## 5.3 Nonstandard and Minimization Problems

Nonstandard problem is any linear programming programming problem which is *not* standard maximum problem. Minimization problem is an example of a nonstandard problem. Nonstandard problem is converted into maximum (not *standard* maximum) problem. Maximum problem satisfies conditions,

- objective function linear and is maximized,
- variables all nonnegative,
- structural constraints all of form $ax + by + \cdots \leq c$,

where, notice, $c$ in structural constraints is *not* necessarily $c \geq 0$, as required by *standard* maximum problem. In other words, $c$ can be *negative*. Maximization problem solved, if a solution exists, using a variety of tools, including *Crown's rules*,

- If there is a row above horizontal bar with only *one* negative entry in right hand side column, stop because no feasible (and so optimal) solution.
- Locate first negative number from top in right hand side column, then select most negative number on that row to left. Column of this number is pivot column.
- Divide right hand side numbers by pivot column numbers if either both are negative numbers or both are positive numbers.
  - If both positive, choose pivot row with *smallest* quotient.
  - If *no* both-positive, use both-negative, choose pivot row with *largest* positive quotient.
- Pivot this way until no negative entries left in right hand side above horizontal column. Then proceed with usual pivoting procedure.

**Exercise 5.3 (Nonstandard and Minimization Problems)**

1. Convert nonstandard into (not necessarily standard) maximization problems.

   (a) Nonstandard problem

$$\begin{array}{lcccccc} \text{Maximize} & x & + & y & & \\ \text{subject to} & 4x & + & 3y & \leq & 20 \\ & -x & + & 3y & \geq & 3 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

transforms to maximum problem by multiplying second constraint by $-1$:

i. maximum problem A

$$\begin{array}{lrcrcr} \text{Maximize} & x & + & y & & \\ \text{subject to} & 4x & + & 3y & \leq & 20 \\ & x & - & 3y & \geq & -3 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

ii. maximum problem B

$$\begin{array}{lrcrcr} \text{Maximize} & x & + & y & & \\ \text{subject to} & 4x & + & 3y & \leq & 20 \\ & x & - & 3y & \leq & -3 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(b) Nonstandard problem

$$\begin{array}{lrcrcr} \text{Minimize} & 8x & + & 10y & & \\ \text{subject to} & x & + & 2y & \geq & 6 \\ & 2x & + & y & \geq & 4 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

transforms to maximum problem by maximizing *negative* of objective function and by multiplying first and second constraints by $-1$:

i. maximum problem A

$$\begin{array}{lrcrcr} \text{Maximize} & -8x & - & 10y & & \\ \text{subject to} & -x & - & 2y & \leq & -6 \\ & -2x & - & y & \leq & -4 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

ii. maximum problem B

$$\begin{array}{lrcrcr} \text{Maximize} & 8x & + & 10y & & \\ \text{subject to} & -x & - & 2y & \leq & -6 \\ & -2x & - & y & \leq & -4 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(c) Nonstandard problem

$$
\begin{array}{rrcrcrcr}
\text{Minimize} & 4x & + & 2y & + & 6z & & \\
\text{subject to} & 2x & + & 2y & + & 2z & \geq & 3 \\
 & 2x & + & y & + & 2z & \geq & 2 \\
 & 3x & + & 2y & + & z & \geq & 4 \\
 & x & & & & & \geq & 0 \\
 & & & y & & & \geq & 0 \\
 & & & & & z & \geq & 0
\end{array}
$$

is equivalent to maximum problem

i. maximum problem A

$$
\begin{array}{rrcrcrcr}
\text{Maximize} & -4x & - & 2y & - & 6z & & \\
\text{subject to} & -2x & - & 2y & - & 2z & \geq & -3 \\
 & -2x & - & y & - & 2z & \geq & -2 \\
 & -3x & - & 2y & - & z & \geq & -4 \\
 & x & & & & & \geq & 0 \\
 & & & y & & & \geq & 0 \\
 & & & & & z & \geq & 0
\end{array}
$$

ii. maximum problem B

$$
\begin{array}{rrcrcrcr}
\text{Maximize} & -4x & - & 2y & - & 6z & & \\
\text{subject to} & -2x & - & 2y & - & 2z & \leq & -3 \\
 & -2x & - & y & - & 2z & \leq & -2 \\
 & -3x & - & 2y & - & z & \leq & -4 \\
 & x & & & & & \geq & 0 \\
 & & & y & & & \geq & 0 \\
 & & & & & z & \geq & 0
\end{array}
$$

(d) Nonstandard problem

$$
\begin{array}{rrcrcr}
\text{Minimize} & -2x & + & y & & \\
\text{subject to} & x & - & y & \leq & 1.5 \\
 & 2x & + & 3y & \leq & 6 \\
 & x & + & y & \geq & 1 \\
 & x & & & \geq & 0 \\
 & & & y & \geq & 0
\end{array}
$$

is equivalent to maximum problem

i. maximum problem A

$$
\begin{array}{rrcrcr}
\text{Maximize} & 2x & - & y & & \\
\text{subject to} & x & - & y & \leq & 1.5 \\
 & 2x & + & 3y & \leq & 6 \\
 & -x & - & y & \leq & 1 \\
 & x & & & \geq & 0 \\
 & & & y & \geq & 0
\end{array}
$$

ii. maximum problem B

$$\begin{array}{lrcrcr}
\text{Maximize} & 2x & - & y & & \\
\text{subject to} & x & - & y & \leq & 1.5 \\
 & 2x & + & 3y & \leq & 6 \\
 & -x & - & y & \leq & -1 \\
 & x & & & \geq & 0 \\
 & & & y & \geq & 0
\end{array}$$

(e) Nonstandard problem

$$\begin{array}{lccccccccccccc}
\text{Minimize} & 20x_1 & + & 8x_2 & + & 10x_3 & + & 12x_4 & + & 22x_5 & + & 18x_6 & & \\
\text{subject to} & x_1 & + & x_2 & + & x_3 & & & & & & & \leq & 400 \\
 & & & & & & & x_4 & + & x_5 & + & x_6 & \leq & 600 \\
 & x_1 & & & + & & & x_4 & & & & & \geq & 200 \\
 & & & x_2 & & & & & + & x_5 & & & \geq & 300 \\
 & & & & & x_3 & & & & & + & x_6 & \geq & 400 \\
 & x_1 \geq 0, & & & & x_2 \geq 0, & & x_3 \geq 0, & x_4 \geq 0, & & x_5 \geq 0, & & x_6 \geq 0 &
\end{array}$$

is equivalent to maximum problem

i. maximum problem A

$$\begin{array}{lccccccccccccc}
\text{Maximize} & -20x_1 & - & 8x_2 & - & 10x_3 & - & 12x_4 & - & 22x_5 & - & 18x_6 & & \\
\text{subject to} & x_1 & + & x_2 & + & x_3 & & & & & & & \leq & 400 \\
 & & & & & & & x_4 & + & x_5 & + & x_6 & \leq & 600 \\
 & -x_1 & & & - & & & x_4 & & & & & \geq & 200 \\
 & & & -x_2 & & & & & - & x_5 & & & \geq & 300 \\
 & & & & & -x_3 & & & & & - & x_6 & \geq & 400 \\
 & x_1 \geq 0, & & & & x_2 \geq 0, & & x_3 \geq 0, & x_4 \geq 0, & & x_5 \geq 0, & & x_6 \geq 0 &
\end{array}$$

ii. maximum problem B

$$\begin{array}{lccccccccccccc}
\text{Maximize} & -20x_1 & - & 8x_2 & - & 10x_3 & - & 12x_4 & - & 22x_5 & - & 18x_6 & & \\
\text{subject to} & x_1 & + & x_2 & + & x_3 & & & & & & & \leq & 400 \\
 & & & & & & & x_4 & + & x_5 & + & x_6 & \leq & 600 \\
 & -x_1 & & & - & & & x_4 & & & & & \leq & -200 \\
 & & & -x_2 & & & & & - & x_5 & & & \leq & -300 \\
 & & & & & -x_3 & & & & & - & x_6 & \leq & -400 \\
 & x_1 \geq 0, & & & & x_2 \geq 0, & & x_3 \geq 0, & x_4 \geq 0, & & x_5 \geq 0, & & x_6 \geq 0 &
\end{array}$$

