Chapter 7

Logic, Sets, and Counting Techniques

In this chapter, we look at logic, sets and how to count. This information is useful for following chapter when we look at basic properties of probability.

7.1 Logic

We look at a *statement* which can be true, false but not both or an *open sentence* which is made true or false by substituting for a pronoun or variable in a response. Compound sentences are constructed using *connectives*

- *conjunction*: p and q, $p \wedge q$,
- *disjunction*: p or q, $p \vee q$,
  - *inclusive or*: one or the other or both (assumed, unless otherwise specified)
  - *exclusive or*: one or the other but *not* both
- *negation*: not p, $\sim p$

An *implication* is discussed, $p \Rightarrow q$, read *if* p, *then* q or p implies q, where p is hypothesis and q is conclusion. We find out

$$(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$$

statement "if p, then q" is equivalent to "not p or q". *Quantifier at least* and its negation, *at most*, are discussed, as is *all* and its negation *some*. We discover "at least" (and "at most") are equivalent to compound statements using connectives "and" and "or".

Exercise 7.1 (Logic)

1. *Statements and connectives.* Consider following statements

   p: Student registered at PNC.
   q: Student disabled.

   (a) $p \wedge q$ means
      i. Student registered at PNC and disabled.
      ii. Student registered at PNC or disabled.
   (b) $p \vee q$ means
      i. Student registered at PNC and disabled.
      ii. Student registered at PNC or disabled.
   (c) $p \wedge \sim q$ means
      i. Student registered at PNC and able-bodied.
      ii. Student not registered at PNC or disabled.
   (d) $p \Rightarrow \sim q$ means
      i. If student registered at PNC, then they are able-bodied.
      ii. Student registered at PNC or able-bodied.
   (e) $\sim p \wedge \sim q$ means
      i. Student not registered at PNC and able-bodied.
      ii. Student not registered at PNC and disabled.
   (f) Student registered at PNC and is disabled:
      i. $p \wedge \sim q$
      ii. $p \wedge q$
   (g) Student is not registered at PNC and is disabled:
      i. $\sim p \wedge q$
      ii. $\sim p \wedge \sim q$
   (h) Student registered at PNC or is able-bodied:
      i. $p \vee \sim q$
      ii. $p \vee q$
   (i) Negation of $p \wedge q$ (choose one or *more*)
      i. $(\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$
      ii. $p \Rightarrow \sim q$
      iii. $\sim p \vee \sim q$

2. *More statements and connectives.* Consider following statements

   p: Student owns iPhone.
   q: Student tweets.

   (a) $\sim p \wedge q$ means
      i. Student owns iPhone and tweets.
      ii. Student does not own iPhone and tweets.

   (b) $p \vee \sim q$ means
      i. Student owns iPhone or does not tweet.
      ii. Student does not own iPhone or tweets.

   (c) $\sim p \vee \sim q$ means
      i. Student does not own iPhone and does not tweet.
      ii. Student does not own iPhone or does not tweet.

   (d) If student owns iPhone, then student tweets:
      i. $p \wedge q$
      ii. $p \Rightarrow q$

   (e) Student does not own iPhone and does not tweet:
      i. $\sim p \wedge q$
      ii. $\sim p \wedge \sim q$

   (f) Student owns iPhone or tweets, inclusively:
      i. $(p \wedge \sim q) \vee (\sim p \wedge q)$
      ii. $p \vee q$ equivalent to $(p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \vee q)$

   (g) Student owns iPhone or tweets, exclusively:
      i. $(p \wedge \sim q) \vee (\sim p \wedge q)$
      ii. $p \vee q$ equivalent to $(p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \vee q)$

   (h) $p \Rightarrow q$ means (choose one or *more*)
      i. If student owns iPhone, then student tweets.
      ii. Student does not own iPhone or tweets, meaning one of three things:
      Student owns iPhone and tweets, or
      Student does not own iPhone and does not tweet, or
      Student does not own iPhone and tweets.

   This demonstrates $p \Rightarrow q$ is equivalent to $\sim p \ \vee q$.

