Final for Mathematics 223
Introductory Analysis I - Fall 2000
Material Covered: Chapters 1–4, B.1, C.1, C.2 of Workbook and text
For: 11th December

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): ____________________ . **ID Number:** ____________.
last first

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1. Consider the function $f(x) = \frac{x^2+4}{(x-3)(x-1)}$.

(a) [1 point] The domain of this function is (circle one)

(i) $\{x|x = 1, x = 3\}$
(ii) $\{x| -\infty < x < \infty\}$
(iii) $\{x| -\infty < x < \infty, x \neq 1, x \neq 3\}$
(iv) $\{x| -\infty < x < \infty, x = 1, x = 3\}$
(v) $\{x| -\infty < x < \infty, x \neq -2, x \neq 2\}$

(b) [1 point] **True** / **False** There is no x such that $f(x) = -2$.
(Hint: Use your calculator to graph $f(x) = \frac{x^2+4}{(x-3)(x-1)}$.)

2. Let x be the study time (in hours) and y be the test grade (out of 100%). For example, $(x, y) = (11, 53)$ means that a person who studies 11 hours gets a test grade of 53%.

(a) [1 point] The linear function that fits the points $(11, 53)$ and $(24, 92)$ is (circle one)

(i) $y = -3x + 20$ (ii) $y = 3x + 20$ (iii) $y = -20x - 3$ (iv) $y = 3x - 20$ (v) $y = 20x + 3$

(b) [1 point] Using the linear function in part (a), the predicted test score for a student who studies 16 hours, is (circle one) **68** / **73** / **74** / **77** / **78**.

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3. Consider the function $f(x) = \frac{(x-1)(x^2+2x+4)}{(x^2+5x-6)}$.

(a) [1 point] $\lim_{x \to 1} f(x) =$ (circle one) **1** / **2** / $-\infty$ / ∞ / **undefined**.

(b) [1 point] $\lim_{x \to -6^+} f(x) =$ (circle one) **1** / **2** / $-\infty$ / ∞ / **undefined**.

(c) [1 point] At $x = 1$, $f(x)$ (circle one)

(i) does not have a limit, is not continuous and is not differentiable.

(ii) does not have a limit, is continuous and is not differentiable.

(iii) has a limit, is not continuous and is differentiable.

(iv) has a limit, is continuous and is not differentiable.

(v) has a limit, is not continuous and is not differentiable.

4. Let $f(x) = -5x^3 + 4x + 1$ and $g(x) = \frac{2x+2}{x^4}$.

(a) [1 point]

$f'(x) =$ ____.

(b) [1 point]

$g'(x) =$ ____.

(c) [1 point]

$\frac{d}{dx}(f \circ g) =$ ____.

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5. Consider the function $f(x) = 2x^3 - 3\sqrt[4]{x}$.

(a) [1 point] The slope of the tangent line to the graph of this function at $x = 3$ is approximately (circle one) **53.67** / **63.16** / **74.00** / **71.23** / **78.55**.

(b) [1 point] The equation of the tangent line to the graph of this function at $x = 3$ is approximately (circle one)

(i) $y - 50.05 = 53.67(x - 3)$.

(ii) $y + 13.65 = 63.17(x - 3)$.

(iii) $y + 100.00 = 74.00(x - 3)$.

(iv) $y + 81.02 = 71.23(x - 3)$.

(v) $y - 17.49 = 78.55(x - 3)$.

6. [3 points] Circle **True** or **False**.

(a) **True** / **False** A *relation* is a correspondence between the domain and the range such that each member of the domain corresponds to exactly one member of the range.

(b) **True** / **False** We say y is *directly proportional* to x if there is some positive constant m such that $y = mx$.

(c) **True** / **False** A *polynomial function* f is given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1$$

where n is a nonnegative integer and a_n, a_{n-1}, $\ldots$, a_1 are real numbers.

(d) **True** / **False** A function f is *continuous* if $f(a)$ exists, $\lim_{x \to a} f(x)$ exists and $\lim_{x \to a} f(x) = f(a)$.

(e) **True** / **False** The *average rate of change* of f with respect to x is also called the difference quotient.

(f) **True** / **False** Suppose that f is a function for which $f'(x)$ exists for every x in an open interval (a, b) contained in its domain and that there is a critical point in (a, b) for which $f'(c) = 0$. Then

1. $f(c)$ is a relative minimum if $f''(c) < 0$
2. $f(c)$ is a relative maximum if $f''(c) > 0$

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7. [2 points] A hardware store sells 250 wheelbarrows per year. It costs \$3 to store one wheelbarrow for one year. To reorder, there is a fixed cost of \$4 for each *lot* of wheelbarrows, plus \$0.50 for each wheelbarrow in the lot. How many times per year should the store order wheelbarrows and in what lot size, in order to minimize inventory costs?

8. Try the following questions on identifying the critical points of a function and on sketching functions.

(a) [2 points] Consider the following table that arises after undertaking the First Derivative method for locating the maximum/minimum of a function $f(x)$, where $f'(-2) = 0$, $f'(4) = 0$ and $f'(7) = 0$.

