Chapter 4

Linear Programming: A Graphical Approach

We will look at how to solve linear programming problems with two variables using a graphical approach. We begin by looking at how to set up a linear programming problem with two or more variables. Then we look at how to interpret systems of linear inequalities with two variables in a graphical way. Finally, we look at how to solve linear programming problems with two variables.

4.1 Modeling Linear Programming Problems

Exercise 4.1 (Modeling Linear Programming Problems)

1. *Acme's Party Service*[1]. Acme's Party service gives birthday parties for \$30 and Halloween parties for \$40. Birthday party requires 25 balloons and 20 noisemakers, while Halloween party requires 15 balloons and 10 noisemakers. Only 250 balloons and 175 noisemakers are available. Acme would like to schedule x birthday parties and y Halloween parties to make as much money as possible.

   (a) If Acme gives 4 birthday parties and 3 Halloween parties, it makes $P = \$30 \times 4 + \$40 \times 3 =$ **200** / **240** / **300** dollars

   (b) If Acme gives 10 birthday parties and 10 Halloween parties, it makes $P = \$30 \times 10 + \$40 \times 10 =$ **500** / **600** / **700** dollars

   (c) If Acme gives x birthday parties and y Halloween parties, it makes

      i. $P = x + y$ dollars

      ii. $P = 30x + 40y$ dollars

   This is the *objective function*.

[1]Spence and Vanden Eynden, 4, p 124, 1990.

(d) Since Acme needs 25 balloons for each of x birthday parties and 15 balloons for each of y Halloween parties and, furthermore, total number of balloons cannot exceed 250, this means

i. $25x + 15y \geq 250$
ii. $15x + 25y < 250$
iii. $25x + 15y \leq 250$
iv. $25x + 15y < 250$

This is a *structural constraint.*

(e) Also, Acme needs 20 noisemakers for each of x birthday party and 10 noisemakers for each y Halloween party and total number of noisemakers cannot exceed 175, this means

i. $20x + 10y \leq 175$
ii. $10x + 10y < 175$
iii. $20x + 10y > 175$
iv. $10x + 20y \leq 175$

This is another *structural constraint.*

(f) Since impossible to have less than zero birthday parties or zero Halloween parties, this means

i. $x < 0$ and $y \geq 0$
ii. $x \geq 0$ and $y < 0$
iii. $x \leq 0$ and $y \leq 0$
iv. $x \geq 0$ and $y \geq 0$

These are *nonnegativity constraints.*

(g) *Linear programming (LP) problem* for this situation is

i. LP Candidate 1

$$\begin{array}{lrcrcr} \text{Maximize} & \text{x} & + & \text{y} & & \\ \text{subject to} & \text{25x} & + & \text{15y} & \leq & 250 \\ & \text{20x} & + & \text{10y} & \leq & 175 \\ & \text{x} & & & \geq & 0 \\ & & & \text{y} & \geq & 0 \end{array}$$

ii. LP Candidate 2

$$\begin{array}{lrcrcr} \text{Minimize} & \text{30x} & + & \text{40y} & & \\ \text{subject to} & \text{25x} & + & \text{15y} & \leq & 250 \\ & \text{20x} & + & \text{10y} & \leq & 175 \\ & \text{x} & & & \geq & 0 \\ & & & \text{y} & \geq & 0 \end{array}$$

iii. LP Candidate 3

$$\begin{array}{lcccccc} \text{Maximize} & 30x & + & 40y & & \\ \text{subject to} & 25x & + & 15y & \leq & 250 \\ & 20x & + & 10y & \leq & 175 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

2. *Trout and Perch*[2] Lake is stocked with trout and perch, which feed on two types of food (A and B) that grow in lake at rates of 6000 units and 2400 units per day, respectively. Each trout consumes 6 units of first food and 2 units of second food per day, and each perch consumes 8 units of first food and 4 units of second food per day. Average trout weighs 1 pound and average perch weighs 1.5 pounds. Lake is stocked so total weight of fish in lake is as great as possible.

   (a) If there are 4 trout and 3 perch, total weight is $W = 1 \times 4 + 1.5 \times 3 =$ **6.5** / **7.5** / **8.5** pounds

   (b) If lake is stocked with x trout and y perch, total weight of fish is

      i. $W = 6000x + 2400y$ pounds
      ii. $W = 6x + 8y$ pounds
      iii. $W = 6x + 1.5y$ pounds
      iv. $W = x + 1.5y$ pounds

   (c) Since each of x trout consume 6 units of food A per day and each of y perch consume 8 units of food A per day and total amount of food A available is 6000 units per day,

      i. $6x + 8y \leq 6000$
      ii. $6x + 8y \geq 6000$
      iii. $8x + 6y \geq 6000$
      iv. $8x + 6y \leq 6000$

   (d) Since each of x trout consume 2 units of food B per day and each of y perch consume 4 units of food B per day and total amount of food B available is 2400 units per day,

      i. $4x + 2y \leq 2400$
      ii. $4x + 4y \leq 2400$
      iii. $2x + 4y \geq 2400$
      iv. $2x + 4y \leq 2400$

   (e) Since impossible to have less than zero trout or zero perch,

      i. $x < 0$ and $y \geq 0$
      ii. $x \geq 0$ and $y < 0$

[2]Grosnick, Chapter 3 Exam, March 1996.