3. *At least and at most; some and all.*

   (a) "At least four of five students tweet" equivalent to (choose *two*!)

i. four students tweet or five students tweet.
ii. (four students tweet and one does not) or (five students tweet and zero do not)
iii. at most three students tweets.

(b) Negation of "At least four of five students tweet" is

i. four students tweet or five students tweet.
ii. (four students tweet and one does not) or (five students tweet and zero do not)
iii. at most three students tweet.

(c) "At most two of five students tweet" equivalent to

i. zero students tweet or one student tweet or two students tweet.
ii. at least three students tweet.

(d) Negation of "All students tweet" (choose *two*!)

i. Some students do not tweet.
(Which could mean all students do not tweet, but at least one does not tweet.)
ii. Not all students tweet.
iii. Some students tweet.
(Which could mean all students tweet, not the negation of "all students tweet".)

(e) Negation of "Some students do not own an iPod" (choose one)

i. Some students own an iPod.
(Which could mean some students do *not* own an iPod,
not the negation of "some students do not own an iPod".)
ii. All students own an iPod.
iii. All students do not own an iPod.
(Which could mean some students do not own an iPod, so not a negation.)

(f) Negation of "Student owns iPod and tweets" (choose one or *more*)

i. (student owns iPod and does not tweet)
or (student does not own iPod and tweets)
or (student does not own iPod and does not tweet).
ii. Student does not own iPod or does not tweet.

In general, $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(g) Negation of "Student owns iPod or tweets"

i. Student owns iPod and tweets.
ii. Student does not own iPod and does not tweet.

In general, $\sim (p \vee q) \equiv \sim p \wedge \sim q$

7.2 Truth Tables

When combining two statements, p and q, which each have two possible values, true (T) or false (F), there are always only four possibilities,

p	q
T	T
T	F
F	T
F	F

Two statements are *logically equivalent* if they have identical truth values and denoted $p \equiv q$. *DeMorgan's Laws of Logic* are given by

- $\sim (p \wedge q) \equiv \sim p \vee \sim q$
- $\sim (p \vee q) \equiv \sim p \wedge \sim q$

Exercise 7.2 (Truth Tables)

1. *True or False?*

   (a) **T** / **F**. If it is a German Shepherd, then it is a dog.

   (b) **T** / **F**. If it is a dog, then it is a German Shepherd.

   (c) **T** / **F**. $2 - 1 = 1$ and $5 \times 2 = 11$.

   (d) **T** / **F**. $2 - 1 = 1$ or $5 \times 2 = 11$.

2. *Truth tables.* Fill in blanks.

   (a) $p \wedge q$

p	q	$p \wedge q$	example
T	T	T	$2 - 1 = 1$ and $5 \times 2 = 10$
T	F	F	$2 - 1 = 1$ and $5 \times 2 = 11$
F	T	______	$2 - 1 = 0$ and $5 \times 2 = 10$
F	F	______	$2 - 1 = 0$ and $5 \times 2 = 11$

   (b) $p \vee q$

p	q	$p \vee q$	example
T	T	______	$2 - 1 = 1$ or $5 \times 2 = 10$
T	F	______	$2 - 1 = 1$ or $5 \times 2 = 11$
F	T	______	$2 - 1 = 0$ or $5 \times 2 = 10$
F	F	______	$2 - 1 = 0$ or $5 \times 2 = 11$

(c) $p \Rightarrow q$

p	q	$p \Rightarrow q$
T	T	______
T	F	______
F	T	______
F	F	______

(d) $\sim p \vee q$

p	q	$\sim p$	$\sim p \vee q$
T	T	F	______
T	F	F	______
F	T	T	______
F	F	T	______

3. *Equivalences using truth tables.*

   (a) $p \vee q$ and $q \vee p$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

   So $p \vee q$ and $q \vee p$ are **equivalent** / **not equivalent**

   (b) $p \Rightarrow q$ and $\sim p \vee q$

p	q	$p \Rightarrow q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

   So $p \Rightarrow q$ and $\sim p \vee q$ are **equivalent** / **not equivalent**

4. *DeMorgan's laws of logic.* Consider following statements

   p: Student owns iPhone.
   q: Student tweets.

