Chapter 6

Mathematics of Finance

We will look at the mathematics of finance.

6.1 Simple and Compound Interest

We will look at two ways interest calculated on money. If principal (present value) amount P invested at interest rate r per year over time t, *simple interest*, I, is

$$I = Prt$$

and *total accumulated amount*, A, is

$$A = P + I = P + Prt = P(1 + rt).$$

If m is interest periods per year, and $n = mt$ is total number of interest periods, total accumulated amount assuming *compound interest* is

$$A = P\left(1 + \frac{r}{m}\right)^{mt} = P(1+i)^n.$$

If interest rate r compounded *continuously*, total accumulated amount after t years

$$A = Pe^{rt}$$

where $e = 2.718...$

Exercise 6.1 (Simple and Compound Interest)

1. *Simple Interest:* $A = P + Prt$.

   (a) If \$700 is invested at 11% *simple* interest, calculate its value after 8 years.
   $A = P + Prt = 700 + 700(0.11)(8) =$ **1116 / 1216 / 1316**

(b) If \$221 is invested at 15% simple interest, its value after 2.5 years is
$A = P + Prt = 221 + 221(0.15)(2.5) =$ **303.88 / 476.2 / 486.2**

(c) If \$5 is invested at 45% simple interest, its value after 13.1 years is
$A = P + Prt = 5 + 5(0.45)(13.1) =$ **34.48 / 47.34 / 86.22**

2. *Compound Interest:* $A = P\left(1 + \frac{r}{m}\right)^{mt} = P(1+i)^n$.

   (a) If \$321 is invested at 2.5% interest compounded *quarterly*, calculate its value after 7 years.
   $A = P\left(1 + \frac{r}{m}\right)^{mt} = 321\left(1 + \frac{0.025}{4}\right)^{4(7)} =$ **372.18 / 382.18 / 392.18**
   Calculator: $321 * (1 + 0.025/4) \wedge (4 * 7)$

   (b) If \$113 is invested at 2.5% interest compounded *monthly*, calculate its value after 3.7 years.
   $A = \left(1 + \frac{r}{m}\right)^{mt} = 113\left(1 + \frac{0.025}{12}\right)^{(12)3.7} =$ **123.94 / 125.81 / 127.81**
   Calculator: $113 * (1 + 0.025/12) \wedge (12 * 3.7)$

   (c) If \$121 is invested at 3% annual interest compounded *daily* (assume 365 days per year), calculate its value after 4 years.
   $A = \left(1 + \frac{r}{m}\right)^{mt} = 121\left(1 + \frac{0.03}{365}\right)^{(365)4} =$ **116.43 / 126.43 / 136.43**
   Calculator: $121 * (1 + 0.03/365) \wedge (365 * 4)$

   (d) If \$700 is invested at 11% interest compounded *yearly* (or annually), calculate its value after 8 years.
   $A = \left(1 + \frac{r}{m}\right)^{mt} = 700\left(1 + \frac{0.11}{1}\right)^{1(8)} =$ **1513.18 / 1613.18 / 1713.18**

   (e) If \$700 is invested at 11% interest compounded *monthly*, calculate its value after 8 years.
   $A = \left(1 + \frac{r}{m}\right)^{mt} = 700\left(1 + \frac{0.11}{12}\right)^{(12)8} =$ **1580.88 / 1680.88 / 1780.88**

3. *Compound Interest (Continuously):* $A = Pe^{rt}$

   (a) If \$2000 is invested at 7% interest compounded *continuously*, calculate its value after 3 years.
   $A = Pe^{rt} = 2000e^{0.07(3)} =$ **2267.36 / 2367.36 / 2467.36**.
   Calculator: $2000e \wedge (0.07 * 3)$

   (b) If \$1500 is invested at 6.5% interest compounded continuously, calculate its value after 3.5 years.
   $A = Pe^{rt} = 1500e^{0.065(3.5)} =$ **1883.19 / 1967.36 / 2267.36**.

   (c) If \$700 is invested at 11% interest compounded continuously, calculate its value after 8 years.
   $A = Pe^{rt} = 700e^{0.11(8)} =$ **1687.63 / 1967.36 / 2267.36**.