iii. $x \leq 0$ and $y \leq 0$
iv. $x \geq 0$ and $y \geq 0$

(f) LP problem for this situation is

i. LP Candidate 1

$$\begin{array}{lrcrcr} \text{Maximize} & x & + & 1.5y & & \\ \text{subject to} & 6x & + & 8y & \leq & 6000 \\ & 2x & + & 4y & \leq & 2400 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

ii. LP Candidate 2

$$\begin{array}{lrcrcr} \text{Maximize} & x & + & 1.5y & & \\ \text{subject to} & 6x & + & 8y & \geq & 6000 \\ & 2x & + & 4y & \leq & 2400 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

iii. LP Candidate 3

$$\begin{array}{lrcrcr} \text{Maximize} & x & + & 1.5y & & \\ \text{subject to} & 6x & + & 8y & \leq & 2400 \\ & 2x & + & 4y & \leq & 6000 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

3. *Insurance.* Westville insurance has a total of \$25 million earmarked for homeowner and business loans. Homeowner loans have a 10% annual rate of return, whereas business loans yield a 12% annual rate of return. As a matter of policy, total amount of homeowner loans will be greater than or equal to four times total amount of business loans. State linear programming problem to determine total amount of loans of each type in order to maximize returns.

(a) For x dollar amount of homeowner loans and y dollar amount of business loans, total return is

i. $R = 25x + y$ dollars
ii. $R = 0.12x + 0.10y$ dollars
iii. $R = 0.10x + 0.12y$ dollars
iv. $R = 0.90x + 0.88y$ dollars

(b) Since Westville insurance has \$25 million earmarked for x homeowner and y business loans,

i. $x + y \geq 25,000,000$
ii. $x + y \leq 25,000,000$
iii. $x - y \leq 25,000,000$

iv. $x - y \geq 25,000,000$

(c) Since x homeowner loans is greater than or equal to four times y business loans, $x \geq 4y$,

i. $x - 4y \leq 0$

ii. $x - 4y \geq 0$

iii. $x + 4y \geq 0.10$

iv. $x + 4y \leq 0.12$

(d) Since impossible to have less than zero homeowner or business loans,

i. $x < 0$ and $y \geq 0$

ii. $x \geq 0$ and $y < 0$

iii. $x \leq 0$ and $y \leq 0$

iv. $x \geq 0$ and $y \geq 0$

(e) LP problem for this situation is

i. LP Candidate 1

$$\begin{array}{lccccc} \text{Maximize} & x & - & y & & \\ \text{subject to} & x & + & y & \leq & 0.10 \\ & x & - & 4y & \leq & 0.12 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

ii. LP Candidate 2

$$\begin{array}{lccccc} \text{Minimize} & 0.10x & + & 0.12y & & \\ \text{subject to} & x & + & y & \leq & 25{,}000{,}000 \\ & x & - & 4y & \leq & 0 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

iii. LP Candidate 3

$$\begin{array}{lccccc} \text{Maximize} & 0.10x & + & 0.12y & & \\ \text{subject to} & x & + & y & \leq & 25{,}000{,}000 \\ & x & - & 4y & \geq & 0 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

4. *Loudspeaker Systems*[3] Shipping costs per loudspeaker system in dollars from two plants of Zoom Loudspeaker Company to three warehouses is given below.

	Warehouse		
Plant	A	B	C
I	20	8	10
II	12	22	18

[3]Tan, Example 4, p 187, 1997.

Maximum output of plants I and II are 400 and 600 units per month. In order for warehouses A, B and C to meet orders, they need to be shipped, at a *minimum*, 200, 300 and 400 units per month respectively. Letting x_1 represent number of loudspeaker systems shipped from plant I to warehouse A and x_2 represent number of loudspeaker systems shipped from plant I to warehouse B, and so on. Information summarized in following table.

	Warehouse			
Plant	A	B	C	max plant output
I	x_1	x_2	x_3	400
II	x_4	x_5	x_6	600
min warehouse require	200	300	400	

What is shipping schedule to minimize shipping costs?

(a) Total shipping costs of sending units from plants to warehouses is

   i. $C = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$ dollars
   ii. $C = 400x_1 + 600x_2 + 200x_3 + 300x_4 + 400x_5 + 400x_6$ dollars
   iii. $C = 10x_1 + 8x_2 + 10x_3 - 12x_4 + 22x_5 - 18x_6$ dollars
   iv. $C = 20x_1 + 8x_2 + 10x_3 + 12x_4 + 22x_5 + 18x_6$ dollars

(b) Since plant I can only produce 400 units per month,

   i. $x_4 + x_5 + x_6 \leq 400$
   ii. $x_1 + x_2 + x_3 \geq 400$
   iii. $x_1 + x_2 \leq 400$
   iv. $x_1 + x_2 + x_3 \leq 400$

(c) Since plant II can only produce 600 units per month,

   i. $x_4 + x_5 + x_6 \leq 600$
   ii. $x_1 + x_2 + x_3 \geq 600$
   iii. $x_1 + x_2 \leq 600$
   iv. $x_1 + x_2 + x_3 \leq 600$

(d) Since warehouse A must be shipped *minimum* of 200 units per month,

   i. $x_2 + x_4 \geq 200$
   ii. $x_2 + x_5 \geq 200$
   iii. $x_1 + x_4 \leq 200$
   iv. $x_1 + x_4 \geq 200$