(f) Nonstandard problem

$$\begin{array}{lrcrcr}
\text{Minimize} & x & - & 5y & & \\
\text{subject to} & -2x & + & y & \leq & 4 \\
 & x & + & y & \leq & 10 \\
 & 0.5x & + & y & \geq & 7 \\
 & x & & & \geq & 0 \\
 & & & y & \geq & 0
\end{array}$$

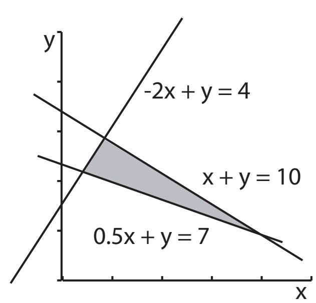

Figure 5.9 (Feasible region)

is equivalent to maximum problem

i. maximum problem A

$$\begin{array}{lrcrcr} \text{Maximize} & \text{-x} & + & \text{5y} & & \\ \text{subject to} & \text{-2x} & + & \text{y} & \leq & 4 \\ & \text{x} & + & \text{y} & \leq & 10 \\ & \text{-0.5x} & - & \text{y} & \leq & -7 \\ & \text{x} & & & \geq & 0 \\ & & & \text{y} & \geq & 0 \end{array}$$

ii. maximum problem B

$$\begin{array}{lrcrcr} \text{Minimize} & \text{x} & - & \text{5y} & & \\ \text{subject to} & \text{-2x} & + & \text{y} & \leq & 4 \\ & \text{x} & + & \text{y} & \leq & 10 \\ & \text{-0.5x} & - & \text{y} & \leq & -7 \\ & \text{x} & & & \geq & 0 \\ & & & \text{y} & \geq & 0 \end{array}$$

(g) Nonstandard problem

$$\begin{array}{lrcrcr} \text{Minimize} & \text{x} & + & \text{5y} & & \\ \text{subject to} & \text{-2x} & + & \text{y} & \leq & 4 \\ & \text{x} & + & \text{y} & = & 10 \\ & \text{0.5x} & + & \text{y} & \geq & 7 \\ & \text{x} & & & \geq & 0 \\ & & & \text{y} & \geq & 0 \end{array}$$

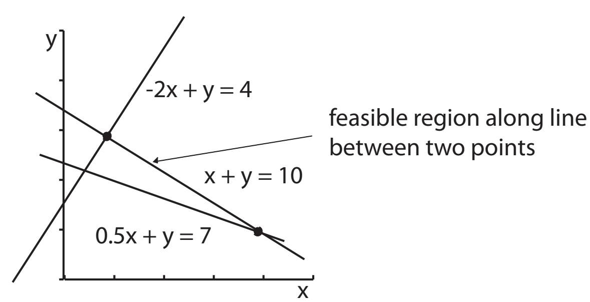


Figure 5.10 (Feasible region)

is equivalent to, since $x + y = 10$ or $y = 10 - x$,

$$\begin{array}{lrcrcr} \text{Maximize} & \text{-x} & - & \text{5(10 - x)} & & \\ \text{subject to} & \text{-2x} & + & \text{(10 - x)} & \leq & 4 \\ & \text{-0.5x} & - & \text{(10 - x)} & \leq & -7 \\ & \text{x} & & & \geq & 0 \\ & & & \text{(10 - x)} & \geq & 0 \end{array}$$

in other words maximization problem

i. maximum problem A

$$\begin{array}{lcccc} \text{Maximize} & 4x & - & 50 & & \\ \text{subject to} & -3x & & & \leq & -6 \\ & 0.5x & & & \leq & 3 \\ & x & & & \geq & 0 \\ & x & & & \leq & 10 \end{array}$$

ii. maximum problem B

$$\begin{array}{lcccc} \text{Maximize} & -6x & - & 50 & & \\ \text{subject to} & -x & & & \leq & 14 \\ & -1.5x & & & \leq & -17 \\ & x & & & \geq & 10 \\ & x & & & \geq & 0 \end{array}$$

2. *Solve nonstandard problem with Crown's rules.*

$$\begin{array}{lcccccc} \text{Maximize} & x & + & y & & \\ \text{subject to} & 4x & + & 3y & \leq & 20 \\ & -x & + & 3y & \geq & 3 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

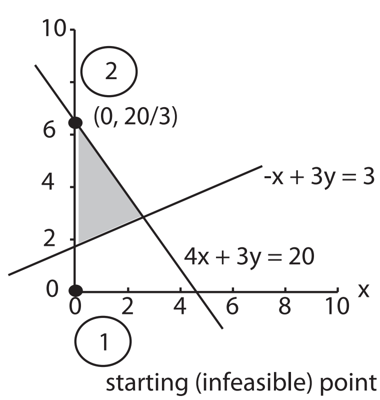


Figure 5.11 (Nonstandard LP problem)

(a) *Geometric solution.*

| corner | $f = x + y$ |
|---|---|
| $(0, 1)$ | $f = 0 + 1 = 1$ |
| $\left(\frac{17}{5}, \frac{32}{15}\right)$ | $f = \frac{17(3)}{15} + \frac{32}{15} =$ ______ |
| $\left(0, \frac{20}{3}\right)$ | $f = 0 + \frac{20}{3} =$ ______ |

Objective function $f$ maximum at $(x, y) = \mathbf{(0, 0)}$ / $\left(\mathbf{\frac{17}{5}, \frac{32}{15}}\right)$ / $\left(\mathbf{0, \frac{20}{3}}\right)$

(b) *Starting infeasible point.* Nonstandard problem transformed to maximum problem by multiplying second constraint by $-1$:

| | | | | | |
|---|---|---|---|---|---|
| Maximize | x | + | y | | |
| subject to | 4x | + | 3y | ≤ | 20 |
| | x | - | 3y | ≤ | -3 |
| | x | | | ≥ | 0 |
| | | | y | ≥ | 0 |

which has initial simplex tableau

| $x$ | $y$ | $s_1$ | $s_2$ | $f$ | | |
|---|---|---|---|---|---|---|
| 4 | 3 | 1 | 0 | 0 | 20 | Quotient ______ |
| 1 | -3 | 0 | 1 | 0 | -3 | Quotient ______ |
| -1 | -1 | 0 | 0 | 1 | 0 | |

Type this 3 by 6 table into MATRIX [A].

Use Crown's rules since -3 above horizontal line in right hand side (rhs).
Pivot column, most negative number ($-3$) on $-3$ rhs row, column $\boldsymbol{x}$ / $\boldsymbol{y}$
Pivot row, only both-positive nonnegative quotient, so row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$
So pivot element: $\mathbf{-3}$ / $\mathbf{1}$ / $\mathbf{3}$

(c) *First and last corner point.*

$\frac{1}{3}R_1 \to R_1, \quad R_2+3R_1 \to R_2, \quad R_3+R_1 \to R_3 \longrightarrow$

| $x$ | $y$ | $s_1$ | $s_2$ | $f$ | |
|---|---|---|---|---|---|
| $\frac{4}{3}$ | 1 | $\frac{1}{3}$ | 0 | 0 | $\frac{20}{3}$ |
| 5 | 0 | 1 | 1 | 0 | 17 |
| $\frac{1}{3}$ | 0 | $\frac{1}{3}$ | 0 | 1 | $\frac{20}{3}$ |

2nd MATRIX MATH *row(1/3,[A],1) STO 2nd MATRIX [B]

2nd MATRIX MATH *row+(3,[B],1,2) STO 2nd MATRIX [C]

2nd MATRIX MATH *row+(1,[C],1,3) STO 2nd MATRIX [D] then MATH ENTER for fractions.