   (a) Since $\sim (p \wedge q) \equiv \sim p \vee \sim q$, this means

      i. Student does not own iPhone or does not tweet.

      ii. Student does not own iPhone and does not tweet.

   (b) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

i. Student does not own iPhone or does not tweet.
ii. Student does not own iPhone and does not tweet.

7.3 Sets

After looking at some definitions related to sets, we look at intersection, union, complement and difference of sets. Venn diagrams help us picture these various operations.

Exercise 7.3 (Sets)

1. *Definitions.*

   (a) **True** / **False**. *Set* is well-defined collection of distinct objects (rule allows us to determine if object belongs to a set or not); for example, "set of letters in English alphabet" is a set but "set of letters" is not a set.

   (b) **True** / **False**. Set A of 1, 2, 3, 4, 5, 6 denoted $A = \{1, 2, 3, 4, 5, 6\}$.

   (c) **True** / **False**. If $A = \{1, 2, 3, 4, 5, 6\}$, then number 6 belongs to A, $6 \in A$.

   (d) **True** / **False**. If $A = \{1, 2, 3, 4, 5, 6\}$, then 7 does not belong to A, $7 \notin A$.

   (e) **True** / **False**. $a \in A$ read *a is an element of A*.

   (f) **True** / **False**. Set A of all numbers between 0 and 5: $A = \{x : 0 < x < 5\}$.

   (g) **True** / **False**. Set B *subset* of A, $B \subseteq A$, if and only if each element of B is also an element of A; for example, $B = \{2, 3\}$ and $A = \{x : 0 \le x \le 5\}$.

   (h) **True** / **False**. Sets B and A equal, $A = B$, if and only if $B \subseteq A$ and $A \subseteq B$, if both sets contain exactly same elements.

   (i) **True** / **False**. If $A = \{x : 0 \le x \le 5\}$ and $B = \{0, 1, 2, 3, 4, 5\}$, then $A \ne B$.

   (j) **True** / **False**. Set containing *no* elements is *empty set* denoted $\emptyset$.

   (k) **True** / **False**. Set containing *all* elements is *universal* set, U.

2. *Venn diagrams: intersection and union.*

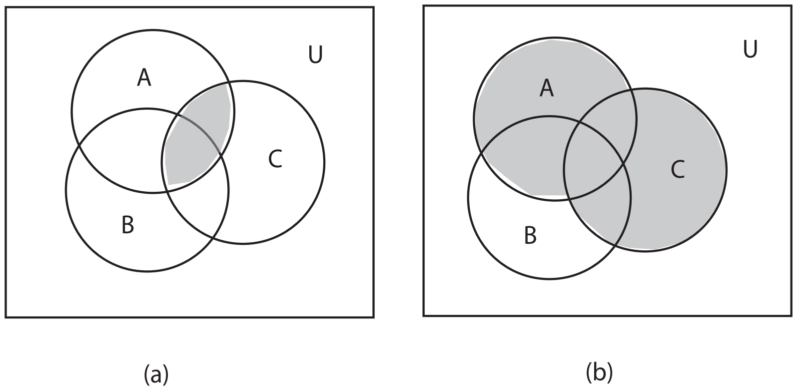

Figure 7.1 (Venn diagrams: intersection and union)

(a) Figure (a), *intersection* of A and C, $A \cap C$, all elements in both A and C,
$\boldsymbol{\{x : (x \in A)}$ or $\boldsymbol{(x \in C)\}}$ / $\boldsymbol{\{x : (x \in A)}$ and $\boldsymbol{(x \in C)\}}$

(b) Figure (b), *union* of A and C, $A \cup C$, elements in either A or C,
$\boldsymbol{\{x : (x \in A)}$ or $\boldsymbol{(x \in C)\}}$ / $\boldsymbol{\{x : (x \in A)}$ and $\boldsymbol{(x \in C)\}}$