(e) Since warehouse B must be shipped minimum of 300 units per month,

   i. $x_2 + x_4 \geq 300$
   ii. $x_2 + x_5 \geq 300$

iii. $x_3 + x_6 \geq 300$
iv. $x_1 + x_4 \geq 300$

(f) Since warehouse C must be shipped minimum of 400 units per month,

i. $x_2 + x_4 \geq 400$
ii. $x_2 + x_5 \geq 400$
iii. $x_3 + x_6 \geq 400$
iv. $x_1 + x_4 \geq 400$

(g) Since impossible to send less than zero loudspeakers from any plant to any warehouse

i. $x_1 \leq 0,\ x_2 \leq 0,\ x_3 \leq 0,\ x_4 \leq 0,\ x_5 \leq 0,\ x_6 \leq 0,$
ii. $x_1 \geq 0,\ x_2 \leq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0,$
iii. $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0,$
iv. $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \leq 0,\ x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0,$

(h) Consequently, LP problem for this situation is

i. LP Candidate 1

$$\begin{array}{lcccccccccccccc}
\text{Maximize} & 20x_1 & + & 8x_2 & + & 10x_3 & + & 12x_4 & + & 22x_5 & + & 18x_6 & & \\
\text{subject to} & x_1 & + & x_2 & + & x_3 & & & & & & & \leq & 6000 \\
 & & & & & & & x_4 & + & x_5 & + & x_6 & \leq & 400 \\
 & x_1 & & & + & & & x_4 & & & & & \geq & 200 \\
 & & & x_2 & & & & & + & x_5 & & & \geq & 300 \\
 & & & & & x_3 & & & & & + & x_6 & \geq & 400 \\
 & x_1 \geq 0, & & & & x_2 \geq 0, & & x_3 \geq 0, & x_4 \geq 0, & & x_5 \geq 0, & & x_6 \geq 0 &
\end{array}$$

ii. LP Candidate 2

$$\begin{array}{lcccccccccccccc}
\text{Minimize} & 20x_1 & + & 8x_2 & + & 10x_3 & + & 12x_4 & + & 22x_5 & + & 18x_6 & & \\
\text{subject to} & x_1 & + & x_2 & + & x_3 & & & & & & & \leq & 400 \\
 & & & & & & & x_4 & + & x_5 & + & x_6 & \leq & 600 \\
 & x_1 & & & + & & & x_4 & & & & & \geq & 200 \\
 & & & x_2 & & & & & + & x_5 & & & \geq & 300 \\
 & & & & & x_3 & & & & & + & x_6 & \geq & 400 \\
 & x_1 \geq 0, & & & & x_2 \geq 0, & & x_3 \geq 0, & x_4 \geq 0, & & x_5 \geq 0, & & x_6 \geq 0 &
\end{array}$$

iii. LP Candidate 3

$$\begin{array}{lcccccccccccccc}
\text{Maximize} & 20x_1 & + & 8x_2 & + & 10x_3 & + & 12x_4 & + & 22x_5 & + & 18x_6 & & \\
\text{subject to} & x_1 & + & x_2 & + & x_3 & & & & & & & \leq & 600 \\
 & & & & & & & x_4 & + & x_5 & + & x_6 & \leq & 400 \\
 & x_1 & & & + & & & x_4 & & & & & \leq & 200 \\
 & & & x_2 & & & & & + & x_5 & & & \leq & 300 \\
 & & & & & x_3 & & & & & + & x_6 & \leq & 400 \\
 & x_1 \geq 0, & & & & x_2 \geq 0, & & x_3 \geq 0, & x_4 \geq 0, & & x_5 \geq 0, & & x_6 \geq 0 &
\end{array}$$

4.2 Linear Inequalities in Two Variables

We look at graphical interpretation of inequalities.

Exercise 4.2 (Linear Inequalities in Two Variables)

1. *Which way to shade: a first look.*

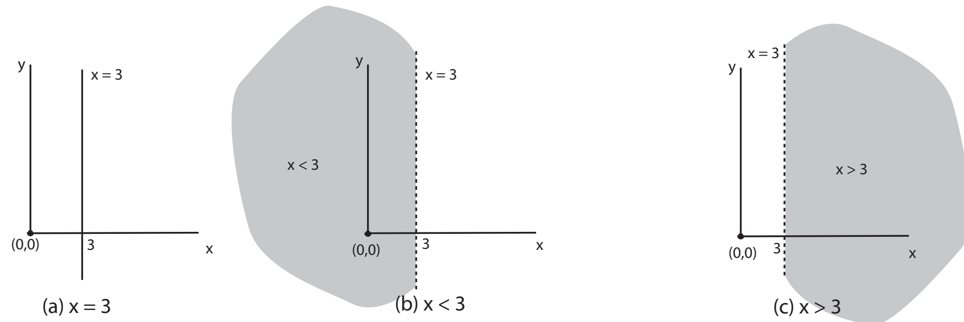


Figure 4.1 (Inequalities related to $x = 3$)

(a) Line $x = 3$ is a **horizontal** / **vertical** line. See figure (a).