Use regular pivot rules since no negatives above horizontal line in rhs
Point $(x, y) = \left(\mathbf{0, \frac{20}{3}}\right)$ / $\left(\mathbf{\frac{23}{6}, \frac{17}{2}}\right)$ / $\left(\mathbf{\frac{17}{2}, \frac{23}{6}}\right)$
with optimal (maximum) $f = \mathbf{0}$ / $\mathbf{\frac{20}{3}}$ / $\mathbf{\frac{37}{3}}$
because all indicators **nonpositive** / **zero** / **nonnegative**

3. *Another nonstandard problem solved with Crown's rules.*

| | | | | | |
|---|---|---|---|---|---|
| Minimize | -2x | + | y | | |
| subject to | x | - | y | ≤ | 1.5 |
| | 2x | + | 3y | ≤ | 6 |
| | x | + | y | ≥ | 1 |
| | x | | | ≥ | 0 |
| | | | y | ≥ | 0 |

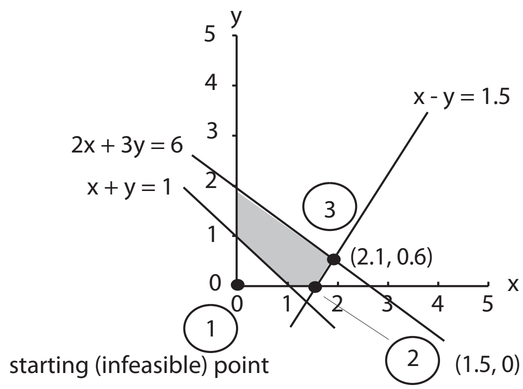


Figure 5.12 (Another nonstandard LP problem)

(a) *Starting infeasible point.* Nonstandard problem transformed to maximum problem by maximizing $-f$ and multiplying third constraint by $-1$:

| | | | | | | |
|---|---|---|---|---|---|---|
| Maximize | 2x | - | y | | |
| subject to | x | - | y | $\leq$ | 1.5 |
| | 2x | + | 3y | $\leq$ | 6 |
| | -x | - | y | $\leq$ | -1 |
| | x | | | $\geq$ | 0 |
| | | | y | $\geq$ | 0 |

which has initial simplex tableau

| $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $f$ | | |
|---|---|---|---|---|---|---|---|
| 1 | -1 | 1 | 0 | 0 | 0 | 1.5 | Quotient ______ |
| 2 | 3 | 0 | 1 | 0 | 0 | 6 | Quotient ______ |
| -1 | -1 | 0 | 0 | 1 | 0 | -1 | Quotient ______ |
| -2 | 1 | 0 | 0 | 0 | 1 | 0 | |

Type this 4 by 7 table into MATRIX [A].

Use Crown's rules since -1 above horizontal line in right hand side (rhs).

Pivot column, either $-1$ on $-1$ rhs row, column $\boldsymbol{x}$ / $\boldsymbol{y}$ / **either** $\boldsymbol{x, y}$

Since either $x$ or $y$ possible, assume $x$ column is pivot column.

Pivot row, smallest both-positive nonnegative quotient, row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$ / $\boldsymbol{R_3}$

So pivot element: $\mathbf{-1}$ / $\mathbf{1}$ / $\mathbf{2}$

(b) *First corner point.*

$$\xrightarrow{R_1 \to R_1, \quad R_2 - 2R_1 \to R_2, \quad R_3 + R_1 \to R_3, \quad R_4 + 2R_1 \to R_4}$$

| $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $f$ | | |
|---|---|---|---|---|---|---|---|
| 1 | -1 | 1 | 0 | 0 | 0 | 1.5 | Quotient ______ |
| 0 | 5 | -2 | 1 | 0 | 0 | 3 | Quotient ______ |
| 0 | -2 | 1 | 0 | 1 | 0 | 0.5 | Quotient ______ |
| 0 | -1 | 2 | 0 | 0 | 1 | 3 | |

2nd MATRIX MATH *row+(-2,[A],1,2) STO 2nd MATRIX [B]

2nd MATRIX MATH *row+(1,[B],1,3) STO 2nd MATRIX [C]

2nd MATRIX MATH *row+(2,[C],1,4) STO 2nd MATRIX [D].

Use regular pivot rules since no negatives above horizontal line in rhs
Pivot column, most negative indicator, is column $\boldsymbol{x}$ / $\boldsymbol{y}$ / $\boldsymbol{s_1}$ / $\boldsymbol{s_2}$
Pivot row, smallest *nonnegative* quotient, is row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$ / $\boldsymbol{R_3}$
So pivot element: $\mathbf{-2}$ / $\mathbf{-1}$ / $\mathbf{5}$

(c) *Last corner point.*

$$\xrightarrow{\frac{1}{5}R_2 \to R_2, \quad R_1+R_2\to R_1, \ R_3+2R_2\to R_3, \ R_4+R_2\to R_4}$$

| $x$ | $y$ | $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|---|---|
| 1 | 0 | 0.6 | 0.2 | 0 | 0 | 2.1 |
| 0 | 1 | -0.4 | 0.2 | 0 | 0 | 0.6 |
| 0 | 0 | 0.2 | 0.4 | 1 | 0 | 1.7 |
| 0 | 0 | 1.6 | 0.2 | 0 | 1 | 3.6 |

2nd MATRIX MATH *row(1/5,[D],2) STO 2nd MATRIX [E]

2nd MATRIX MATH*row+(1,[E],2,1) STO 2nd MATRIX [F]

2nd MATRIX MATH *row+(2,[F],2,3) STO 2nd MATRIX [G]

2nd MATRIX MATH *row+(1,[G],2,4) STO 2nd MATRIX [H].

Point $(x, y) = \mathbf{(0.6, 2.1)}$ / $\mathbf{(2.1, 0.6)}$ / $\mathbf{(0.6, 1.7)}$
with optimal (maximum) $-f = -(3.6) = \mathbf{-3.6}$ / $\mathbf{3.6}$ / $\mathbf{4}$
minimum of $f$ is equivalent to maximum of $-f$
because all indicators **nonpositive** / **zero** / **nonnegative**

4. *Another nonstandard problem solved with Crown's rules.*

$$\begin{array}{lcccccc} \text{Minimize} & 8x & + & 10y & & \\ \text{subject to} & x & + & 2y & \geq & 6 \\ & 2x & + & y & \geq & 4 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

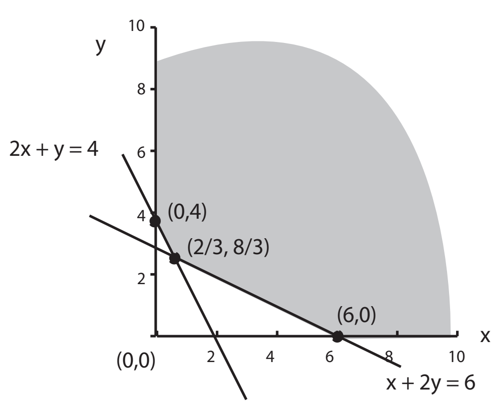


Figure 5.13 (Another nonlinear problem)

(a) *Starting infeasible point.* Nonstandard problem transformed to maximum problem by maximizing $-f$ and multiplying both constraints by $-1$:

| | | | | | |
|---|---|---|---|---|---|
| Maximize | -8x | - | 10y | | |
| subject to | -x | - | 2y | $\leq$ | -6 |
| | -2x | - | y | $\leq$ | -4 |
| | x | | | $\geq$ | 0 |
| | | | y | $\geq$ | 0 |

which has initial simplex tableau

| $x$ | $y$ | $s_1$ | $s_2$ | $f$ | | |
|---|---|---|---|---|---|---|
| -1 | -2 | 1 | 0 | 0 | -6 | Quotient ______ |
| -2 | -1 | 0 | 1 | 0 | -4 | Quotient ______ |
| 8 | 10 | 0 | 0 | 1 | 0 | |

Type this 3 by 6 table into MATRIX [A].