(c) *General intersection.*
Intersection $A_1 \cap A_2 \cap ... \cap A_n$ equivalent to
$\boldsymbol{\{x : x}$ belongs to every set $\boldsymbol{\}}$
$\boldsymbol{\{x : x}$ belongs to at least one of the sets $\boldsymbol{\}}$

(d) *General union.*
Union $A_1 \cup A_2 \cup ... \cup A_n$ equivalent to
$\boldsymbol{\{x : x}$ belongs to every set $\boldsymbol{\}}$
$\boldsymbol{\{x : x}$ belongs to at least one of the sets $\boldsymbol{\}}$

(e) *Notation.* Often, "$\cap$" dropped, so $A \cap B$ becomes AB for example.
Consequently, $(A \cap B) \cup (A \cap C)$ equivalent to
$\boldsymbol{(AC') \cup (BC') \cup (AB)}$ / $\boldsymbol{(AB) \cup (AC)}$ / $\boldsymbol{(A \cup B)'}$

3. *Venn diagrams: difference and complement.*

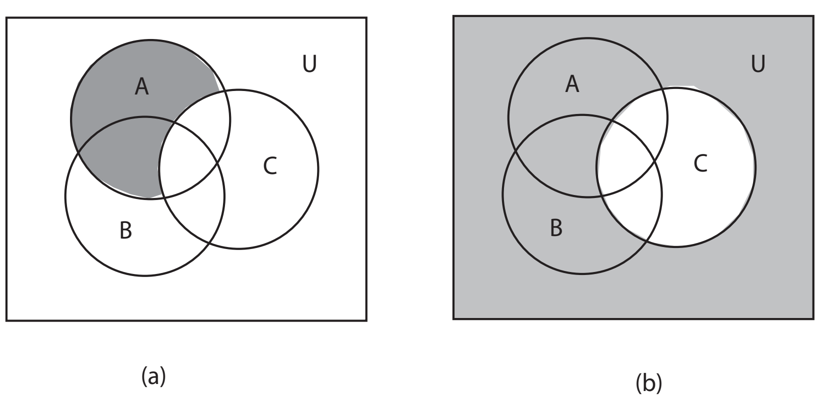

Figure 7.2 (Venn diagrams: difference and complement)

(a) Figure (a), *difference* $A - C$, elements of A not in C:
$\{x : (x \in A)$ and $(x \notin C)\}$ / $\{x : (x \in U)$ and $(x \notin C)\}$

(b) Figure (b), *complement* of C, C', elements of U not in C:
$\{x : (x \in A)$ and $(x \notin C)\}$ / $\{x : (x \in U)$ and $(x \notin C)\}$

4. *More Venn diagrams.*

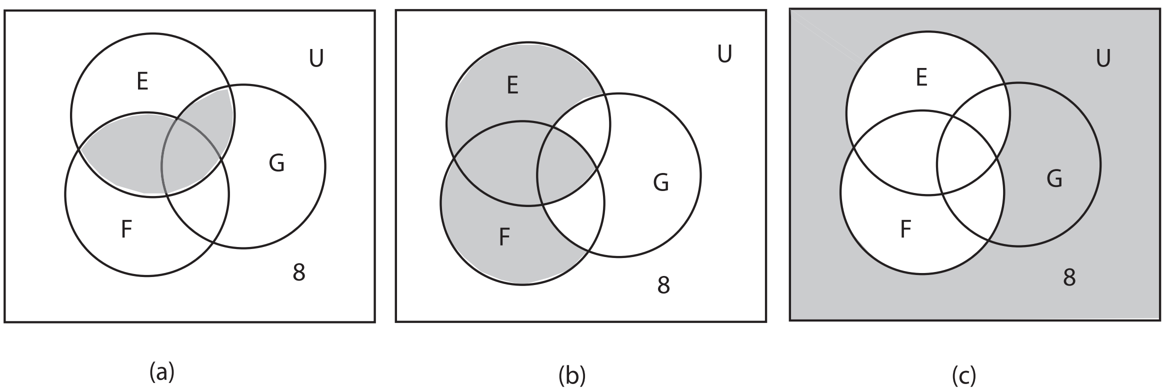

Figure 7.3 (More Venn diagrams)