(b) Inequality $x < 3$ is region **left** / **right** of line $x = 3$. See figure (b).
Point $(x, y) = (0, 0)$ left of $x = 3$, **satisfies** / **violates** inequality $x = 0 < 3$.

(c) Inequality $x > 3$ is region **left** / **right** of line $x = 3$. See figure (c).
Point $(x, y) = (0, 0)$ left of $x = 3$, **satisfies** / **violates** inequality $x > 3$.

(d) *Using A Test Point To Identify Inequality Region.* **True** / **False**

  i. Draw (graph) *equality* part of inequality.

  ii. Determine if test point (often (0,0)) satisfies inequality or not.

  iii. If test point *does* satisfy inequality, point must be *inside* inequality region and so shade from *equality towards* test point. If test point violates inequality, point must be outside inequality region and so shade from equality *away* from test point.

2. *Which way to shade: a second look.*

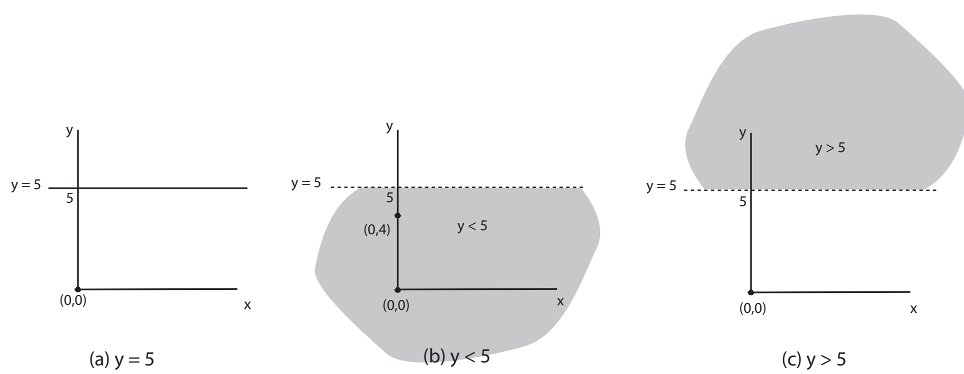


Figure 4.2 (Inequalities related to $y = 5$)

(a) Line $y = 5$ is a **horizontal** / **vertical** line. See figure (a).
Press Y=, clear everything. Type 5 beside Y_1 =, then WINDOW -10 10 1 0 10 1 1, then GRAPH

(b) Point $(x, y) = (0, 0)$ below $y = 5$, **satisfies** / **violates** inequality $y = 0 < 5$.
So inequality $y < 5$ is region **below** / **above** line $y = 5$. See figure (b).
Press Y=, choose "shade down" symbol to *left* of Y_1, then GRAPH

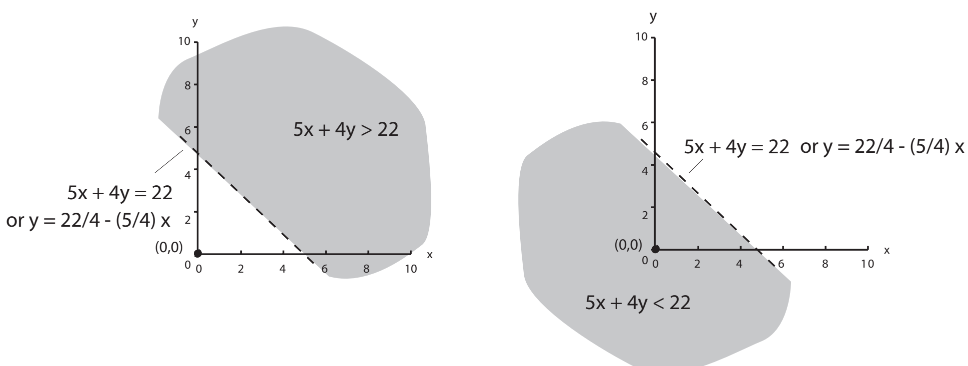
(c) Point $(x, y) = (0, 4)$ below $y = 5$, **satisfies** / **violates** inequality $y = 4 < 5$.
So inequality $y < 5$ is region **below** / **above** line $y = 5$. See figure (b).
Comment: Any test point like $(0, 4)$ can be used, although point $(0, 0)$ often easiest to use.

(d) Point $(x, y) = (0, 0)$ below $y = 5$, **satisfies** / **violates** inequality $y > 5$.
So inequality $y > 5$ is region **below** / **above** line $y = 5$. See figure (c).
Press Y=, choose "shade up" symbol to *left* of Y_1, then GRAPH

(e) Inequality $y \geq 5$ is region
below / **below and equal** / **above** / **above and equal** to line $y = 5$.

(f) Inequality $y \leq 5$ is region
below / **below and equal** / **above** / **above and equal** to line $y = 5$.

