Final for Statistics 213
Probability and Decision Theory - Spring 2002
Material Covered: Chapters 1–9 of Workbook and text
29th April

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this final. A calculator may also be used. No other aids are permitted.

Name (please print): ______________________ . **ID Number:** __________.
last first

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1. Consider the following questions on matrix manipulations.

(a) [1 point] Consider the following system of equations.

$$\begin{bmatrix} 3 & \text{x} \\ \text{y} & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$$

Then $(x, y) =$ (circle closest one)
$\mathbf{(\frac{1}{4}, 23)}$ / $\mathbf{(\frac{2}{4}, -10)}$ / $\mathbf{(\frac{3}{4}, -13)}$ / $\mathbf{(\frac{5}{4}, -23)}$ / $\mathbf{(\frac{7}{4}, -33)}$.

(b) [1 point] Consider the following system of equations

$$\begin{array}{rcrcrcr} 3x & - & 2y & + & 2z & = & 7 \\ 2x & + & 4y & + & 5z & = & 4 \\ x & + & 2y & - & z & = & 10 \end{array}$$

The solution to this system of equations is

$(x, y, z) =$ (______, ______, ______)

(c) [1 point] **True** / **False** For 2×2 matrix A and 2×2 matrix B,

$$(A + B)^2 = A^2 + 2AB + B^2$$

2. Solve the following linear programming problems using a graphical procedure. (Use your graphing calculator.)

(a) [1 point]

$$\begin{array}{lcccccc} \text{Maximize} & 3x & + & 4y & & \\ \text{subject to} & x & + & y & \leq & 4 \\ & 2x & + & y & \geq & 10 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

Either the solution is (circle at most one) **unbounded** / **inconsistent**

or $(x, y) =$ (_______, _______)

(b) [1 point]

$$\begin{array}{lcccccc} \text{Maximize} & 3x & + & 4y & & \\ \text{subject to} & x & + & y & \leq & 4 \\ & 2x & + & y & \leq & 10 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

Either the solution is (circle at most one) **unbounded** / **inconsistent**

or $(x, y) =$ (_______, _______)

(c) [1 point]

$$\begin{array}{lcccccc} \text{Maximize} & 3x & + & 4y & & \\ \text{subject to} & x & + & y & \geq & 4 \\ & 2x & + & y & \geq & 10 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

Either the solution is (circle at most one) **unbounded** / **inconsistent**

or $(x, y) =$ (_______, _______)

3. Consider the following linear programming problem.

$$\begin{array}{lcccccccc} \text{Maximize} & 4\text{x} & + & 2\text{y} & + & \text{z} & & \\ \text{subject to} & \text{x} & + & 3\text{y} & + & 2\text{z} & \leq & 3 \\ & 2\text{x} & + & 2\text{y} & + & 2\text{z} & \leq & 5 \\ & 3\text{x} & + & \text{y} & + & \text{z} & \leq & 4 \\ & \text{x} & & & & & \geq & 0 \\ & & & \text{y} & & & \geq & 0 \\ & & & & & \text{z} & \geq & 0 \end{array}$$

(a) [1 point] Give the initial simplex tableau.

x	y	z	u	v	w	P	
______	______	______	______	______	______	______	______
______	______	______	______	______	______	______	______
______	______	______	______	______	______	______	______
______	______	______	______	______	______	______	______

(b) [1 point] Identify the pivot point in the initial simplex tableau and then pivot to the next simplex tableau.

x	y	z	u	v	w	P	
______	______	______	______	______	______	______	______
______	______	______	______	______	______	______	______
______	______	______	______	______	______	______	______
______	______	______	______	______	______	______	______

(c) [1 point] The optimal solution is

$(x, y, z) =$ ____,

with $P =$ ____.

4. Purdue University North Central will match Professor Bee's contribution of \$150 into his retirement account at the end of each month for the next seventeen (17) years. This account earns interest at the rate of 7.5% per year compounded monthly.

(a) [1 point] Professor Bee will have, in seventeen (17) years, (circle closest one)
121,097.43 / **122,097.43** / **123,097.43** / **124,097.43** / **125,097.43**.

(b) [1 point] Professor Bee will have, in seventeen (17) years, *in present dollars*, (circle closest one)
34,533.99 / **34,633.99** / **34,733.99** / **34,833.99** / **34,933.99**.

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5. Try the following counting questions.

(a) [1 point] The number of ways that 3 students and 4 faculty members can sit in a row if the students must sit together at the head of the row is
(circle closest one) **144** / **288** / **432** / **576** / **720**.

(b) [1 point] The number of ways that 3 students and 4 faculty members can sit in a row if the students are separated by one faculty member is
(circle closest one) **144** / **288** / **432** / **576** / **720**.

(c) [1 point] The number of different arrangements that can be made from the letters PURDUE is
(circle closest one) **360** / **720** / **1080** / **1440** / **1800**.

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6. A STAT 213 student estimates she will achieve an A grade with an 80 percent chance if she studies 10 hours per week, with a 60 percent chance if she studies 7 hours per week and with a 40 percent chance if she studies 4 hours per week. She further believes that she will study 10 hours, 7 hours and 4 hours per week with probability 0.1, 0.2 and 0.7, respectively.

(a) [1 point] What is the chance she will achieve an A grade?
(circle closest one) **0.18** / **0.28** / **0.38** / **0.48** / **0.58**.