(d) An amount \$700 invested at 11% *simple* interest (\$1316) is
lesser / greater
than \$700 invested at 11% interest *compounded annually* (\$1613.18)
lesser / greater
than \$700 invested at 11% interest *compounded monthly* (\$1680.88)
lesser / greater
than \$700 invested at 11% interest *compounded continuously* (\$1687.63)
after 8 years.

4. *Related questions.*

(a) *Interest rate, r?*

i. If $A = 700$, $P = 15$, $t = 10$ years, interest compounded yearly
Since $A = P\left(1 + \frac{r}{m}\right)^{mt}$, then $700 = 15\left(1 + \frac{r}{1}\right)^{1(10)}$ or $(1+r)^{10} = \frac{700}{15}$
or taking tenth root of both sides,
$1 + r = \left(\frac{700}{15}\right)^{1/10}$ or $r = \left(\frac{700}{15}\right)^{1/10} - 1 \approx \mathbf{0.15} \ / \ \mathbf{0.39} \ / \ \mathbf{0.47}$.
Calculator: $(700/15) \wedge (0.1) - 1$

ii. If $A = 700$, $P = 15$, $t = 10$ years, interest compounded monthly
Since $A = P\left(1 + \frac{r}{m}\right)^{mt}$, $700 = 15\left(1 + \frac{r}{12}\right)^{12(10)}$ or $\left(1 + \frac{r}{12}\right)^{120} = \frac{700}{15}$
or taking 120th root of both sides,
$1 + \frac{r}{12} = \left(\frac{700}{15}\right)^{1/120}$ or $r = 12\left(\left(\frac{700}{15}\right)^{1/120} - 1\right) \approx \mathbf{0.15} \ / \ \mathbf{0.39} \ / \ \mathbf{0.47}$.
Calculator: $12 * ((700/15) \wedge (1/120) - 1)$

(b) *Number of interest periods, $n = mt$?*

i. If $A = 700$, $P = 15$, $r = 0.08$ interest compounded yearly
Since $A = P\left(1 + \frac{r}{m}\right)^{mt}$, $700 = 15\left(1 + \frac{0.08}{1}\right)^{mt}$ or $(1 + 0.08)^{mt} = \frac{700}{15}$
or taking natural logs of both sides,
$\ln(1 + 0.08)^{mt} = \ln\frac{700}{15}$ or $mt\ln(1 + 0.08) = \ln\frac{700}{15}$
or $n = mt = \frac{\ln\frac{700}{15}}{\ln 1.08} \approx \mathbf{48} \ / \ \mathbf{50} \ / \ \mathbf{52}$.
Calculator: $\ln(700/15)/\ln(1.08)$

ii. If $A = 700$, $P = 15$, $r = 0.08$ interest compounded monthly
Since $A = P\left(1 + \frac{r}{m}\right)^{mt}$, $700 = 15\left(1 + \frac{0.08}{12}\right)^{mt}$ or $\left(1 + \frac{0.08}{12}\right)^{mt} = \frac{700}{15}$
or taking natural logs of both sides,
$\ln\left(1 + \frac{0.08}{12}\right)^{mt} = \ln\frac{700}{15}$ or $12t\ln\left(1 + \frac{0.08}{12}\right) = \ln\frac{700}{15}$
or $n = 12t = \frac{\ln\frac{700}{15}}{\ln\left(1 + \frac{0.08}{12}\right)} \approx \mathbf{563} \ / \ \mathbf{578} \ / \ \mathbf{589}$.
Calculator: $\ln(700/15)/\ln(1 + 0.08/12)$

(c) *Principal, P?*

i. If $A = 700$, $t = 5$ years, $r = 0.08$ interest compounded yearly
Since $A = P\left(1 + \frac{r}{m}\right)^{mt}$, $700 = P\left(1 + \frac{0.08}{1}\right)^{1(5)}$

or $P = 700(1+0.08)^{-5} \approx \mathbf{476.41} \ / \ \mathbf{500.00} \ / \ \mathbf{528.89}$.