3. *Which way to shade: another example.*



(a) 5x + 4y > 22
or y > 22/4 - (5/4) x

(b) 5x + 4y < 22
or y < 22/4 - (5/4) x

Figure 4.3 (Inequalities related to $5x + 4y = 22$ or equivalently $y = \frac{22}{4} - \frac{5}{4}x$)

(a) The line $5x + 4y = 22$
intersects y-axis at $y = \mathbf{\frac{22}{4}}$ / $\mathbf{\frac{4}{22}}$ / $\mathbf{\frac{22}{5}}$. Hint: when $x = 0$, $5(0) + 4y = 22$, so $y = ?$
and intersects x-axis at $x = \mathbf{\frac{22}{4}}$ / $\mathbf{\frac{4}{22}}$ / $\mathbf{\frac{22}{5}}$. Hint: when $y = 0$, $5x + 4(0) = 22$, so $x = ?$

(b) Point $(x, y) = (0, 0)$, below $5x + 4y = 22$,
satisfies / **violates** inequality $5x + 4y > 22$ because $5(0) + 4(0) = 0$.
So inequality $5x + 4y > 22$ is region, excluding $(0, 0)$,
below / **above** line $5x + 4y = 22$. See figure (a).

(c) Point $(x, y) = (0, 0)$, below $5x + 4y = 22$,
satisfies / **violates** inequality $5(0) + 4(0) = 0 < 22$.
So inequality $5x + 4y < 22$ is region, including $(0, 0)$,
below / **above** line $5x + 4y = 22$. See figure (b).

4. *Region* $5x + 4y \geq 22$ *and* $3x + 7y \leq 27$.

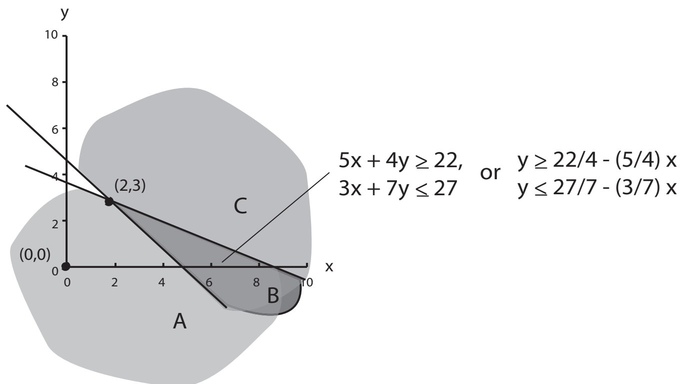


Figure 4.4 (Region $5x + 4y \geq 22$ and $3x + 7y \leq 27$)

(a) Point $(x, y) = (0, 0)$, below $5x + 4y = 22$,
satisfies / **violates** inequality $5x + 4y \geq 22$ because $5(0) + 4(0) = 0$.
So inequality $5x + 4y \geq 22$ is region, excluding $(0, 0)$,
above / **above and equal to** line $5x + 4y = 22$. See figure.

(b) Point $(x, y) = (0, 0)$, below $3x + 7y = 27$,
satisfies / **violates** inequality $3(0) + 7(0) = 0 \leq 27$.
So inequality $3x + 7y \leq 27$ is region, including $(0, 0)$,
below / **below and equal to** line $3x + 7y = 27$. See figure.

(c) Inequality $5x + 4y \geq 22$ and $3x + 7y \leq 27$ is region **A** / **B** / **C** in figure.

(d) Region B is

i. feasible and bounded
ii. feasible and unbounded
iii. unfeasible and bounded
iv. unfeasible and unbounded

(e) Intersection point of equalities, $5x + 4y = 22$ and $3x + 7y = 27$, is
(1, 3) / **(0, 3)** / **(2, 3)** / **(3, 2)**.
Let $Y_1 = \frac{22}{4} - \frac{5}{4}X$ and $Y_2 = \frac{27}{7} - \frac{3}{7}X$, then GRAPH, then 2nd CALC intersect.

5. *Region* $5x + 4y \geq 22$, $3x + 7y \leq 27$, $y \geq 0$ *and* $x \geq 0$.

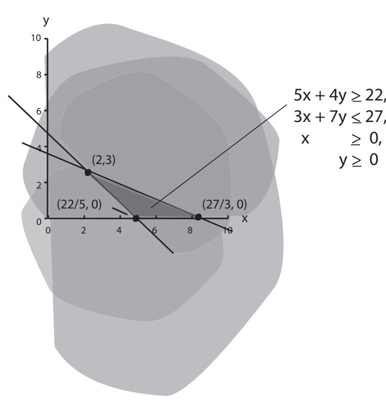


Figure 4.5 (Region $5x + 4y \geq 22$, $3x + 7y \leq 27$, $y \geq 0$ and $x \geq 0$)

(a) Region $5x + 4y \geq 22$, $3x + 7y \leq 27$, $y \geq 0$ and $x \geq 0$ is
  i. feasible and bounded
  ii. feasible and unbounded
  iii. unfeasible and bounded
  iv. unfeasible and unbounded

(b) The three vertices of feasible bounded region are
  i. (2,3), ($\frac{27}{3}$,1) and ($\frac{22}{5}$,0)
  ii. (2,3), ($\frac{27}{3}$,0) and ($\frac{22}{5}$,0)
  iii. (2,3), (0,$\frac{27}{3}$) and ($\frac{22}{5}$,0)
  iv. (2,3), (5,1) and ($\frac{22}{5}$,0)

(c) One (unnecessary) inequality *not* bounding the feasible region is
  i. $5x + 4y \geq 22$
  ii. $3x + 7y \leq 27$
  iii. $y \geq 0$
  iv. $x \geq 0$