(a) Shading in figure (a):
$(E - G) \cup (F - G) \cup (E \cap F)$ / $(E \cap F) \cup (E \cap G)$ / $(E \cup F)'$

(b) Shading in figure (b):
$(E - G) \cup (F - G) \cup (E \cap F)$ / $(E \cap F) \cup (E \cap G)$ / $(E \cup F)'$

(c) Shading in figure (c):
$(E - G) \cup (F - G) \cup (E \cap F)$ / $(E \cap F) \cup (E \cap G)$ / $(E \cup F)'$

(d) **True** / **False**. $E - G = E \cap G'$ and $F - G = F \cap G'$

5. *Word problem.* Of students registered at PNC, let D represent those who are disabled, I those who own an iPhone and T those who tweet.

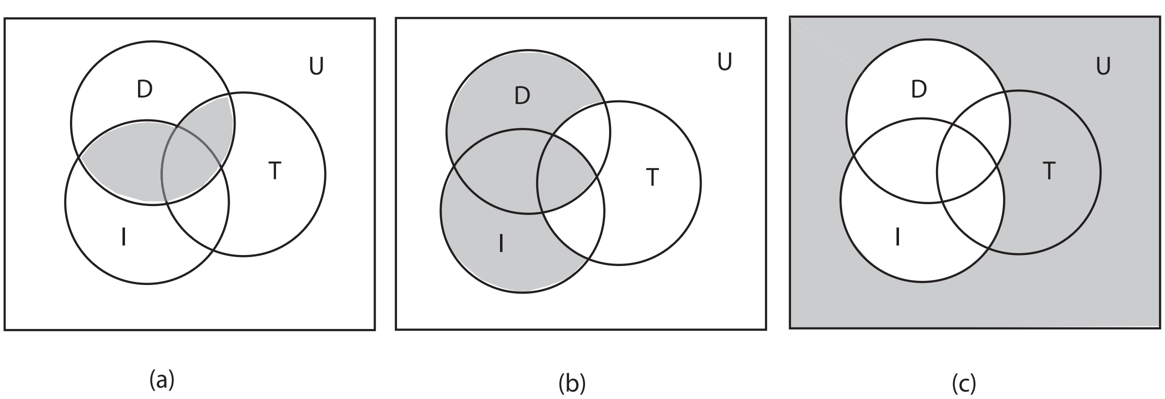

Figure 7.4 (Students at PNC)

(a) Students registered at PNC represented by set $\boldsymbol{D}$ / $\boldsymbol{I}$ / $\boldsymbol{T}$ / $\boldsymbol{U}$

(b) Shaded region of figure (a) represents students

   i. disabled who own iPhones or disabled who tweet.

   ii. disabled who do not tweet, or iPhone owners who do not tweet or disabled who own iPhones

   iii. registered at PNC who are not (disabled or own iPhones)

(c) Shaded region of figure (b) represents students

   i. disabled who own iPhones or disabled who tweet.

   ii. disabled who do not tweet, or iPhone owners who do not tweet or disabled who own iPhones

   iii. registered at PNC who are not (disabled or own iPhones)

(d) Shaded region of figure (c) represents students

   i. disabled who own iPhones or disabled who tweet.

   ii. disabled who do not tweet, or iPhone owners who do not tweet or disabled who own iPhones

   iii. registered at PNC who are not (disabled or own iPhones)

6. *Set operations.* Let
$E_1 = \{a, b, c, d, e, f\}$,
$E_2 = \{e, f, g, h\}$,
$E_3 = \{i\}$
and the universal set is $U = \{a, b, c, d, e, f, g, h, i, j\}$.