Calculator: $700 * 1.08 \wedge (-5)$

ii. If $A = 700$, $t = 5$ years, $r = 0.08$ interest compounded monthly

Since $A = P\left(1+\frac{r}{m}\right)^{mt}$, $700 = P\left(1+\frac{0.08}{12}\right)^{12(5)}$

or $P = 700\left(1+\frac{0.08}{12}\right)^{-60} \approx \mathbf{469.85} \ / \ \mathbf{499.00} \ / \ \mathbf{518.89}$.

Calculator: $700 * (1+0.08/12) \wedge (-60)$

(d) *Other.* Two hundred dollars (\$200) is deposited monthly into account paying 6.25% compounded monthly. After 3 years, accumulated amount is put into 2-year certificate which pays 8% compounded quarterly. Determine final accumulated amount.

$A = \left(1+\frac{r}{m}\right)^{mt} = 200\left(1+\frac{0.0625}{12}\right)^{12(3)} \approx \mathbf{241.13} \ / \ \mathbf{375.89}$

Calculator: $200 * (1+0.0625/12) \wedge (36)$

$A = \left(1+\frac{r}{m}\right)^{mt} = 241.13\left(1+\frac{0.08}{4}\right)^{4(2)} \approx \mathbf{282.52} \ / \ \mathbf{375.89}$

Calculator: $241.13 * (1+0.08/4) \wedge (8)$

5. *Using the TI–83 Calculator: Compound Interest.* Determine the future value of \$700 which is invested at 11% interest which is compounded monthly after 8.3 years.

- Press APPS ENTER FINANCE ENTER TVM Solver ENTER.
- Set the TVM Solver parameters as N = 8.3, I% = 11, PV = −700, PMT = 0, FV = 0, P/Y = 1, C/Y = 12. Arrow back to FV and then press ALPHA ENTER. The answer FV = 1737.01 appears.

N stands for number of years. I% is the yearly interest rate. PV stands for "present value" and is typed in as a *negative* number because it is considered as an "outflow" of cash. PMT is the "payment amount", which, in this case, does not apply and so is set to zero. FV is "future value" and is the variable we are trying to determine in this question. P/Y is the "number of payment periods per year", which, in this case, does not apply and so is set to one. C/Y is the "number of compounding periods per year".

6.2 Ordinary Annuities

We will look at annuities (a sequence of payments made at regular time intervals); more specifically, ordinary annuities (annuity where interest on payments compounded at same time payment made). If principal (present value) amount P invested at interest rate r per year over time t, m is interest periods per year, and $n = mt$ is total number of interest periods, *future value of an ordinary annuity*,

$$A = p\left[\frac{\left(1+\frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}}\right] = p\left[\frac{(1+i)^n - 1}{i}\right]$$

payments to a sinking fund,

$$p = A\left[\frac{\left(\frac{r}{m}\right)}{\left(1+\frac{r}{m}\right)^{mt} - 1}\right] = A\left[\frac{i}{(1+i)^n - 1}\right]$$

ordinary annuity formula,

$$A = P\left(1+\frac{r}{m}\right)^{mt} + p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right] = P(1+i)^n + p\left[\frac{(1+i)^n-1}{i}\right]$$

present value of an ordinary annuity,

$$P = p\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right] = p\left[\frac{1-(1+i)^{-n}}{i}\right].$$

Exercise 6.2 (Ordinary Annuities)

1. *Future value of an annuity:* $A = p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right] = p\left[\frac{(1+i)^n-1}{i}\right]$

   (a) Future value of 5 year term annuity, \$100 paid each quarter, earning interest at 8.5% annually, compounded quarterly, is
   $A = p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right] = 100\left[\frac{\left(1+\frac{0.085}{4}\right)^{4(5)}-1}{\frac{0.085}{4}}\right] \approx$ **2260.21 / 2460.21**
   Calculator: $100 * ((1+0.085/4) \wedge (20) - 1)/(0.085/4)$

   (b) Future value of 3 year term annuity, \$120 paid each month, earning interest at 9.5% annually, compounded monthly, is
   $A = p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right] = 120\left[\frac{\left(1+\frac{0.095}{12}\right)^{12(3)}-1}{\frac{0.095}{12}}\right] \approx$ **4975.89 / 5075.89**
   Calculator: $120 * ((1+0.095/12) \wedge (36) - 1)/(0.095/12)$