6. Choose system of linear inequalities which describes shaded region.

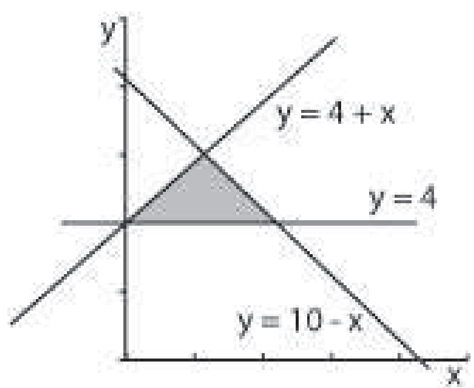


Figure 4.6 (Linear inequalities and shaded region)

(a) system A
$y \geq 4 + x$
$y \geq 4$
$y \leq 10 - x$

(b) system B
$y \leq 4 + x$
$y \geq 4$
$y \geq 10 - x$

(c) system C
$y \leq 4 + x$
$y \geq 4$
$y \leq 10 - x$

7. Choose system of linear inequalities which describes shaded region.

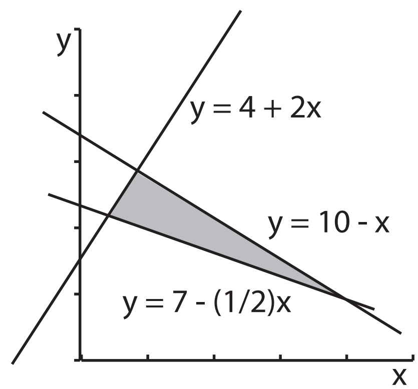


Figure 4.7 (Linear inequalities and shaded region)

(a) system A
$y \leq 4 + 2x$
$y \leq 10 - x$
$y \geq 7 - \frac{1}{2}x$

(b) system B
$y \leq 4 + 2x$
$y \geq 10 - x$
$y \geq 7 - \frac{1}{2}x$

(c) system C
$y \geq 4 + 2x$
$y \leq 10 - x$
$y \leq 7 - \frac{1}{2}x$

8. Choose system of linear inequalities which describes shaded region.

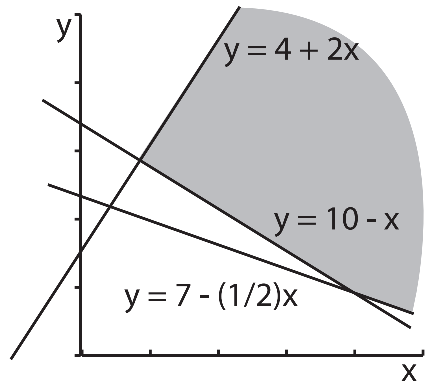


Figure 4.8 (Linear inequalities and shaded region)

(a) system A
$y \leq 4 + 2x$
$y \leq 10 - x$
$y \geq 7 - \frac{1}{2}x$

(b) system B
$y \leq 4 + 2x$
$y \geq 10 - x$
$y \geq 7 - \frac{1}{2}x$

(c) system C
$y \geq 4 + 2x$
$y \leq 10 - x$
$y \leq 7 - \frac{1}{2}x$

4.3 Solving Linear Programming Problems Graphically

Graphical method of solving linear programming problems involves identifying all possible vertices of feasible set (corner points), calculating value of objective function at these points, then choosing point which optimizes objective function.

Exercise 4.3 (Solving Linear Programming Problems Graphically)

1. *Erasers and pen kits.* L&R company sells Sally kits at 6 dollars a piece and Tommy kits at 4 dollars a piece. Sally kit requires 1 eraser and 2 pens, while Tommy kit requires 2 erasers and 1 pen. Only 8 erasers and 10 pen are left. How many Sally kits (x) and Tommy kits (y) should be sold to maximize profit?

$$\begin{array}{lrcrcr} \text{Maximize} & 6x & + & 4y & & \\ \text{subject to} & x & + & 2y & \le & 8 \\ & 2x & + & y & \le & 10 \\ & x & & & \ge & 0 \\ & & & y & \ge & 0 \end{array}$$

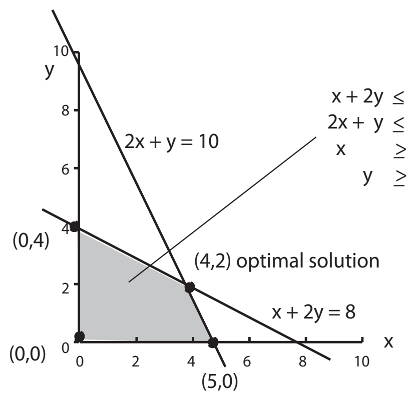


Figure 4.9 (Solving Linear Programming Problems Graphically)

Determine y-intercept corner point: when $x = 0$, $x + 2y = 0 + 2y = 8$, so $y = 4$

and x-intercept corner point: when $y = 0$, $2x + y = 2x + 0 = 10$, so $x = 5$

and then: GRAPH $Y_1 = 4 - (1/2)X$ and $Y_2 = 10 - 2x$ using WINDOW 0 10 1 0 10 1 1 and find intersection (corner) point using GRAPH TRACE, 2nd CALC intersect.