Use Crown's rules since -4,-6 above horizontal line in right hand side (rhs).
Pivot column, -2 most negative on row of topmost rhs -6, so column $\boldsymbol{x}$ / $\boldsymbol{y}$
Pivot row, *largest* both-negative nonnegative quotient, row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$
So pivot element: $\mathbf{-10}$ / $\mathbf{-2}$ / $\mathbf{-1}$

(b) *First corner point.*

$$-R_2 \to R_2, \quad R_1 + 2R_2 \to R_1, \quad R_3 - 10R_1 \to R_3 \quad \longrightarrow$$

| $x$ | $y$ | $s_1$ | $s_2$ | $f$ | | |
|---|---|---|---|---|---|---|
| 3 | 0 | 1 | -2 | 0 | 2 | Quotient ______ |
| 2 | 1 | 0 | -1 | 0 | 4 | Quotient ______ |
| -12 | 0 | 0 | 10 | 1 | -40 | |

2nd MATRIX MATH *row(-1,[A],2) STO 2nd MATRIX [B]

2nd MATRIX MATH *row+(2,[B],2,1) STO 2nd MATRIX [C]

2nd MATRIX MATH *row+(-10,[C],2,3) STO 2nd MATRIX [D].

Use regular pivot rules since no negatives above horizontal line in rhs
Pivot column, most negative indicator, is column $\boldsymbol{x}$ / $\boldsymbol{y}$ / $\boldsymbol{s_1}$ / $\boldsymbol{s_2}$
Pivot row, smallest *nonnegative* quotient, is row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$ / $\boldsymbol{R_3}$
So pivot element: **1** / **2** / **3**

(c) *Last corner point.*

$$\tfrac{1}{3}R_1 \to R_1, \quad R_2 - 2R_1 \to R_2, \quad R_3 + 12R_1 \to R_3 \quad \longrightarrow$$

| $x$ | $y$ | $s_1$ | $s_2$ | $f$ | |
|---|---|---|---|---|---|
| 1 | 0 | $\frac{1}{3}$ | $-\frac{2}{3}$ | 0 | $\frac{2}{3}$ |
| 0 | 1 | $-\frac{2}{3}$ | $\frac{1}{3}$ | 0 | $\frac{8}{3}$ |
| 0 | 0 | 4 | 2 | 1 | -32 |

2nd MATRIX MATH *row(1/3,[D],1) STO 2nd MATRIX [E]

2nd MATRIX MATH*row+(-2,[E],1,2) STO 2nd MATRIX [F]

2nd MATRIX MATH *row+(12,[F],1,3) STO 2nd MATRIX [G] then MATH ENTER for fractions.

Point $(x, y) = \left(\mathbf{\frac{2}{3}, \frac{8}{3}}\right)$ / $\left(\mathbf{\frac{8}{3}, \frac{2}{3}}\right)$

with optimal (maximum) $-f = -(-32) = \mathbf{-32}$ / **32**

minimum of $f$ is equivalent to maximum of $-f$

because all indicators **nonpositive** / **zero** / **nonnegative**

5. *Nonstandard problem with no solution using Crown's rules.*

| | | | | | |
|---|---|---|---|---|---|
| Maximize | 8x | - | 10y | | |
| subject to | x | + | 2y | $\leq$ | 6 |
| | 2x | + | y | $\leq$ | -4 |
| | x | | | $\geq$ | 0 |
| | | | y | $\geq$ | 0 |

or, equivalently,

| $x$ | $y$ | $s_1$ | $s_2$ | $f$ | |
|---|---|---|---|---|---|
| 1 | 2 | 1 | 0 | 0 | 6 |
| 2 | 1 | 0 | 1 | 0 | -4 |
| -8 | 10 | 0 | 0 | 1 | 0 |

**True** / **False**. No feasible (and so optimal solution) because, according to Crown's rules, only one negative, $-4$, in second row in rhs. On one hand, $x \geq 0$ and $y \geq 0$, but, on other hand, $2x + y \leq -4$. This is not possible: for example, $2(0) + 0 = 0 > -4$. Any larger $(x, y)$ is even more contradictory.

6. *Solve nonlinear problem using EXCEL.*

| | | | | | |
|---|---|---|---|---|---|
| Maximize | x | + | y | | |
| subject to | 4x | + | 3y | $\leq$ | 20 |
| | -x | + | 3y | $\geq$ | 3 |
| | x | | | $\geq$ | 0 |
| | | | y | $\geq$ | 0 |

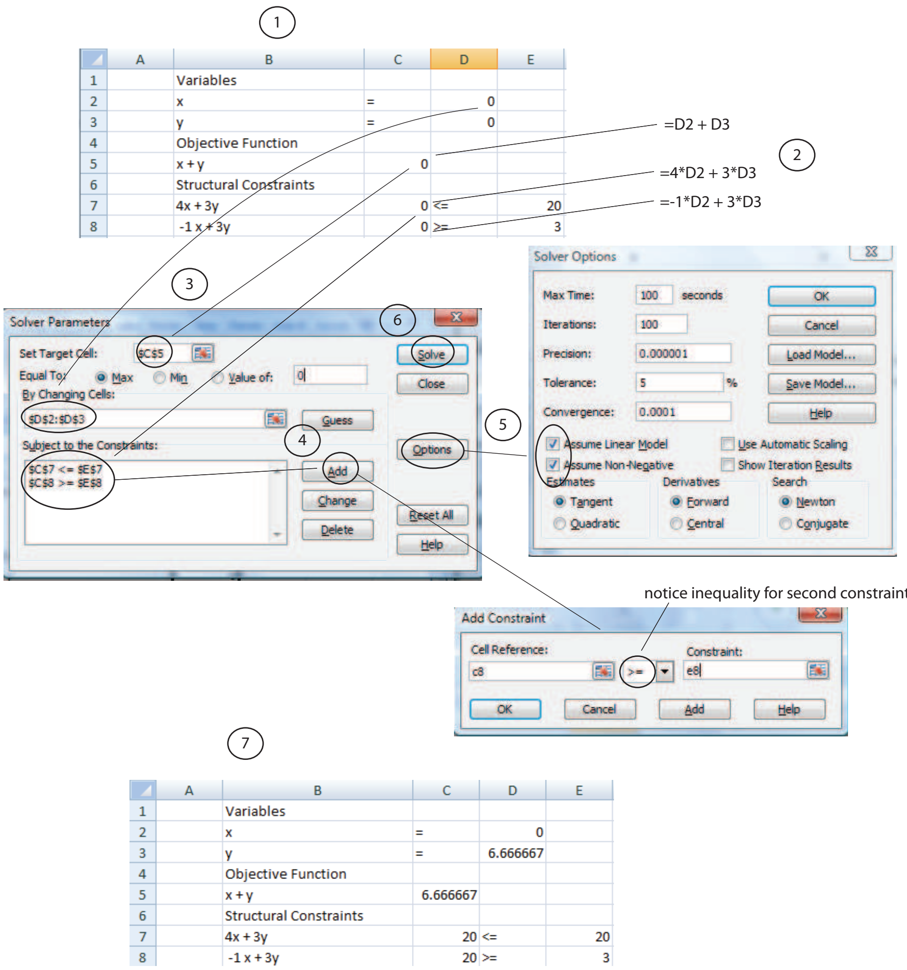


Figure 5.14 (Solving nonstandard LP problem using EXCEL)

Following steps in figure,
Point $(x, y) = \left(\mathbf{0}, \frac{\mathbf{20}}{\mathbf{3}}\right) \;/\; \left(\frac{\mathbf{20}}{\mathbf{3}}, \mathbf{0}\right)$
with optimal (maximum) $f = \frac{\mathbf{20}}{\mathbf{3}} \;/\; \frac{\mathbf{23}}{\mathbf{3}}$

## 5.4 The Simplex Method: Standard Minimization Problems

Read differently, final simplex tableau contains solution to not only maximization problem, but also minimization problem as well. In fact, maximization problem and minimization problem are *dual* to one another. More specifically, if original (or *primal*) problem is maximization problem,

- maximization problem
- nonnegativity constraints

- structural constraints written as $\leq$

then *dual* of maximization problem is minimization problem,

- minimization problem
- nonnegativity constraints
- structural constraints written as $\geq$

Reverse is also true: if original (or primal) problem is minimization problem, then dual is maximization problem. Duality, in addition to Crown's rules, can be used to solve linear programming problems.