   (c) Future value of 3.2 year term annuity, \$105 paid each day, earning interest at 6.5% annually, compounded daily (365 days), is
   $A = p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right] = 105\left[\frac{\left(1+\frac{0.065}{365}\right)^{365(3.2)}-1}{\frac{0.065}{365}}\right] \approx$ **135,313.40 / 136,313.40**
   Calculator: $105 * ((1+0.065/365) \wedge (365 * 3.2) - 1)/(0.065/365)$

2. *Payments to sinking fund:* $p = A\left[\frac{\left(\frac{r}{m}\right)}{\left(1+\frac{r}{m}\right)^{mt}-1}\right] = A\left[\frac{i}{(1+i)^n-1}\right]$

   (a) Lab of computers replaced in 3 years time for anticipated (future) cost of \$25,000 where \$25,000 accumulated over 3 year period through equal installments made at end of each month. If yearly interest rate is 8.5%, size of each installment is
   $p = A\left[\frac{\left(\frac{r}{m}\right)}{\left(1+\frac{r}{m}\right)^{mt}-1}\right] = 25000\left[\frac{\left(\frac{0.085}{12}\right)}{\left(1+\frac{0.085}{12}\right)^{12(3)}-1}\right] \approx$ **612.11 / 613.11 / 614.11**
   Calculator: $25000 * (0.085/12)/((1+0.085/12) \wedge (36) - 1)$

(b) Quarterly annuity required (future) sinking fund of \$30,000, needed after 5 years, if yearly interest rate is 7.5%, is
$p = A\left[\frac{\left(\frac{r}{m}\right)}{\left(1+\frac{r}{m}\right)^{mt}-1}\right] = 30000\left[\frac{\left(\frac{0.075}{4}\right)}{\left(1+\frac{0.075}{4}\right)^{4(5)}-1}\right] \approx \mathbf{1250.14} \; / \; \mathbf{1350.14}$
Calculator: 30000 ∗ (0.075/4)/((1 + 0.075/4) ∧ (20) − 1)

3. *Ordinary annuity formula:* $A = P\left(1+\frac{r}{m}\right)^{mt} + p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right]$

(a) If \$700 is invested now at 11% interest compounded quarterly and also \$120 is added each quarter, calculate value of investment after 8 years.
$A = \left(1+\frac{r}{m}\right)^{mt} + p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right] =$
$700\left(1+\frac{0.11}{4}\right)^{4(8)} + 120\left[\frac{\left(1+\frac{0.11}{4}\right)^{4(8)}-1}{\frac{0.11}{4}}\right] \approx \mathbf{7700.08} \; / \; \mathbf{8075.89}$
Calculator: 700 ∗ (1 + 0.11/4) ∧ (32) + 120 ∗ ((1 + 0.11/4) ∧ (32) − 1)/(0.11/4)

(b) If \$600 is invested now at 1% interest compounded semiannually and also \$100 is added every six months, calculate value of investment after 5 years.
$A = \left(1+\frac{r}{m}\right)^{mt} + p\left[\frac{\left(1+\frac{r}{m}\right)^{mt}-1}{\frac{r}{m}}\right] =$
$600\left(1+\frac{0.01}{2}\right)^{2(5)} + 100\left[\frac{\left(1+\frac{0.01}{2}\right)^{2(3)}-1}{\frac{0.01}{2}}\right] \approx \mathbf{700.08} \; / \; \mathbf{1653.49}$
Calculator: 600 ∗ (1 + 0.01/2) ∧ (10) + 100 ∗ ((1 + 0.01/2) ∧ (10) − 1)/(0.01/2)

4. *Present value of an annuity:* $P = p\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right] = p\left[\frac{1-(1+i)^{-n}}{i}\right]$

(a) Present value of 5 year term annuity, \$100 paid each quarter, earning 8.5% yearly interest, compounded quarterly, is
$P = p\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right] = 100\left[\frac{1-\left(1+\frac{0.085}{4}\right)^{-4(5)}}{\frac{0.085}{4}}\right] \approx \mathbf{1415.59} \; / \; \mathbf{1615.59}$
Calculator: 100 ∗ (1 − (1 + 0.085/4) ∧ (−20))/(0.085/4)

