations is

$(x, y, z) =$ (_______, _______, _______)

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3. The dosage of medicine for a child psychiatric patient is sometimes based on the patient's surface area. If a is the adult dosage and S is the child's surface area, then the child dosage, D, is given by

$$D(S) = \left(\frac{S + 0.5}{1.3}\right) a$$

(a) [1] Dosage D is a linear function of child surface area, S, where the

slope is: ____________________________________.

(b) [1] If the adult dosage is 250 mg, a child whose surface area is 0.3 square meter should

receive: ____________________________________.

4. Consider the following standard maximization problem.

$$\begin{array}{lrcrcr} \text{Maximize} & 321x & - & 12y & & \\ \text{subject to} & 60x & + & 12y & \leq & 102 \\ & 19x & + & 10y & \leq & 90 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(a) [1] Give the initial simplex tableau of this problem by filling in the following table.

x	y	u	v	P	
______	______	1	0	0	______
______	______	0	1	0	______
______	______	0	0	1	______

(b) [2] The optimal solution to this problem is given by

$(x, y) = ($______, ______$)$ where $P =$ ______.

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5. Try the following problems.

(a) [1] If \$655 is invested at a 11% yearly interest, which is compounded monthly, what will be its value after 10 years? Circle closest one.

(i) 1937.69 (ii) 1947.71 (iii) 1957.89 (iv) 2056.65 (v) 2114.50

(b) [1] A car loan of \$32,000 is to be repaid over a 3.5 year period through equal installments made at the end of each month. If the yearly interest rate is 7.5%, then the size of each installment is (circle closest one)

(i) 868.64 (ii) 905.25 (iii) 1114.99 (iv) 1187.24 (v) 1200.01

6. Each glass of blended juice is to contain at least 1200 International Units (I.U.) of vitamin A and 200 I.U. of vitamin C. One ounce of orange juice contains 50 I.U. of vitamin A, 15 I.U. of vitamin C, and 14 calories; each ounce of pink grapefruit juice contains 120 I.U. of vitamin A, 12 I.U. of vitamin C, and 11 calories. We are interested in determining how many ounces of each juice should a glass of the blend contain if it is to meet the minimum vitamin requirements and at the same time contain a minimum number of calories?

(a) [2] Complete the following (restricted) standard minimization or primal problem by filling in the blanks.

$$\begin{array}{lrcrcr} \text{Minimize} & \_\_\_\_\,x & + & \_\_\_\_\,y & & \\ \text{subject to} & \_\_\_\_\,x & + & \_\_\_\_\,y & \geq & \_\_\_\_ \\ & \_\_\_\_\,x & + & \_\_\_\_\,y & \geq & \_\_\_\_ \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(b) [2] Rewrite the primal problem as a dual problem by filling in the blanks of the following standard maximization problem.

$$\begin{array}{lrcrcr} \text{Maximize} & \_\_\_\_\,u & + & \_\_\_\_\,v & & \\ \text{subject to} & \_\_\_\_\,u & + & \_\_\_\_\,v & \leq & \_\_\_\_ \\ & \_\_\_\_\,u & + & \_\_\_\_\,v & \leq & \_\_\_\_ \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

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7. The members of a hospital review board comprising a president, three vice–presidents and five department heads are to be selected from a group of three candidates for president, seven candidates for vice–president and ten candidates for department heads.

(a) [1] How many ways can the hospital review board be formed? Circle closest one.

(i) $C(1,3) \times C(3,7) \times C(5,10)$ (ii) $P(1,3) \times P(3,7) \times P(5,10)$
(iii) $P(3,1) \times P(7,3) \times P(10,5)$
(iv) $C(3,1) \times C(7,3) \times C(10,5)$ (v) $C(3,1) + C(7,3) + C(10,5)$

(b) [2] In how many ways can the hospital review board be formed, if one of the candidates for vice president is designated in charge of the Intensive Unit ward and one is designated in charge of the Mental Illness ward? Circle closest one.

(i) 157325 (ii) 158125 (iii) 158760 (iv) 159160 (v) 159250

8. Among 500 teenagers in a mental institution, 245 are schizophrenics, 190 have bipolar disorders and 105 are both schizophrenics and have bipolar disorders. What is the probability that a teenager, selected at random from this group,

(a) [1] has schizophrenia or a bipolar disorder or both? Circle closest one.

(i) $\frac{245+190+105}{500}$ (ii) $\frac{245-190-105}{500}$ (iii) $\frac{245-190-105}{500}$

(iv) $\frac{245+190-105}{500}$ (v) $\frac{-245+190-105}{500}$

(b) [1] exactly one of these two types of mental illness? Circle closest one.

(i) $\frac{100}{500}$ (ii) $\frac{155}{500}$ (iii) $\frac{215}{500}$ (iv) $\frac{225}{500}$ (v) $\frac{305}{500}$

(c) [1] neither schizophrenia nor a bipolar disorder? Circle closest one.

(i) 0.13 (ii) 0.22 (iii) 0.34 (iv) 0.42 (v) 0.48

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9. Consider the following payoff matrix of a 2×2 game.

$$\begin{bmatrix} 2 & -3 \\ -7 & 6 \end{bmatrix}$$

(a) [1] This (circle one) **is** / **is not** a strictly determined game. *If* it was a strictly determined game, then

it is said to be ____.

(b) [1] If the column player's mixed strategy is

$$\begin{bmatrix} 0.3 \\ 0.7 \end{bmatrix}$$

and the row player's mixed strategy is

$$\begin{bmatrix} 0.2 & 0.8 \end{bmatrix}$$

then the value of the game is
(circle closest one) **−2.34** / **−1.57** / **0.43** / **1.38** / **3.24**.

(c) [1] The optimal mixed strategy for the row player

is: (_______, _______)

10. Consider the following problems concerning the Normal distribution.

(a) [1] For $\mu = 32$ and $\sigma = 3$, $P(X < 28.3) =$
(circle closest one) **0.09** / **0.11** / **0.13** / **0.15** / **0.17**.

(b) [1] The IQ of depressed females is normally distributed with a mean of 95 and a standard deviation of 0.7. What is the IQ of a female at the 34th percentile? Circle closest one. **91.2** / **92.3** / **92.9** / **93.4** / **94.7**

1. (a) (i), (ii), (iii), (iv) (b) (iv) (c) (i)

2. (a) (iv), (v) (b) (iv) (c) -20, -28, 13

3. (a) $\frac{a}{1.3}$ (b) 153.8 mg

4. (a) 60, 12, 102; 19, 10, 90; −321, 12, 0;
   (b) (90/19,0), 28890/19

5. (a) (iii); (b) (i)

6. (a) 14, 11; 50, 120, 1200; 15, 12, 200;
   (b) 1200, 200; 50, 15, 14; 120, 12, 11.

7. (a) (iv); (b) (iii)

8. (a) (iv) (b) (iv) (c) (iii)

9. (a) **is not**, fair (b) **1.38** (c) $\left(\frac{13}{18}, \frac{5}{18}\right) = (0.73, 0.27)$

10. (a) **0.11** (b) **94.7**