**Exercise 5.4 (The Dual Problem)**

1. *Comparing primal and dual problems.*

   (a) *Transform minimum (primal) problem to maximum (dual) problem.*

$$\begin{array}{lcccccc} \text{Minimize} & 8x & + & 10y & & \\ \text{subject to} & x & + & 2y & \geq & 6 \\ & 2x & + & y & \geq & 4 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

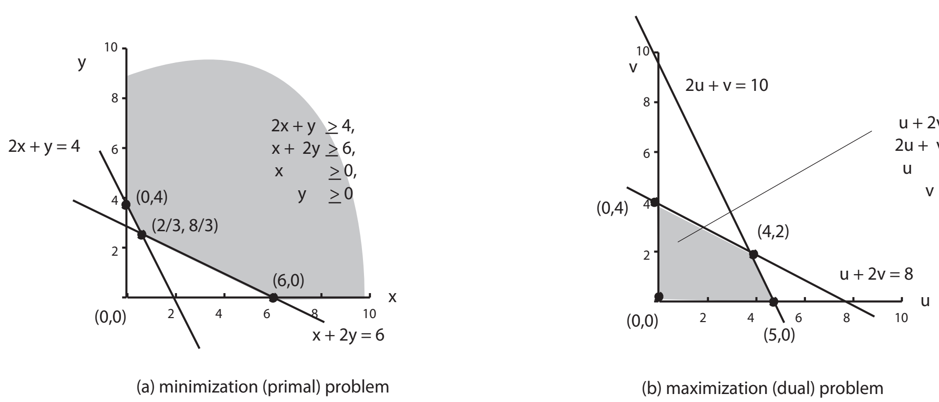


Figure 5.15 (Minimum and maximum problems)

Matrix form of minimization (primal, $P$) problem without slack variables,

$$P = \left[\begin{array}{cc|c} 1 & 2 & 6 \\ 2 & 1 & 4 \\ \hline 8 & 10 & 0 \end{array}\right]$$

Transpose of primal is dual ($D$), a maximization problem without slack variables,

$$D = P^T = \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 2 & 1 & 10 \\ \hline 6 & 4 & 0 \end{array}\right]$$

or

i. Maximum (dual) problem A

$$\begin{array}{rcccccc} \text{Maximize} & 6u & + & 4v & & \\ \text{subject to} & u & + & 2v & \leq & 6 \\ & 2u & + & v & \leq & 4 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

ii. Maximum (dual) problem B

$$\begin{array}{rcccccc} \text{Maximize} & 6u & + & 4v & & \\ \text{subject to} & u & + & 2v & \leq & 8 \\ & 2u & + & v & \leq & 10 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

(b) *Transform minimum (primal) problem to maximum (dual) problem.*

$$\begin{array}{rcccccccc} \text{Minimize} & 4x & + & 2y & + & 6z & & \\ \text{subject to} & 2x & + & 2y & + & 2z & \geq & 3 \\ & 2x & + & y & + & 2z & \geq & 2 \\ & 3x & + & 2y & + & z & \geq & 4 \\ & x & & & & & \geq & 0 \\ & & & y & & & \geq & 0 \\ & & & & & z & \geq & 0 \end{array}$$

Matrix form of minimization (primal, $P$) problem without slack variables,

$$P = \left[\begin{array}{ccc|c} 2 & 2 & 2 & 3 \\ 2 & 1 & 2 & 2 \\ 3 & 2 & 1 & 4 \\ \hline 4 & 2 & 6 & 0 \end{array}\right]$$

Transpose of primal is dual ($D$), a maximization problem,

$$D = P^T = \left[\begin{array}{ccc|c} 2 & 2 & 3 & 4 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 6 \\ \hline 3 & 2 & 4 & 0 \end{array}\right]$$

or

i. Maximum (dual) problem A

$$\begin{array}{rrcrcrcr} \text{Maximize} & 3u & + & 2v & + & 4w & & \\ \text{subject to} & 3u & + & 2v & + & 4w & \leq & 4 \\ & 2u & + & v & + & 2w & \leq & 2 \\ & 2u & + & 2v & + & w & \leq & 6 \\ & u & & & & & \geq & 0 \\ & & & v & & & \geq & 0 \\ & & & & & w & \geq & 0 \end{array}$$

ii. Maximum (dual) problem B

$$\begin{array}{rrcrcrcr} \text{Maximize} & 3u & + & 2v & + & 4w & & \\ \text{subject to} & 2u & + & 2v & + & 3w & \leq & 4 \\ & 2u & + & v & + & 2w & \leq & 2 \\ & 2u & + & 2v & + & w & \leq & 6 \\ & u & & & & & \geq & 0 \\ & & & v & & & \geq & 0 \\ & & & & & w & \geq & 0 \end{array}$$

(c) *Transform maximum (primal) problem to minimum (dual) problem.*

$$\begin{array}{rrcrcr} \text{Maximize} & x & + & y & & \\ \text{subject to} & 25x & + & 15y & \leq & 250 \\ & 20x & + & 10y & \leq & 175 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

Matrix form of maximization (primal, $P$) problem without slack variables,

$$P = \left[\begin{array}{cc|c} 25 & 15 & 250 \\ 20 & 10 & 175 \\ \hline 1 & 1 & 0 \end{array}\right]$$

Transpose of primal is dual ($D$), a minimization problem,

$$D = P^T = \left[\begin{array}{cc|c} 25 & 20 & 1 \\ 15 & 10 & 1 \\ \hline 250 & 175 & 0 \end{array}\right]$$

or

i. Minimum (dual) problem A

$$\begin{array}{rrcrcr} \text{Minimize} & 250u & + & 175v & & \\ \text{subject to} & 25u & + & 20v & \geq & 1 \\ & 15u & + & 10v & \geq & 1 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

ii. Minimum (dual) problem B

$$\begin{array}{lrcrcl} \text{Maximize} & 250u & + & 175v & & \\ \text{subject to} & 25u & + & 20v & \leq & 1 \\ & 15u & + & 10v & \leq & 1 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

(d) *Duality and nonstandard problem.* Nonstandard problem

$$\begin{array}{lrcrcl} \text{Minimize} & -2x & + & y & & \\ \text{subject to} & x & - & y & \leq & 1.5 \\ & 2x & + & 3y & \leq & 6 \\ & x & + & y & \geq & 1 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

is equivalent to minimum problem

$$\begin{array}{lrcrcl} \text{Minimize} & -2x & + & y & & \\ \text{subject to} & -x & + & y & \geq & -1.5 \\ & -2x & - & 3y & \geq & -6 \\ & x & + & y & \geq & 1 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

Matrix form of minimization (primal, $P$) problem without slack variables,

$$P = \left[\begin{array}{cc|c} -1 & 1 & -1.5 \\ -2 & -3 & -6 \\ 1 & 1 & 1 \\ \hline -2 & 1 & 0 \end{array}\right]$$

Transpose of primal is dual ($D$), a maximization problem,

$$D = P^T = \left[\begin{array}{ccc|c} -1 & -2 & 1 & -2 \\ 1 & -3 & 1 & 1 \\ \hline -1.5 & -6 & 1 & 0 \end{array}\right]$$

or

i. Maximum (dual) problem A

$$\begin{array}{lrcrcrcl} \text{Maximize} & -1.5u & - & 6v & + & w & & \\ \text{subject to} & -u & - & 2v & + & w & \leq & -2 \\ & u & - & 3v & + & w & \leq & 1 \\ & u & & & & & \geq & 0 \\ & & & v & & & \geq & 0 \\ & & & & & w & \geq & 0 \end{array}$$

ii. Maximum (dual) problem B

$$\begin{array}{lccccccc}
\text{Maximize} & \text{-1.5u} & - & \text{6v} & + & \text{w} & & \\
\text{subject to} & \text{-u} & - & \text{2v} & + & \text{w} & \geq & -2 \\
& \text{u} & - & \text{3v} & + & \text{w} & \geq & 1 \\
& \text{u} & & & & & \geq & 0 \\
& & & \text{v} & & & \geq & 0 \\
& & & & & \text{w} & \geq & 0
\end{array}$$