(b) Present value of 3 year term annuity, \$120 paid monthly, earning 9.5% yearly interest, compounded monthly, is
$P = p\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right] = 120\left[\frac{1-\left(1+\frac{0.095}{12}\right)^{-12(3)}}{\frac{0.095}{12}}\right] \approx \mathbf{3546.14} \; / \; \mathbf{3746.14}$
Calculator: 120 ∗ (1 − (1 + 0.095/12) ∧ (−36))/(0.095/12)

(c) Present value of 7 year term annuity, \$97 paid monthly, earning 9.5% yearly interest, compounded daily (365 days), is
$P = p\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right] = 97\left[\frac{1-\left(1+\frac{0.095}{365}\right)^{-365(7)}}{\frac{0.095}{365}}\right] \approx \mathbf{180,006} \; / \; \mathbf{181,006}$
Calculator: 97 ∗ (1 − (1 + 0.095/365) ∧ (−365 ∗ 7))/(0.095/365)

6.3 Consumer Loans and APR

Annual percentage rate (APR) or *effective interest rate* allows consumers to compare different interest rates. Assuming principal (present value) amount is P, interest rate is r per year, m is interest periods per year, APR is

$$APR = \left(1 + \frac{r}{m}\right)^m - 1 = (1+i)^m - 1$$

amortization, amount of payments to retire a load,

$$p = P\left[\frac{\left(\frac{r}{m}\right)}{1 - \left(1 + \frac{r}{m}\right)^{-mt}}\right] = P\left[\frac{i}{1-(1+i)^{-n}}\right]$$

amortization, number of payments to retire a load,

$$n = \frac{\ln\left(\frac{p}{p - P\left(\frac{r}{m}\right)}\right)}{\ln\left(1+\frac{r}{m}\right)} = \frac{\ln\left(\frac{p}{p-Pi}\right)}{\ln(1+i)}$$

Amortization table or *amortization schedule* is also discussed.

Exercise 6.3 (Consumer Loans and APR)

1. $APR = \left(1 + \frac{r}{m}\right)^m - 1 = (1+i)^m - 1$.
   Which is larger: 10% compounded monthly or 10.2% compounded quarterly?

   (a) After 1 year, \$1 invested 10% compounded *monthly*,
   $A = P\left(1+\frac{r}{m}\right)^{mt} = 1\left(1 + \frac{0.10}{12}\right)^{12(1)} \approx$ **1.084713** / **1.094713** / **1.104713**
   Calculator: $1 * (1 + 0.10/12) \wedge (12)$
   so *interest* earned in one year is this amount subtract \$1,
   $APR = \left(1+\frac{r}{m}\right)^m - 1 \approx$ **0.1047** / **0.1147** / **0.1247** or 10.47%
   Calculator: $(1 + 0.10/12) \wedge (12) - 1$

   (b) After 1 year, \$1 invested 10.2% compounded *quarterly*,
   $A = P\left(1+\frac{r}{m}\right)^{mt} = 1\left(1 + \frac{0.102}{4}\right)^{4(1)} \approx$ **1.085968** / **1.095968** / **1.105968**
   Calculator: $(1 + 0.102/4) \wedge (4)$
   so *interest* earned in one year,
   $APR = \left(1+\frac{r}{m}\right)^m - 1 =$ **0.105968** / **0.115968** / **0.125968** or 10.60%
   Calculator: $(1 + 0.102/4) \wedge (4) - 1$

   (c) Consequently, 10% compounded monthly (APR: 10.47%) is
   less / **more** than 10.2% compounded quarterly (APR: 10.60%).