(a) Complete table: calculate value of objective function at four corners.

corner	$R = 6x + 4y$
(0,0)	$R = 6(0) + 4(0) = 0$
(5,0)	$R = 6(5) + 4(0) = 30$
(4,2)	$R = 6(4) + 4(2) = 32$
(0,4)	$R = 6(0) + 4(4) =$ ______

(b) Optimal (maximal) solution is at $(x, y) =$ $\mathbf{(0,4)}$ / $\mathbf{(5,0)}$ / $\mathbf{(4,2)}$ where value of objective function is a *maximum* of **0** / **16** / **30** / **32**.

2. *Acme's party service.* Acme would like to schedule x birthday parties and y Halloween parties to make as much money as possible, taking into account limited number of balloons and noisemakers as given in following LP.

$$\begin{array}{lrcrcr} \text{Maximize} & x & + & y & & \\ \text{subject to} & 25x & + & 15y & \le & 250 \\ & 20x & + & 10y & \le & 175 \\ & x & & & \ge & 0 \\ & & & y & \ge & 0 \end{array}$$

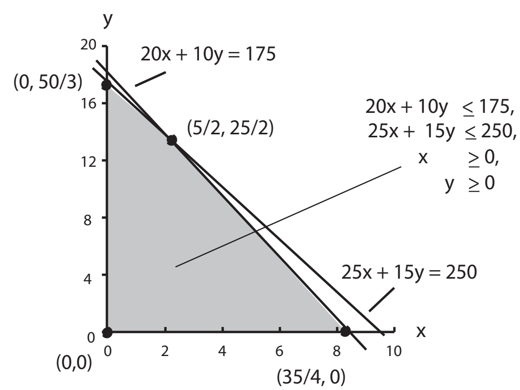


Figure 4.10 (Solving System of Linear Inequalities Graphically)

Determine x-intercept and y-intercept corner points, then
GRAPH $\mathrm{Y}_1 = 250/15 - (25/15)X$ and $\mathrm{Y}_2 = 175/10 - (20/10)x$ using WINDOW 0 10 1 0 20 1 1,
find intersection points using GRAPH TRACE, 2nd CALC intersect.

(a) Complete following table.

vertex	$z = x + y$
(0,0)	$z = 0 + 0 =$ _______
$(0, \frac{50}{3})$	$z = 0 + \frac{50}{3} =$ _______
$(\frac{5}{2}, \frac{25}{2})$	$z = \frac{5}{2} + \frac{25}{2} =$ _______
$(\frac{35}{4}, 0)$	$z =$ ______________

In STAT EDIT, type x coordinates into L_1, y coordinates into L_2, then define $L_3 = L_1 + L_2$.

(b) Optimal (maximal) solution $(x, y) = \mathbf{(0,0)}$ / $\mathbf{(\frac{35}{4}, 0)}$ / $\mathbf{(\frac{5}{2}, \frac{25}{2})}$ / $\mathbf{(0, \frac{50}{3})}$
where value of objective function is a *maximum* of **0** / **8.75** / **15** / **16.7**.

(c) *If* objection function *minimum* (rather than *maximum*),
optimal solution $(x, y) = \mathbf{(0,0)}$ / $\mathbf{(\frac{35}{4}, 0)}$ / $\mathbf{(\frac{5}{2}, \frac{25}{2})}$ / $\mathbf{(0, \frac{50}{3})}$
where value of objective function is a *minimum* of **0** / **8.75** / **15** / **16.7**.

(d) **True** / **False** Any LP problem with nonempty bounded feasible region R and objective function f has both minimum at corner of R and maximum at (possible same, most likely different) corner of R.

3. *Edge (rather than corner) of feasible region.*

$$\begin{array}{lcccccc} \text{Maximize} & 6\text{x} & + & 3\text{y} & & \\ \text{subject to} & \text{x} & + & 2\text{y} & \leq & 8 \\ & 2\text{x} & + & \text{y} & \leq & 10 \\ & \text{x} & & & \geq & 0 \\ & & & \text{y} & \geq & 0 \end{array}$$

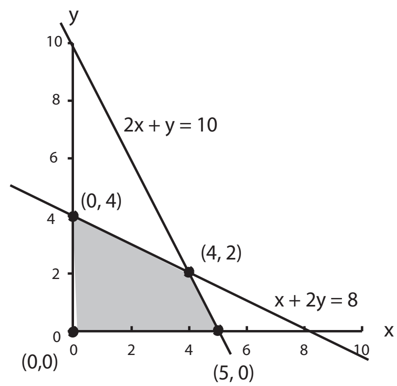


Figure 4.11 (Optimal Solution: Edge of Feasible Region)

GRAPH $Y_1 = 4 - (1/2)X$ and $Y_2 = 10 - 2x$ using WINDOW 0 10 1 0 10 1 1
and find intersection points using GRAPH TRACE, 2nd CALC intersect.

(a) Complete table.

corner	$z = 6x + 3y$
(0, 0)	$z =$ ______
(0, 4)	$z =$ ______
(4, 2)	$z =$ ______
(5, 0)	$z =$ ______

In STAT EDIT, type x coordinates into L_1, y coordinates into L_2, then define $L_3 = 6L_1 + 3L_2$.

(b) Optimal solution is

i. either corner $(x, y) = (4, 2)$ or $(x, y) = (5, 0)$

ii. edge of points between and including $(x, y) = (4, 2)$ and $(x, y) = (5, 0)$

where objective function has value **0** / **12** / **30**.