(e) *Duality and nonstandard problem.* Nonstandard problem

$$\begin{array}{lccccc}
\text{Minimize} & \text{-2x} & + & \text{y} & & \\
\text{subject to} & \text{x} & - & \text{y} & \leq & 1.5 \\
& \text{2x} & + & \text{3y} & \leq & 6 \\
& \text{x} & + & \text{y} & \geq & 1 \\
& \text{x} & & & \geq & 0 \\
& & & \text{y} & \geq & 0
\end{array}$$

is equivalent to maximum problem

$$\begin{array}{lccccc}
\text{Maximize} & \text{2x} & - & \text{y} & & \\
\text{subject to} & \text{x} & - & \text{y} & \leq & 1.5 \\
& \text{2x} & + & \text{3y} & \leq & 6 \\
& \text{-x} & - & \text{y} & \leq & -1 \\
& \text{x} & & & \geq & 0 \\
& & & \text{y} & \geq & 0
\end{array}$$

Matrix form of maximization (primal, $P$) problem without slack variables,

$$P = \left[\begin{array}{cc|c} 1 & -1 & 1.5 \\ 2 & 3 & 6 \\ -1 & -1 & -1 \\ \hline 2 & -1 & 0 \end{array}\right]$$

Transpose of primal is dual ($D$), a minimization problem,

$$D = P^T = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ -1 & 3 & -1 & -1 \\ \hline 1.5 & 6 & -1 & 0 \end{array}\right]$$

or

i. Minimum (dual) problem A

$$\begin{array}{lccccccc}
\text{Minimize} & \text{1.5u} & + & \text{6v} & - & \text{w} & & \\
\text{subject to} & \text{u} & + & \text{2v} & - & \text{w} & \leq & 2 \\
& \text{-u} & + & \text{3v} & - & \text{w} & \leq & -1 \\
& \text{u} & & & & & \geq & 0 \\
& & & \text{v} & & & \geq & 0 \\
& & & & & \text{w} & \geq & 0
\end{array}$$

ii. Minimum (dual) problem B

$$
\begin{array}{lrcrcrcl}
\text{Minimize} & 1.5u & + & 6v & - & w & & \\
\text{subject to} & u & + & 2v & - & w & \geq & 2 \\
 & -u & + & 3v & - & w & \geq & -1 \\
 & u & & & & & \geq & 0 \\
 & & & v & & & \geq & 0 \\
 & & & & & w & \geq & 0
\end{array}
$$

2. *Solve minimization problem using duality.*

$$
\begin{array}{lrcrcl}
\text{Minimize} & 8x & + & 10y & & \\
\text{subject to} & x & + & 2y & \geq & 6 \\
 & 2x & + & y & \geq & 4 \\
 & x & & & \geq & 0 \\
 & & & y & \geq & 0
\end{array}
$$

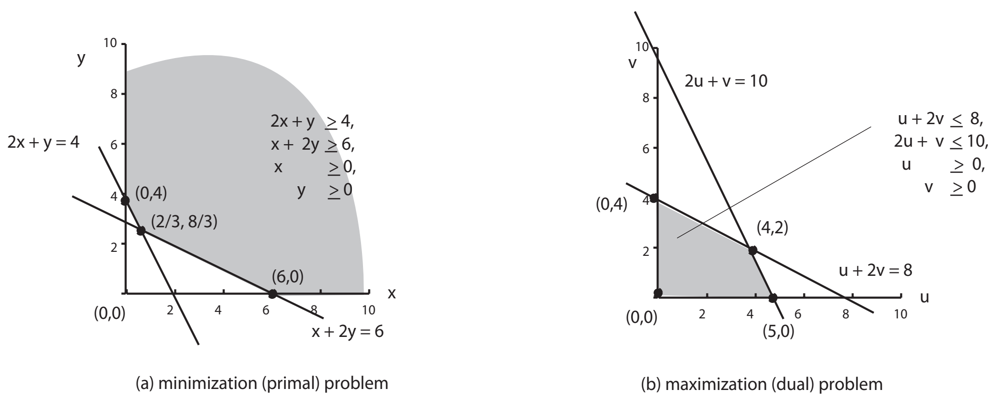


Figure 5.16 (Minimum (primal) and maximum (dual) problem)

(a) *Geometric solution.*

| corner | $f = 8x + 10y$ |
|---|---|
| (0,4) | $f = 8(0) + 10(4) = 40$ |
| (2/3, 8/3) | $f = 8(2/3) + 10(8/3) = 32$ |
| (6, 0) | $f = 8(6) + 10(0) = 48$ |

Objective function $f$ *minimum* at $(x, y) = \mathbf{(0, 4)}$ / $\left(\mathbf{\frac{2}{3}, \frac{8}{3}}\right)$ / $\mathbf{(6, 0)}$

(b) *Starting point.* Matrix form of minimization (primal, $P$) problem,

$$
P = \left[\begin{array}{cc|c} 1 & 2 & 6 \\ 2 & 1 & 4 \\ \hline 8 & 10 & 0 \end{array}\right]
$$

Transpose of primal is dual ($D$), a maximization problem,

$$D = P^T = \left[\begin{array}{cc|c} 1 & 2 & 8 \\ 2 & 1 & 10 \\ \hline 6 & 4 & 0 \end{array}\right]$$

or

$$\begin{array}{lccccc} \text{Maximize} & 6u & + & 4v & & \\ \text{subject to} & u & + & 2v & \leq & 8 \\ & 2u & + & v & \leq & 10 \\ & u & & & \geq & 0 \\ & & & v & \geq & 0 \end{array}$$

which has initial simplex tableau

| $u$ | $v$ | $s_1$ | $s_2$ | $f$ | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 0 | 0 | 8 | Quotient ______ |
| 2 | 1 | 0 | 1 | 0 | 10 | Quotient ______ |
| -6 | -4 | 0 | 0 | 1 | 0 | |

Type this 3 by 6 table into MATRIX [A].
Pivot column, most negative indicator, column $\boldsymbol{u}$ / $\boldsymbol{v}$
Pivot row, smallest quotient, so row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$
So pivot element: **1** / **2** / **8**

(c) *Next corner.*

$$\xrightarrow{\frac{1}{2}R_2 \to R_2, \quad R_1 - R_2 \to R_1, \; R_3 + 6R_2 \to R_3}$$

| $u$ | $v$ | $s_1$ | $s_2$ | $f$ | | |
|---|---|---|---|---|---|---|
| 0 | 1.5 | 1 | -0.5 | 0 | 3 | Quotient ______ |
| 1 | 0.5 | 0 | 0.5 | 0 | 5 | Quotient ______ |
| 0 | -1 | 0 | 3 | 1 | 30 | |

2nd MATRIX MATH *row(1/2,[A],2) STO 2nd MATRIX [B]
2nd MATRIX MATH *row+(−1,[B],2,2) STO 2nd MATRIX [C]
2nd MATRIX MATH *row+(6,[C],2,3) STO 2nd MATRIX [D].
Pivot column, most negative indicator, column $\boldsymbol{u}$ / $\boldsymbol{v}$
Pivot row, smallest quotient, so row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$
So pivot element: **0.5** / **1** / **1.5**

(d) *Last corner.*

$$\xrightarrow{\frac{1}{1.5}R_1 \to R_1, \quad R_2 - 0.5R_1 \to R_2, \; R_3 + R_1 \to R_3}$$

| $u$ | $v$ | $s_1$ | $s_2$ | $f$ | |
|---|---|---|---|---|---|
| 0 | 1 | $\frac{2}{3}$ | $-\frac{1}{3}$ | 0 | 2 |
| 1 | 0 | $-\frac{1}{3}$ | $\frac{2}{3}$ | 0 | 4 |
| 0 | 0 | $\boxed{\frac{2}{3}}$ | $\boxed{\frac{8}{3}}$ | 1 | 32 |

2nd MATRIX MATH *row(1/1.5,[D],2) STO 2nd MATRIX [E]

2nd MATRIX MATH *row+(−1,[E],2,2) STO 2nd MATRIX [F]

2nd MATRIX MATH *row+(6,[F],2,3) STO 2nd MATRIX [G] then MATH ENTER for fractions.