2. *Amortization, amount of payments to retire a loan:* $p = P\left[\frac{\left(\frac{r}{m}\right)}{1-\left(1+\frac{r}{m}\right)^{-mt}}\right]$

(a) Car loan of \$25,000 repaid monthly over 3 year period, yearly interest 8.5%. Amount of each installment

$$p = P\left[\frac{\left(\frac{r}{m}\right)}{1-\left(1+\frac{r}{m}\right)^{-mt}}\right] = 25000\left[\frac{\left(\frac{0.085}{12}\right)}{1-\left(1+\frac{0.085}{12}\right)^{-(12)3}}\right] \approx \mathbf{769.19} \ / \ \mathbf{789.19}$$

Calculator: $25000 * (0.085/12)/(1 - (1 + 0.085/12) \wedge (-36))$

(b) House loan of \$125,000 repaid quarterly over 20 year period, yearly interest 7.5%. Amount of each installment

$$p = P\left[\frac{\left(\frac{r}{m}\right)}{1-\left(1+\frac{r}{m}\right)^{-mt}}\right] = 125000\left[\frac{\left(\frac{0.075}{4}\right)}{1-\left(1+\frac{0.075}{4}\right)^{-(4)20}}\right] \approx \mathbf{1906.99} \ / \ \mathbf{3029.08}$$

Calculator: $125000 * (0.075/4)/(1 - (1 + 0.075/4) \wedge (-80))$

(c) *Amortization table.*
Loan of \$5,000 repaid quarterly over 1.5 year period, yearly interest 8.5%. Amount of each installment

$$p = P\left[\frac{\left(\frac{r}{m}\right)}{1-\left(1+\frac{r}{m}\right)^{-mt}}\right] = 5000\left[\frac{\left(\frac{0.085}{4}\right)}{1-\left(1+\frac{0.085}{4}\right)^{-(4)1.5}}\right] \approx \mathbf{896.40} \ / \ \mathbf{989.19}$$

Calculator: $5000 * (0.085/4)/(1 - (1 + 0.085/4) \wedge (-6))$

payment	amount	interest	principal	balance
1	896.40	106.25	790.15	4209.85
2	896.40	89.45	806.95	3402.90
3	896.40	72.31	824.09	2578.81
4	896.40	54.80	841.60	1737.21
5	896.40	36.92	859.48	877.73
6	896.40	18.65	877.75	0

To begin, interest $= 5000 \times \frac{0.085}{4} = 106.25$, then principal $= 896.40 - 106.25 = 790.15$, and balance $= 5000 - 790.15 = 4209.85$, then interest $= 4209.85 \times \frac{0.085}{4} \approx 89.45$ and so on.

(d) **True** / **False**. Amortization determines sequence of payments (annuity) equivalent to *present* lump sum, whereas sinking fund determines annuity equivalent to *future* lump sum.

3. *Amortization, number of payments to retire a loan:* $n = \frac{\ln\left(\frac{p}{p-P\left(\frac{r}{m}\right)}\right)}{\ln\left(1+\frac{r}{m}\right)} = \frac{\ln\left(\frac{p}{p-Pi}\right)}{\ln(1+i)}$

(a) Number of quarterly \$1000 payments to repay car loan of \$25,000, 8.5%:

$$n = \frac{\ln\left(\frac{p}{p-P\left(\frac{r}{m}\right)}\right)}{\ln\left(1+\frac{r}{m}\right)} = \frac{\ln\left(\frac{1000}{1000-25000\left(\frac{0.085}{4}\right)}\right)}{\ln\left(1+\frac{0.085}{4}\right)} \approx \mathbf{36.03} \ / \ \mathbf{36.89} \text{ or 37 payments}$$

Calculator: $\ln(1000/(1000 - 25000 * 0.085/4))/\ln(1 + 0.085/4)$

(b) Number of monthly \$1000 payments to repay house loan of \$125,000, 7%:

$$n = \frac{\ln\left(\frac{p}{p-P\left(\frac{r}{m}\right)}\right)}{\ln\left(1+\frac{r}{m}\right)} = \frac{\ln\left(\frac{1000}{1000-125000\left(\frac{0.07}{12}\right)}\right)}{\ln\left(1+\frac{0.07}{12}\right)} \approx \mathbf{224.58} \ / \ \mathbf{225.34.45} \text{ or 225 payments}$$

Calculator: $\ln(1000/(1000 - 125000 * 0.07/12))/\ln(1 + 0.07/12)$