

Final for Statistics 213
Probability and Decision Theory - Spring 2001
Material Covered: Chapters 1–9 of Workbook and text
For: 2nd May

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): _____ . ID Number: _____
last first

1. Consider the following system of equations.

$$\begin{array}{rclcl} x & + & 3y & + & z & = & 3 \\ 7x & + & 7y & + & 9z & = & 8 \\ x & + & 3y & + & 5z & = & 6 \end{array}$$

(a) [1 point] This system can also be described by $AX = B$, where

$$A = \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix}$$

and

$$X = \begin{bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{bmatrix}$$

and

$$B = \begin{bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{bmatrix}$$

(b) [1 point] The solution to this system of equations is

$$(x, y, z) = (\underline{\hspace{2cm}}, \underline{\hspace{2cm}}, \underline{\hspace{2cm}})$$

2. Infestation of crops by insects has long been of great concern to farmers and agricultural scientists. Below, is data on the age of a cotton plant (days), x , and percentage of damaged squares, y .

x	9	12	12	15	18	18	21	21	27	30	30	33
y	11	12	23	30	29	52	41	65	60	72	84	93

(a) [1 point] The least squares line is

(b) [1 point] The predicted percentage of damaged squares when the age is 20 days is (circle closest one) **23** / **32** / **37** / **40** / **46**.

3. [2 points] Westville insurance has a total of \$25 million earmarked for homeowner and business loans. The homeowner loans have a 10% annual rate of return, whereas the business loans yield a 12% annual rate of return. As a matter of policy, the total amount of homeowner loans will be greater than or equal to four times the total amount of business loans. *State* (do not work out!) the linear programming problem which would be used to determine the total amount of loans of each type in order to maximize returns.

4. Consider the following linear programming problem.

$$\begin{array}{rcll}
 \text{Minimize} & 4x & + & 2y & + & 6z & & \\
 \text{subject to} & 2x & + & 2y & + & 2z & \geq & 3 \\
 & 2x & + & y & + & 2z & \geq & 2 \\
 & 3x & + & 2y & + & z & \geq & 4 \\
 & x & & & & & \geq & 0 \\
 & & & y & & & \geq & 0 \\
 & & & & & z & \geq & 0
 \end{array}$$

(a) [1 point] Give the dual of this problem.

(b) [1 point] Give the initial simplex tableau of the dual.

u	v	w	x	y	z	P

(c) [1 point] Identify the pivot point in the initial simplex tableau and then pivot to the next simplex tableau.

u	v	w	x	y	z	P

(d) [1 point] The optimal solution to the primal problem is

$(x, y, z) =$ _____,

with $P =$ _____.

5. Try the following problems.

(a) [1 point] Find the sum of the even integers between and including 16 and 92.
Circle closest one. **2014** / **2106** / **2210** / **2290** / **2320**.

(b) [1 point] Determine the sum of the first eight terms of the sequence

$$3.1, 7.75, 19.375, 48.4375, \dots$$

Circle closest one. **3151.4** / **3168.3** / **3172.2** / **3183.8** / **3199.6**.

6. In how many ways can five married couples be arranged in a row of ten seats if:

(a) [1 point] each married couple is seated together?
Circle closest one. **120** / **240** / **720** / **3,840** / **3,628,800**.

(b) [1 point] the members of each gender are seated together?
Circle closest one. **4800** / **6200** / **8600** / **24000** / **28800**.

7. The registrar's office releases the following information concerning the majors of sophomores at Purdue University North Central.

	% of sophomores in this major	% of females	% of males
Liberal Arts	0.55	0.75	0.25
Education	0.25	0.85	0.15
Technology	0.10	0.25	0.75
Sciences	0.05	0.50	0.50
Other	0.05	0.65	0.35

(a) [1 point] The probability a student, selected at random, is a female is
(circle closest one) **0.60** / **0.65** / **0.71** / **0.75** / **0.79**.

(b) [1 point] Given that a technology student is selected at random, the chance this student is a female is
(circle closest one) **0.04** / **0.10** / **0.15** / **0.19** / **0.25**.

(c) [1 point] Given that a female is selected at random, the chance this student is majoring in technology is
(circle closest one) **0.04** / **0.10** / **0.15** / **0.19** / **0.25**.

10. A basketball player has a 65% chance of sinking a free throw. Use the appropriate normal distribution to approximate the resulting binomial distribution.

(a) [1 point] The probability of making at least 100 baskets in 150 free throws, is (circle closest one) **0.09** / **0.19** / **0.23** / **0.28** / **0.37**.

(b) [1 point] In 150 free throws, how many free throws does the basketball player make at the 77th percentile?
Circle closest one. **95** / **102** / **110** / **116** / **132**

(1) system of equations

(a) [1 point] This system can also be described by $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 7 & 7 & 9 \\ 1 & 3 & 5 \end{bmatrix}$$

and

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and

$$B = \begin{bmatrix} 3 \\ 8 \\ 6 \end{bmatrix}$$

(b) $(x, y, z) = \left(-\frac{6}{7}, \frac{29}{28}, \frac{3}{4}\right) = (-0.86, 1.034, 0.75)$

(2) (a) $\hat{y} \approx 3.28x - 19.67$ (b) 46

(3) Westville insurance

$$\begin{array}{rcllcl} \text{Maximize} & 0.10x & + & 0.12y & & \\ \text{subject to} & x & + & y & \leq & 25,000,000 \\ & x & - & 4y & \geq & 0 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(4) linear programming problem

(a) the dual of this problem.

$$\begin{array}{rcllcllcl} \text{Maximize} & 3u & + & 2v & + & 4w & & & \\ \text{subject to} & 2u & + & 2v & + & 3w & + & x & = & 4 \\ & 2u & + & v & + & 2w & & + & y & = & 2 \\ & 2u & + & 2v & + & w & & & + & z & = & 6 \\ & u & & & & & & & & & \geq & 0 \\ & & & v & & & & & & & \geq & 0 \\ & & & & & w & & & & & \geq & 0 \end{array}$$

(b) initial simplex tableau of the dual

u	v	w	x	y	z	P	
2	2	3	1	0	0	0	4
2	1	2	0	1	0	0	2
2	2	1	0	0	1	0	6
-3	-2	-4	0	0	0	1	0

(c) next simplex tableau.

u	v	w	x	y	z	P	
-1	0.5	0	1	-1.5	0	0	1
1	0.5	1	0	0.5	0	0	1
1	1.5	0	0	-0.5	1	0	5
1	0	0	0	2	0	1	4

(d) $(x, y, z) = (0, 2, 0)$, with $P = 4$.

(5) (a) **2106** (b) **3151.4**

(6) (a) **3,840** (b) **28800**

(7) (a) **0.71** (b) **0.25** (c) **0.04**

(8) (a) **33,500** (b) **5,723** (c) **13 to 7**

(9) (a) **-3** (b) **0.345** (c) **B**

(10) (a) **0.37** (b) **102**