Final for Mathematics 223
Introductory Analysis I - Fall 2001
Material Covered: Chapters 1–4, B.1, C.1, C.2 of workbook and text
For: 10th December

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): ____________________ . **ID Number:** ____________.

last first

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1. Consider the following differential equations.

(a) [1 point] Solve $y' = -4y$, $y(0) = -7$. Circle one.
$\boldsymbol{y = -4e^{\ 7y}}$ / $\boldsymbol{y = -e^{\ 4t}}$ / $\boldsymbol{y = -7e^{\ y}}$ / $\boldsymbol{y = -7e^{\ 4t}}$ / $\boldsymbol{y = 7e^{\ 4t}}$

(b) [1 point] Solve $y' = 5y(4-y)$, $y(0) = 2$. Circle one.
$\boldsymbol{y = \frac{2}{5+4e^{\ 20t}}}$ / $\boldsymbol{y = \frac{4}{5e^{\ 20t}}}$ / $\boldsymbol{y = \frac{4}{4e^{\ 20t}}}$ / $\boldsymbol{y = \frac{8}{2+2e^{\ 20t}}}$ / $\boldsymbol{y = \frac{4}{2+2e^{\ 10t}}}$

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2. Consider the following questions on trigonometric functions.
Assume x is in radians, not degrees.

(a) [1 point] $\lim_{x\to 0} \frac{3x-\sin^2 x}{x}$ = (circle closest one) **0 / 1 / 2 / 3 / 4**

(b) [1 point] Find $\frac{d}{dx}\left(\frac{3x-\sin^2 x}{x}\right)$. Circle one.

- **(i)** $\frac{\sin^2(x)-2x\sin x\cos x}{x^2}$
- **(ii)** $\frac{-\sin^2(x)-2x\sin x\cos x}{x^2}$
- **(iii)** $\frac{\sin^2(x)+2x\sin x\cos x}{x^2}$
- **(iv)** $\frac{\sin^2(x)-x\sin x\cos x}{x^2}$
- **(v)** $\frac{\sin^2(x)-2x\sin x\cos x}{x}$

3. Consider the following demand and supply functions,
$D(x) = (x-5)^2$, $S(x) = 2x^2 + 4x$, where $0 \le x \le 15$.

(a) [1 point] The equilibrium point is (circle closest one)
$\mathbf{(1.60, 8.54)}$ / $\mathbf{(1.60, 9.54)}$ / $\mathbf{(1.60, 10.54)}$ / $\mathbf{(1.60, 11.54)}$ / $\mathbf{(1.60, 12.54)}$.

(b) [1 point] $\lim_{x\to\infty} \frac{D(x)}{S(x)}$ = (circle closest one)
0.25 / **0.50** / **0.75** / **1.00** / **1.25**.

(c) [1 point] At $x = -1$, $\frac{D(x)}{S(x)}$ (circle one)

- **(i)** does not have a limit, is not continuous and is not differentiable.
- **(ii)** does not have a limit, is continuous and is not differentiable.
- **(iii)** has a limit, is not continuous and is differentiable.
- **(iv)** has a limit, is not continuous and is not differentiable.
- **(v)** has a limit, is continuous and is differentiable.

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4. Consider the following piecewise function.

$$f(x) = \begin{cases} 3x - 4 & \text{if } x \le -3 \\ 2x^4 + 4x & \text{if } -3 < x \le 4 \\ x^2 + 2x + 2 & \text{if } 4 < x \end{cases}$$

(a) [1 point] $\lim_{x\to -3^-} f(x)$ = (circle closest one)
−13 / **−12** / **−11** / **100** / **150**.

(b) [1 point] The function $f(x)$ is not continuous when (circle none, one or more)
$\boldsymbol{x = -6}$ / $\boldsymbol{x = -3}$ / $\boldsymbol{x = 0}$ / $\boldsymbol{x = 2}$ / $\boldsymbol{x = 4}$

(c) [1 point] $\lim_{x\to -3^+} \frac{f(x)}{x^2}$ = (circle closest one)
13.3 / **14.6** / **15.5** / **16.7** / **17.2**.