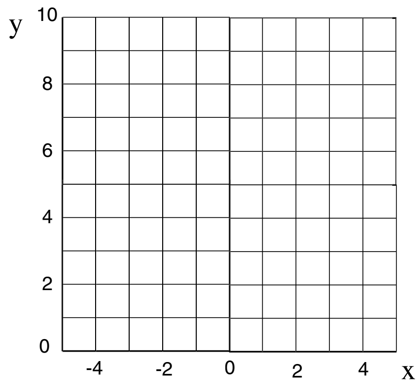
interval	$(-\infty, -2)$	$(-2, 4)$	$(4, 7)$	$(7, \infty)$
test value	$x = -4$	$x = 1$	$x = 6$	$x = 15$
sign of $f'(x)$	$f'(-4) > 0$	$f'(1) < 0$	$f'(6) > 0$	$f'(15) > 0$

For each of the four points below, circle whether they are either relative maximum, relative minimum, inflection points or none of these.

$x = -2$	**maximum**	**minimum**	**inflection**	**none of these**
$x = 1$	**maximum**	**minimum**	**inflection**	**none of these**
$x = 4$	**maximum**	**minimum**	**inflection**	**none of these**
$x = 7$	**maximum**	**minimum**	**inflection**	**none of these**

(b) [2 points] Sketch a function on the coordinate system below which satisfies the following conditions.

1. f is continuous everywhere, except at $x = 0$
2. $f(-3) = 7$, $f(2) = 3$
3. $f'(-3) = 0$, $f'(2) = 0$
4. $f'(x) > 0$ on $(-\infty, -3)$ and $f'(x) < 0$ on $(2, \infty)$
5. $\lim_{x \to 0^-} = -\infty$ and $\lim_{x \to 0^+} = -\infty$



9. Try a couple of derivative–of–trigonometric–functions questions.

(a) [1 point] Consider the equation $y = \cos^2 x$.

Then $\frac{dy}{dx} =$ ____.

(b) [1 point] Consider the equation $y = \sin(2x^3 + 5x)\cos^3 x$.

Then $\frac{dy}{dx} =$ ____.

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10. [2 points] If the half–life of radioactive northcentrillium is 345 years, how much of 42 grams of northcentrillium will be left after 799 years? [Hint: Use the exponential decay model.] (Circle closest one)

(i) 7.97 (ii) 8.32 (iii) 8.43 (iv) 8.88 (v) 9.01

1. (a) **(iii)** $\{x| -\infty < x < \infty, x \neq 1, x \neq 3\}$;
(b) **True** There is no x such that $f(x) = -2$

2. (a) **(ii)** $y = 3x + 20$
(b) **(i) 68**

3. (a) **(i) 1**;
(b) **(iv)** $\boldsymbol{\infty}$
(c) **(v)** has a limit, is not continuous and is not differentiable

4. (a) $f'(x) = -15x^2 + 4$; (b) $g'(x) = \frac{-6x^4+8x^3}{x^8} = \frac{-6x+8}{x^5}$;
(c) $\frac{d}{dx}(f \circ g) = \left(-15\left(\frac{2x+2}{x^4}\right)^2 + 4\right)\left(\frac{-6x+8}{x^5}\right)$.

5. (a) **(i) 53.67**;
(b) **(i)** $y - 50.05 = 53.67(x - 3)$

6. (a) **False** (at least one, not exactly one)
(b) **True**
(c) **False** (forgot a_0)
(d) **True**
(e) **True**
(f) **False** ($f''(c) > 0$ and $f''(c) < 0$ are reversed)

7. minimize $f(x) = \frac{x}{2}(3) + \left(4 + \frac{1}{2}x\right)\left(\frac{250}{x}\right) = 1.5x + 1000x^{-1} + 125, \quad 0 < x < 250$
since $f'(x) = 1.5 - 1000x^{-2} = 0$ at $x \approx 25.82$
and since $f(0)$ is undefined, $f(25.82) \approx 202.46$ and $f(250) = 504$,
lot size should be $x = 25.82$ wheelbarrows and $\frac{250}{25.82} \approx 9.68$ lots should be ordered per year to minimize inventory costs

8. (a) **maximum** (since $f'(-4) > 0$ then $f'(1) < 0$)
none of these (although $f'(1) < 0$)
minimum (since $f'(1) < 0$ then $f'(6) > 0$)
none of these (since $f'(6) > 0$ and $f'(15) > 0$)
(b) outline of two hills, left higher than right, with vertical asymptote at $x = 0$

9. (a) $\boldsymbol{-2 \sin x \cos x}$
(b) $\boldsymbol{(\cos^3 x)(\cos(2x^3 + 5x)(6x^2 + 5)) + \sin(2x^3 + 5x)(-3\cos^2 x \sin x)}$

10. **(iii) 8.43**
at $t = 345$, $P = 42e^{-k(345)} = \frac{42}{2} = 21$
so $-345k = \ln 0.5$ or $k \approx 0.002009$
and so at $t = 799$, $P = 42e^{-0.002009(799)} \approx 8.43$ grams is left