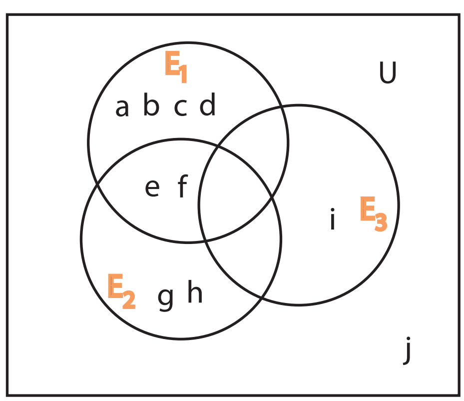

Figure 7.5 (Set operations)

(a) Notice $E_1' = \{g, h, i, j\}$,
so $E_2' = \boldsymbol{\{g, h, i, j\}}$ / $\boldsymbol{\{a, b, c, d\}}$ / $\boldsymbol{\{a, b, c, d, i, j\}}$

(b) Since $E_1 \cup E_2 = \{a, b, c, d, e, f, g, h\}$
and $E_2 \cup E_1 = \boldsymbol{\{g, h, i, j\}}$ / $\boldsymbol{\{a, b, c, d\}}$ / $\boldsymbol{\{a, b, c, d, e, f, g, h\}}$
then $E_1 \cup E_2 = E_2 \cup E_1$ (commutative law)

(c) Since $E_1 \cap E_2 = E_1E_2 = \{e, f\}$
and $E_2E_1 = \boldsymbol{\{e, f\}}$ / $\boldsymbol{\{a, b, c, d\}}$ / $\boldsymbol{\{a, b, c, d, e, f, g, h\}}$
$E_1E_2 = E_2E_1$ (commutative law)

(d) **True** / **False** Intersection of set E_1 and complement, E_1', is empty set, $E_1E_1' = \emptyset$. (Can anything be both in a set and not in a set at same time?)

(e) Notice $E_2 - E_1 = \{g, h\}$
and $E_1 - E_2 = \boldsymbol{\{e, f\}}$ / $\boldsymbol{\{a, b, c, d\}}$ / $\boldsymbol{\{a, b, c, d, e, f, g, h\}}$
so $E_2 - E_1 \neq E_1 - E_2$

7. *Sets and Logic.*
True / **False** If set P consists of elements which make statement p true and set Q consists of elements which make statement q true, sets and logic translate as follows

statement	truth sets
and: $p \wedge q$	intersection: $P \cap Q$
or: $p \vee q$	union: $P \cup Q$
negation: $\sim p$	complement: P'
$p \wedge \sim q$	difference: $P - Q = P \cap Q'$

7.4 Application of Venn Diagrams

We look at how to carefully analysis Venn diagrams, and at various properties of sets, as well how some counting principles:

- *Inclusion-exclusion principle.* "Or" means "add", but do not double-count:

$$\begin{aligned} n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ n(A \cap B) &= n(A) + n(B) - n(A \cup B) \end{aligned}$$

- *Complement principle.*

$$n(A) = n(U) - n(A')$$

Exercise 7.4 (Application of Venn Diagrams)

1. *Labeling Venn diagrams.*

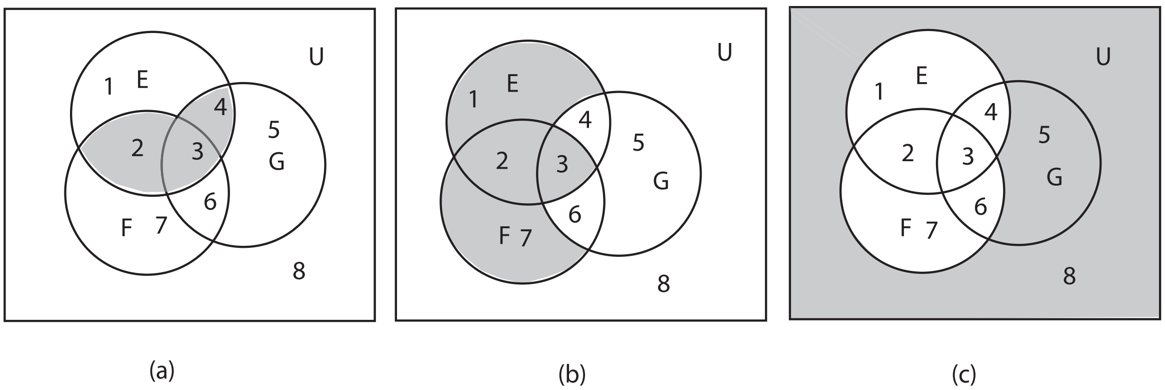

Figure 7.6 (Venn diagrams)

(a) *Shaded region figure (a).* Fill in blank.