*Dual* point $(u, v) = \mathbf{(4, 2)}$ / $\left(\mathbf{\frac{2}{3}, \frac{8}{3}}\right)$
with optimal (maximum) $f = \mathbf{0}$ / $\mathbf{-32}$ / $\mathbf{32}$
because all indicators **nonpositive** / **zero** / **nonnegative**

Dual point is *not* solution to original (primal) linear programming problem.

*Primal* point $(x, y) = \mathbf{(4, 2)}$ / $\left(\mathbf{\frac{2}{3}, \frac{8}{3}}\right)$ under slack $(s_1, s_2)$
with optimal (*minimum*) $f = \mathbf{0}$ / $\mathbf{-32}$ / $\mathbf{32}$

Optimal maximum and minimum points are different but have same objective function value.

3. *Solve another minimization problem using duality.*

$$\begin{array}{lrcrcrcr} \text{Minimize} & 4x & + & 2y & + & 6z & & \\ \text{subject to} & 2x & + & 2y & + & 2z & \geq & 3 \\ & 2x & + & y & + & 2z & \geq & 2 \\ & 3x & + & 2y & + & z & \geq & 4 \\ & x & & & & & \geq & 0 \\ & & & y & & & \geq & 0 \\ & & & & & z & \geq & 0 \end{array}$$

(a) *Starting point.* Matrix form of minimization (primal, $P$) problem,

$$P = \left[\begin{array}{ccc|c} 2 & 2 & 2 & 3 \\ 2 & 1 & 2 & 2 \\ 3 & 2 & 1 & 4 \\ \hline 4 & 2 & 6 & 0 \end{array}\right]$$

Transpose of primal is dual ($D$), a maximization problem,

$$D = P^T = \left[\begin{array}{ccc|c} 2 & 2 & 3 & 4 \\ 2 & 1 & 2 & 2 \\ 2 & 2 & 1 & 6 \\ \hline 3 & 2 & 4 & 0 \end{array}\right]$$

or

$$\begin{array}{lrcrcrcr} \text{Maximize} & 3u & + & 2v & + & 4w & & \\ \text{subject to} & 2u & + & 2v & + & 3w & \leq & 4 \\ & 2u & + & v & + & 2w & \leq & 2 \\ & 2u & + & 2v & + & w & \leq & 6 \\ & u & & & & & \geq & 0 \\ & & & v & & & \geq & 0 \\ & & & & & w & \geq & 0 \end{array}$$

which has initial simplex tableau

| $u$ | $v$ | $w$ | $s_1$ | $s_2$ | $s_3$ | $f$ | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 3 | 1 | 0 | 0 | 0 | 4 | Quotient ____ |
| 2 | 1 | 2 | 0 | 1 | 0 | 0 | 2 | Quotient ____ |
| 2 | 2 | 1 | 0 | 0 | 1 | 0 | 6 | Quotient ____ |
| -3 | -2 | -4 | 0 | 0 | 0 | 1 | 0 | |

Type this 4 by 8 table into MATRIX [A].

Pivot column, most negative indicator, column ***u*** / ***v*** / ***w***
Pivot row, smallest quotient, so row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$ / $\boldsymbol{R_3}$
So pivot element: **1** / **2** / **3**

(b) *Next and last point.*

$$\frac{1}{2}R_2 \to R_2, \quad R_1 - 3R_2 \to R_1, \ R_3 - R_2 \to R_3, \ R_4 + 4R_2 \to R_4 \longrightarrow$$

| $u$ | $v$ | $w$ | $s_1$ | $s_2$ | $s_3$ | $f$ | |
|---|---|---|---|---|---|---|---|
| -1 | 0.5 | 0 | 1 | -1.5 | 0 | 0 | 1 |
| 1 | 0.5 | 1 | 0 | 0.5 | 0 | 0 | 1 |
| 1 | 1.5 | 0 | 0 | -0.5 | 1 | 0 | 5 |
| 1 | 0 | 0 | 0 | 2 | 0 | 1 | 4 |

2nd MATRIX MATH *row(1/2,[A],2) STO 2nd MATRIX [B]

2nd MATRIX MATH *row+(−3,[B],2,1) STO 2nd MATRIX [C]

2nd MATRIX MATH *row+(−1,[C],2,3) STO 2nd MATRIX [D]

2nd MATRIX MATH *row+(4,[D],2,4) STO 2nd MATRIX [E].

Primal point $(x, y, z) = \mathbf{(0, 0, 0)}$ / $\mathbf{(0, 2, 0)}$ under slack $(s_1, s_2, s_3)$
with optimal (*minimum*) $f =$ **0** / **1** / **4**
because all indicators **nonpositive** / **zero** / **nonnegative**

4. *Solve nonstandard problem using duality and Crown's rules.*

$$\begin{array}{lrcrcr} \text{Minimize} & -2x & + & y & & \\ \text{subject to} & x & - & y & \leq & 1.5 \\ & 2x & + & 3y & \leq & 6 \\ & x & + & y & \geq & 1 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

is equivalent to minimum problem

$$\begin{array}{lrcrcr} \text{Minimize} & -2x & + & y & & \\ \text{subject to} & -x & + & y & \geq & -1.5 \\ & -2x & - & 3y & \geq & -6 \\ & x & + & y & \geq & 1 \\ & x & & & \geq & 0 \\ & & & y & \geq & 0 \end{array}$$

(a) *Starting point.* Matrix form of minimization (primal, $P$) problem,

$$P = \left[\begin{array}{cc|c} -1 & 1 & -1.5 \\ -2 & -3 & -6 \\ 1 & 1 & 1 \\ \hline -2 & 1 & 0 \end{array}\right]$$

Transpose of primal is dual ($D$), a maximization problem,

$$D = P^T = \left[\begin{array}{ccc|c} -1 & -2 & 1 & -2 \\ 1 & -3 & 1 & 1 \\ \hline -1.5 & -6 & 1 & 0 \end{array}\right]$$

or

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Maximize | -1.5u | - | 6v | + | w | | | | |
| subject to | -u | - | 2v | + | w | $\leq$ | -2 | | |
| | u | - | 3v | + | w | $\leq$ | 1 | | |
| | u | | | | | $\geq$ | 0 | | |
| | | | v | | | $\geq$ | 0 | | |
| | | | | | w | $\geq$ | 0 | | |

which has initial simplex tableau

| $u$ | $v$ | $w$ | $s_1$ | $s_2$ | $f$ | | |
|---|---|---|---|---|---|---|---|
| -1 | -2 | 1 | 1 | 0 | 0 | -2 | Quotient ______ |
| 1 | -3 | 1 | 0 | 1 | 0 | 1 | Quotient ______ |
| 1.5 | 6 | -1 | 0 | 0 | 1 | 0 | |

Type this 3 by 7 table into MATRIX [A].
Use Crown's rules since -2 above horizontal line in right hand side (rhs).
Pivot column, most negative no. ($-2$) on $-2$ rhs row, column ***u*** / ***v*** / ***w***
Pivot row, only both-negative nonnegative quotient, so row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$
So pivot element: **−3** / **−2** / **6**

(b) *Next point.*

$$\xrightarrow{-\frac{1}{2}R_1\to R_1,\quad R_2+3R_1\to R_2,\ R_3-6R_1\to R_3}$$

| $u$ | $v$ | $w$ | $s_1$ | $s_2$ | $f$ | | |
|---|---|---|---|---|---|---|---|
| 0.5 | 1 | -0.5 | -0.5 | 0 | 0 | 1 | Quotient ______ |
| 2.5 | 0 | -0.5 | -1.5 | 1 | 0 | 4 | Quotient ______ |
| -1.5 | 0 | 2 | 3 | 0 | 1 | -6 | |

2nd MATRIX MATH *row(-1/2,[A],1) STO 2nd MATRIX [B]

2nd MATRIX MATH *row+(3,[B],1,2) STO 2nd MATRIX [C]

2nd MATRIX MATH *row+(−6,[C],1,3) STO 2nd MATRIX [D].