(b) [1 point] Given that she achieves an A grade, what is the chance she studied for four (4) hours?
(circle closest one) $\mathbf{\frac{6}{12}}$ / $\mathbf{\frac{7}{12}}$ / $\mathbf{\frac{8}{12}}$ / $\mathbf{\frac{9}{12}}$ / $\mathbf{\frac{10}{12}}$.

(c) [1 point] Given that she does *not* achieve an A grade, what is the chance she studied for four (4) hours?
(circle closest one) $\mathbf{\frac{17}{26}}$ / $\mathbf{\frac{18}{26}}$ / $\mathbf{\frac{19}{26}}$ / $\mathbf{\frac{20}{26}}$ / $\mathbf{\frac{21}{26}}$.

7. A small store owner buys loaves of bread at \$1.05 per loaf wholesale, and sells them for \$1.45 per loaf. His daily demand, X, is a binomial random variable where $n = 5$ and $p = \frac{2}{3}$.

(a) [1 point] The expected number of loaves, $E(X)$, sold per week is:
(circle closest one) $\mathbf{\frac{10}{3}}$ / $\mathbf{\frac{11}{3}}$ / $\mathbf{\frac{12}{3}}$ / $\mathbf{\frac{13}{3}}$ / $\mathbf{\frac{14}{3}}$.

(b) [1 point] Complete the following probability distribution of the number of loaves of bread, X, sold per day.

X	0	1	2	3	4	5
$P(X = x)$	____	____	____	____	____	____

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8. Consider the following questions on stochastic matrices.

(a) [1 point] Consider the following matrices.

$$A = \begin{bmatrix} 1 & 0.2 \\ 0 & 0.8 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0.7 \\ 1 & 0.3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.5 & 1 \\ 0.5 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0.3 & 0 \\ 0.7 & 1 \end{bmatrix}.$$

Which are regular stochastic matrices?
(Circle none, one or more) ***A*** / ***B*** / ***C*** / ***D*** / **none**.
(It is assumed you are *not* able to switch rows and columns.)

(b) [1 point] For matrix A, $S(I - R)^{-1} =$
(circle closest one) **0** / **0.2** / **0.6** / **0.8** / **1**.
(You can switch rows and columns if necessary.)

(c) [1 point] For matrix B, $S(I - R)^{-1} =$
(circle closest one) **0** / **0.3** / **0.5** / **0.7** / **1**.
(You can switch rows and columns if necessary.)

9. Consider the following three linear equations.

$$y = -\frac{2}{3}x - 3 \quad (1)$$

$$y = \frac{3}{2}x + 4 \quad (2)$$

$$3y = -2x - 9 \quad (3)$$

(a) [1 point] This system of linear equations has
(circle one) **no** / **a unique** / **two** / **three** / **many** solution(s).

(b) [1 point] Linear equation (1) intersects linear equation (2) at an angle of
(circle closest one) **30^o** / **45^o** / **60^o** / **80^o** / **90^o**.

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10. [1 point] Janice has a total of $100,000 to invest in either stocks (x), mutual funds (y) or treasury bills (z) at 15%, 10% and 5% return, respectively. Being a cautious person, she has decided to invest at most 10% of her investment in stocks ($x \leq 0.10(x+y+z)$), at least 35% in mutual funds and at least 40% in treasury bills. *State* (do not work out!) the linear programming problem which would be used to determine the total amount of investment of each type in order to maximize Janice's return. (The x, y, z should all be on the left side of the inequalities in your answer!)

(1) (a) $(\mathbf{\frac{5}{4}, -23})$ (b) $(x, y, z) = (135/28, 81/56, -16/7) = (4.82, 1.45, -2.29)$ (c) **False**

(2) (a) **inconsistent** (b) $\boldsymbol{(x, y) = (0, 4)}$ (c) **unbounded**

(3) simplex tableau problem

(a) initial simplex tableau.

x	y	z	u	v	w	P	
1	3	2	1	0	0	0	3
2	2	2	0	1	0	0	5
3	1	1	0	0	1	0	4
-4	-2	-1	0	0	0	1	0

(b) next simplex tableau.

x	y	z	u	v	w	P	
1	3	2	1	0	0	0	3
0	-4	-2	-2	1	0	0	-1
0	-8	-5	-3	0	1	0	-5
0	10	7	4	0	0	1	12

(c) $(x, y, z) = (3, 0, 0)$ with $P = 12$

(4) (a) $\mathbf{123,097.43}$ (b) $\mathbf{34,533.99}$

(5) (a) **144** (b) **432** (c) **360**

(6) (a) **0.48** (b) $\mathbf{\frac{7}{12}}$ (c) $\mathbf{\frac{21}{26}}$

(7) binomial distribution

(a) $\mathbf{\frac{10}{3}}$

(b) probability distribution

X	0	1	2	3	4	5
$P(X = x)$	1/243	10/243	40/243	80/243	80/243	32/243
	0.004	0.041	0.164	0.329	0.329	0.132

(c) **4**

(8) (a) $\boldsymbol{B}$, $\boldsymbol{C}$ (b) **1** (c) **1**

(9) (a) **a unique** (b) $\mathbf{90^o}$

(10) linear programming problem statement

Maximize	0.15x	+	0.10y	+	0.05z		
subject to	x	+	y	+	z	$\leq$	100,000
	0.90x	-	0.10y	-	0.10z	$\leq$	0
	-0.35x	+	0.65y	-	0.35z	$\geq$	0
	-0.40x	-	0.40y	+	0.60z	$\geq$	0
	x					$\geq$	0
			y			$\geq$	0
					z	$\geq$	0