(c) *If* objection function is $z = 5x + 4y$ instead of $z = 6x + 3y$,
optimal (maximal) solution is at $(x, y) =$ $\mathbf{(0, 0)}$ / $\mathbf{(0, 4)}$ / $\mathbf{(4, 2)}$ / $\mathbf{(5, 0)}$
where objective function has value **16** / **28** / **30**.

4. *Another problem.*

$$
\begin{array}{lcccccc}
\text{Minimize} & 3\text{x} & + & 2\text{y} & & \\
\text{subject to} & -\frac{3}{2}\text{x} & + & \text{y} & \leq & \frac{5}{2} \\
 & \frac{2}{3}\text{x} & + & \text{y} & \leq & 9 \\
 & \frac{3}{2}\text{x} & + & \text{y} & \geq & \frac{11}{2} \\
 & -\frac{4}{3}\text{x} & + & \text{y} & \geq & -3 \\
 & \text{x} & & & \geq & 0 \\
 & & & \text{y} & \geq & 0
\end{array}
$$

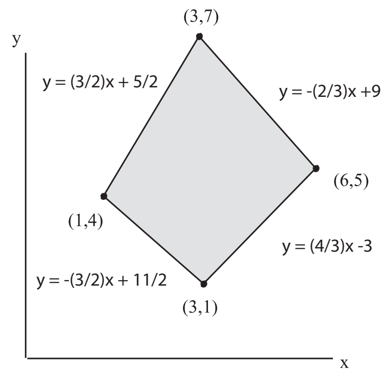


Figure 4.12 (Another graphical linear programming problem)

(a) Complete table.

corner	$z = 3x + 2y$
(1, 4)	$z =$ ______
(3, 7)	$z =$ ______
(6, 5)	$z =$ ______
(3, 1)	$z =$ ______

In STAT EDIT, type x coordinates into L_1, y coordinates into L_2, then define $L_3 = 3L_1 + 2L_2$.

(b) Optimal (*minimal*) solution is

i. $(x, y) = (1, 4)$
ii. $(x, y) = (3, 1)$
iii. either corner $(x, y) = (1, 4)$ or $(x, y) = (3, 1)$
iv. edge of points between and including $(x, y) = (1, 4)$ and $(x, y) = (3, 1)$

where objective function has value **0** / **11** / **28**.

5. *And another problem.*

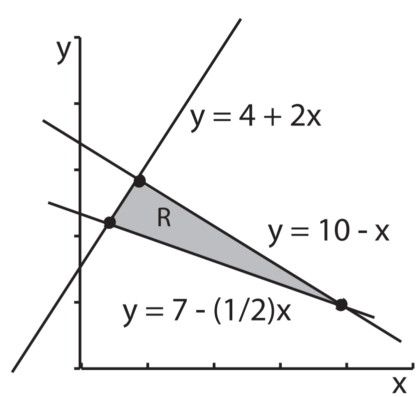


Figure 4.13 (Another graphical linear programming problem)

GRAPH $Y_1 = 4 + 2X$, $Y_2 = 10 - X$ and $Y_3 = 7 - 0.5X$, then ZOOM ZoomFit ENTER TRACE, find intersection points using 2nd CALC intersect.

(a) Corner points are (choose three)
(1.2, 6.4) / (2, 8) / (6, 4) / (7, 5) / (5, 10)

(b) Corner point which minimizes $Z = x - 5y$ on feasible set R is

i. point (1.2, 6.4) with value -30.8.
ii. point (2,8) with value -38.
iii. point (6,4) with value -14.
iv. point (7,5) with value -18.
v. point (5,10) with value -45.

In STAT EDIT, type x coordinates into L_1, y coordinates into L_2, then define $L_3 = L_1 - 5L_2$.

6. *Last problem.*

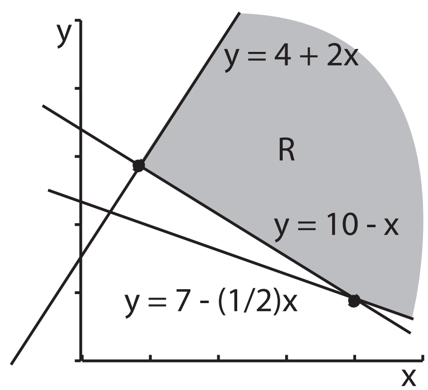


Figure 4.14 (Another graphical linear programming problem)

GRAPH $Y_1 = 4 + 2X$, $Y_2 = 10 - X$ and $Y_3 = 7 - 0.5X$, then ZOOM ZoomFit ENTER TRACE, find intersection points using 2nd CALC intersect.

(a) Corner points are (choose two)
(1.2, 6.4) / (2, 8) / (6, 4) / (7, 5) / (5, 10)

(b) Corner point which minimizes $Z = 4x + 3y$ on feasible set R is

i. point (1.2, 6.4) with value 24.
ii. point (2,8) with value 32.
iii. point (6,4) with value 36.
iv. point (7,5) with value 43.
v. point (5,10) with value 50.

In STAT EDIT, type x coordinates into L_1, y coordinates into L_2, then define $L_3 = 4L_1 + 3L_2$.

(c) **True** / **False**. There is no *maximum* since feasible set R is unbounded. Optimal solution does not occur at a corner point in this case. Optimal maximum and minimum solutions *always* occur for bounded regions but do not necessarily occur for unbounded regions.