Set	region labels
E	1,2,3,4
F	2,3,6,7
G	3,4,5,6
$(E \cap F) \cup (E \cap G)$	______

(b) *Shaded region figure (b).* Fill in blank.

Set	region labels
E	1,2,3,4
F	2,3,6,7
G	3,4,5,6
$(E - G) \cup (F - G) \cup (E \cap F)$	______

(c) *Shaded region figure (c).* Fill in blank.

Set	region labels
E	1,2,3,4
F	2,3,6,7
G	3,4,5,6
$(E \cup F)'$	______

2. *More Venn diagrams and labeling.* Let
   $E_1 = \{a, b, c, d, e, f\}$,
   $E_2 = \{e, f, g, h\}$,
   $E_3 = \{i\}$
   and the universal set is $U = \{a, b, c, d, e, f, g, h, i, j\}$.

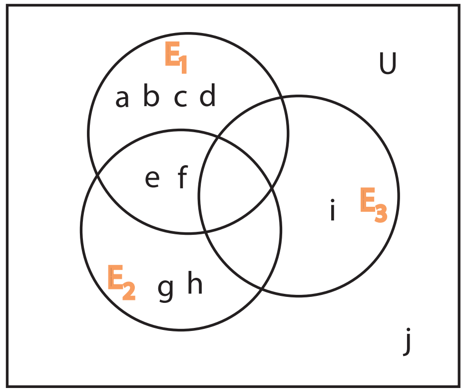

Figure 7.7 (Venn diagrams and counting)

(a) Since $E_1 \cap E_2 = E_1E_2 =$
$\{e, f\}$ / $\{a, b, c, d\}$ / $\{a, b, c, d, i, j\}$
then $(E_1E_2)E_3 =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\emptyset$
On the other hand, since
$E_2E_3 =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\emptyset$
then $E_1(E_2E_3) =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\emptyset$
So $(E_1E_2)E_3 = E_1(E_2E_3)$ (associative law)

(b) **True** / **False** $E_1 \cup (E_2 \cup E_3) = (E_1 \cup E_2) \cup E_3$ (associative law)

(c) Since $E_1 \cup E_2 =$
$\{e, f\}$ / $\{a, b, c, d\}$ / $\{a, b, c, d, e, f, g, h\}$
then $(E_1 \cup E_2)E_3 =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\emptyset$
on the other hand, since
$E_1E_3 =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\emptyset$
and $E_2E_3 =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\emptyset$
then $E_1E_3 \cup E_2E_3 =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\emptyset$
and so $(E_1 \cup E_2)E_3 = E_1E_3 \cup E_2E_3$ (distributive law)

(d) **True** / **False**

$$\begin{aligned}(E_1 \cup E_3)(E_2 \cup E_3) &= E_1E_2 \cup E_1E_3 \cup E_3E_2 \cup E_3E_3 \\ &= E_1E_2 \cup \emptyset \cup \emptyset \cup E_3 \\ &= E_1E_2 \cup E_3\end{aligned}$$

(e) Since $E_1 \cup E_2 =$
$\{e, f\}$ / $\{a, b, c, d\}$ / $\{a, b, c, d, e, f, g, h\}$
then $(E_1 \cup E_2)' =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\{i, j\}$
on the other hand, since
$E_1' =$ $\{g, h, i, j\}$ / $\{a, b, c, d\}$ / $\emptyset$
and $E_2' =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\{a, b, c, d, i, j\}$
then $E_1'E_2' =$ $\{e, f\}$ / $\{a, b, c, d\}$ / $\{i, j\}$
and so $(E_1 \cup E_2)' = E_1'E_2'$ (DeMorgan's Laws).

(f) **True** / **False** $(E_1E_2)' = E_1' \cup E_2'$ (DeMorgan's Laws)

3. *Venn diagrams and counting.*
   Let $A = \{a, b, c, d, e, f\}$,
   $B = \{b, d, e, h\}$,
   $C = \{a, e\}$
   and the universal set is $U = \{a, b, c, d, e, f, g, h, i, j\}$. Let n be "number".