Use regular pivoting because no negatives above horizontal line in rhs.
Pivot column, most negative indicator, column ***u*** / ***v*** / ***w***

Pivot row, smallest quotient, so row $\boldsymbol{R_1}$ / $\boldsymbol{R_2}$
So pivot element: **0.5** / **1.5** / **2.5**

(c) *Next and last point.*

$$\underrightarrow{-\frac{1}{2.5}R_2 \to R_2, \quad R_1 - 0.5R_2 \to R_2, \quad R_3 + 1.5R_2 \to R_3}$$

| $u$ | $v$ | $w$ | $s_1$ | $s_2$ | $f$ | |
|---|---|---|---|---|---|---|
| 0 | 1 | -0.4 | -0.2 | -0.2 | 0 | 0.2 |
| 1 | 0 | -0.2 | -0.6 | 0.4 | 0 | 1.6 |
| 0 | 0 | 1.7 | 2.1 | 0.6 | 1 | -3.6 |

2nd MATRIX MATH *row(1/2.5,[D],2) STO 2nd MATRIX [E]

2nd MATRIX MATH *row+(-0.5,[E],2,1) STO 2nd MATRIX [F]

2nd MATRIX MATH *row+(1.5,[F],2,3) STO 2nd MATRIX [G].

Primal point $(x, y) = \mathbf{(0.2, 1.6)}$ / $\mathbf{(2.1, 0.6)}$ under slack $(s_1, s_2)$
with optimal (*minimum*) $f = \mathbf{-3.6}$ / **3.6** / **4** which matches answer above.
because all indicators **nonpositive** / **zero** / **nonnegative**

5. *Word problem and EXCEL.* Number of rooms[7] World Travel needs at Lodge for each of months January through March are:

| Month | January | February | March |
|---|---|---|---|
| Number of rooms needed | 25 | 12 | 20 |

Lodge leases its rooms to World Travel at a discount rate of \$800 for two consecutive months or \$1000 for three consecutive months. What leasing plan is least expensive?

(a) Let $x$ be number of rooms leased for two months, beginning January 1; let $y$ be number of rooms leased for two months, beginning February 1; let $z$ be number of rooms leased for *three* months, beginning January 1. Since rooms in January could be leased under either a two or three month lease (both beginning in January), and *at least* 25 rooms are needed in January, this means

i. $x + z \geq 25$
ii. $x + y + z \geq 12$
iii. $y + z \geq 20$
iv. $x + y + z \geq 37$

(b) Since rooms in February could also be leased under either a two month lease (beginning in either January or February) or three month lease (beginning in January), and *at least* 12 rooms are needed in February, this means

i. $x + z \geq 25$

[7] Spence and Vanden Eynden, Example 3.14, pp 163–165, 1990.

ii. $x + y + z \geq 12$
iii. $y + z \geq 20$
iv. $x + y + z \geq 37$

(c) Since rooms in March can be leased under a two month lease (beginning in February) or three month lease (beginning in January), and *at least* 20 rooms are needed in March, this means

i. $x + z \geq 25$
ii. $x + y + z \geq 12$
iii. $y + z \geq 20$
iv. $x + y + z \geq 37$

(d) Since Lodge leases its rooms to World Travel at a discount rate of \$800 for two consecutive months or \$1000 for three consecutive months, this means objective function is given by

i. $P = 800x + 800y + 1000z$
ii. $P = x + z$
iii. $P = 25x + 12y + 20z$
iv. $P = 800x - 1000y - 800z$

(e) LP problem is given as

$$\begin{array}{lccccccc}
\text{Minimize} & 800\text{x} & + & 800\text{y} & + & 1000\text{z} & & \\
\text{subject to} & \text{x} & & & + & \text{z} & \geq & 25 \\
 & \text{x} & + & \text{y} & + & \text{z} & \geq & 12 \\
 & & & \text{y} & + & \text{z} & \geq & 20 \\
 & \text{x} & & & & & \geq & 0 \\
 & & & \text{y} & & & \geq & 0 \\
 & & & & & \text{z} & \geq & 0
\end{array}$$

Following steps in figure,

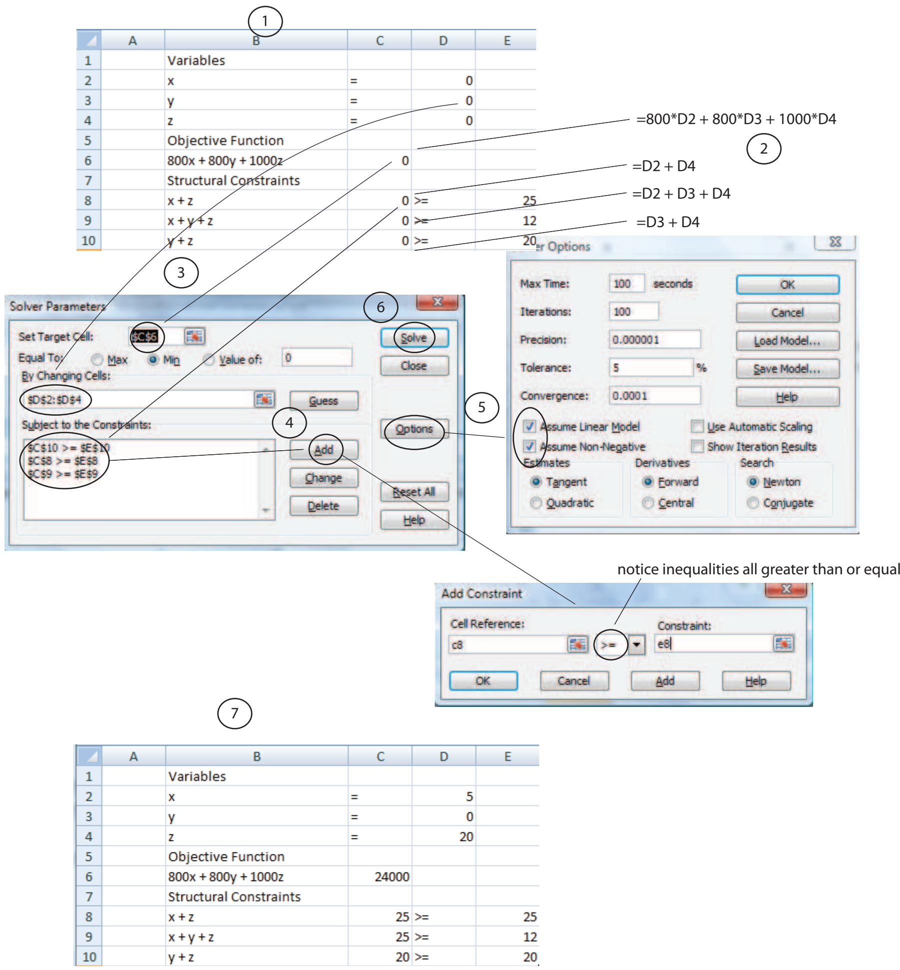


Figure 5.17 (Solve LP problem using EXCEL)

Point $(x, y, z) = \mathbf{(5, 0, 20)}$ / $\mathbf{(25, 12, 20)}$
with optimal (minimum) $f = \mathbf{24,000}$ / $\mathbf{30,000}$