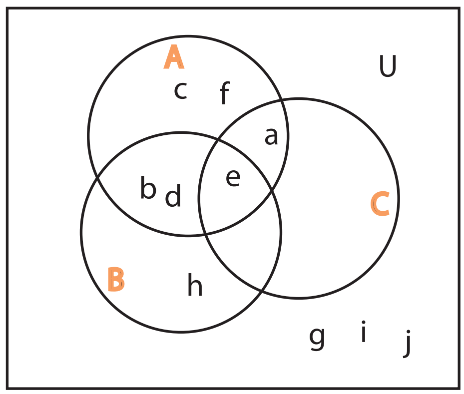

Figure 7.8 (Venn diagrams and counting)

(a) $n(A) = n\{a, b, c, d, e, f\} =$ **5** / **6**

(b) $n(B) = n\{b, d, e, h\} =$ **1** / **2** / **3** / **4**,

(c) $n(C) = n\{a, e\} =$ **1** / **2** / **3** / **4**,

(d) $n(A \cap B) = n\{b, d, e\} =$ **1** / **2** / **3** / **4**,

(e) $n(A \cap C) = n\{a, e\} =$ **1** / **2** / **3** / **4**,

(f) $n(B \cap C) = n\{e\} =$ **1** / **2** / **3** / **4**,

(g) $n(A \cap B \cap C) = n\{e\} =$ **1** / **2** / **3** / **4**,

(h) $n(A \cup B) = n(A) + n(B) - n(A \cap B) =$
$= n\{a, b, c, d, e, f\} + n\{b, d, e, h\} - n\{b, d, e\} =$ **5** / **6** / **7** / **8**,

(i) $n(A \cup C) = n\{a, b, c, d, e, f\} =$ **5** / **6** / **7** / **8**,

(j) $n(B \cup C) = n\{a, b, d, e, h\} =$ **5** / **6** / **7** / **8**,

(k) $n(A \cup B \cup C) =$
$= n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) =$
$= n\{a, b, d, c, e, f, h\} =$ **5** / **6** / **7** / **8** / **9**,

(l) $n(A') = n(U) - n(A) = n\{g, h, i, j\} =$ **1** / **2** / **3** / **4**,

(m) $n(B') = n\{a, c, f, g, i, j\} =$ **5** / **6** / **7** / **8**,

(n) $n(C') = n\{b, c, d, f, g, h, i, j\} =$ **5** / **6** / **7** / **8**,

(o) $n(A' \cap B') = n\{g, i, j\} =$ **1** / **2** / **3** / **4**,

(p) $n(A' \cup B') = n(A') + n(B') - n(A' \cap B') =$
$= n\{g,h,i,j\} + n\{a,c,f,g,i,j\} - n\{g,i,j\} =$ **5** / **6** / **7** / **8**,

(q) $n(A' \cup C') = n(A') + n(C') - n(A' \cap C') =$
$= n\{g,h,i,j\} + n\{b,c,d,f,g,h,i,j\} - n\{g,h,i,j\} =$ **5** / **6** / **7** / **8**.

(r) $n(A' \cup C) = n(A') + n(C) - n(A' \cap C) =$
$= n\{g,h,i,j\} + n\{a,e\} - n\{\emptyset\} =$ **5** / **6** / **7** / **8**,

4. *More counting.*
Study investigates effect of nutritional level on plant growth.

	nutritional level →	poor	adequate	excellent	row totals
plant	below average	100	75	65	240
growth	above average	60	50	40	150
	column totals	160	125	105	390

(a) n(below average growth) = **100** / **160** / **240**

(b) n(poor nutrition) = **100** / **160** / **240**

(c) n(below average growth *and* poor nutrition) = **100** / **160** / **240**

(d) n(below average growth *or* poor nutrition) =
n(below ave growth) + n(poor nutr) − n(below aver growth *and* poor nutr)
= **100** / **300** / **400**