5. Consider the following piecewise function.

$$f(x) = \begin{cases} 3x^2 + 4 & \text{if } x < 4 \\ \frac{5}{x} & \text{if } 4 \leq x \end{cases}$$

(a) [1 point] The difference quotient, $\frac{f(x+h)-f(x)}{h}$, is (circle closest one)

(i)

$$\frac{f(x+h) - f(x)}{h} = \begin{cases} 6xh + 3h & \text{if } x < 4 \\ \frac{-5}{x^2+xh} & \text{if } 4 \leq x \end{cases}$$

(ii)

$$\frac{f(x+h) - f(x)}{h} = \begin{cases} 6x + 3h & \text{if } x < 4 \\ \frac{-5}{x^2+xh} & \text{if } 4 \leq x \end{cases}$$

(iii)

$$\frac{f(x+h) - f(x)}{h} = \begin{cases} 6x + 3h^2 & \text{if } x < 4 \\ \frac{-5h}{x^2+xh} & \text{if } 4 \leq x \end{cases}$$

(iv)

$$\frac{f(x+h) - f(x)}{h} = \begin{cases} 6x + 3h & \text{if } x < 4 \\ \frac{-5}{x^2h+xh} & \text{if } 4 \leq x \end{cases}$$

(v)

$$\frac{f(x+h) - f(x)}{h} = \begin{cases} 6x + 3 & \text{if } x < 4 \\ \frac{-5}{x^2+xh} & \text{if } 4 \leq x \end{cases}$$

(b) [1 point] The limit, as h tends to zero, of the difference quotient is (circle one)

(i)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \begin{cases} 6 & \text{if } x < 4 \\ \frac{-5}{x^2} & \text{if } 4 \leq x \end{cases}$$

(ii)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \begin{cases} 6x & \text{if } x < 4 \\ \frac{5}{x^2} & \text{if } 4 \leq x \end{cases}$$

(iii)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \begin{cases} 6 & \text{if } x < 4 \\ \frac{5}{x^2} & \text{if } 4 \leq x \end{cases}$$

(iv)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \begin{cases} 6x & \text{if } x < 4 \\ \frac{-5}{x^2} & \text{if } 4 \leq x \end{cases}$$

(v)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \begin{cases} 6x & \text{if } x < 4 \\ \frac{-5}{x} & \text{if } 4 \leq x \end{cases}$$

6. Consider the function $f(x) = 3x^5 - 5x^3$.

(a) [1 point] The tangent line of $f(x)$ at $x = -1$ is (circle one)

- **(i)** $y = 2$
- **(ii)** $y - 1 = 2(x - 2)$
- **(iii)** $y + 1 = 2(x - 2)$
- **(iv)** $y - 1 = 2(x + 2)$
- **(v)** $y - 1 = -2(x - 2)$

(b) [1 point] The horizontal tangent lines of $f(x)$ are at (circle none, one or more)
$\mathbf{(-1,-1)}$ / $\mathbf{(-1,2)}$ / $\mathbf{(0,0)}$ / $\mathbf{(1,-1)}$ / $\mathbf{(1,-2)}$.

(c) [1 point] The tangent lines with slope -3 are at (circle none, one or more)
$\boldsymbol{x = -0.851}$ / $\boldsymbol{x = -0.526}$ / $\boldsymbol{x = 0}$ / $\boldsymbol{x = 0.526}$ / $\boldsymbol{x = 0.851}$.

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7. Consider the function $y = 4(-3x^2 + 5)^4 + 7(-3x^2 + 5)^3 - (-3x^2 + 5) + 5$.

(a) [1 point] Let $y = f \circ g(x)$. Then $f(x) =$ (circle one)

- **(i)** $-3x^2 + 5$
- **(ii)** $4x^4 + 7x^3 - x$
- **(iii)** $4x^4 + 7x^3 - x - 5$
- **(iv)** $4(-3x^2 + 5)^4 + 7(-3x^2 + 5)^3 - (-3x^2 + 5) + 5$
- **(v)** $4x^4 + 7x^3 - x + 5$

(b) [1 point] $\frac{dy}{dx} = f'(g)g' =$ (circle one)

- **(i)** $[4(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 - 1]$
- **(ii)** $[16(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 - 1](6x)$
- **(iii)** $[16(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 + 1](-6x)$
- **(iv)** $[16(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 - 1]$
- **(v)** $[16(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 - 1](-6x)$

(c) [1 point] $\frac{d}{dx}\left(\frac{\frac{dy}{dx}}{-6x}\right) =$ (circle one)

- **(i)** $[4(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 - 1]$
- **(ii)** $[16(-3x^2 + 5)^2 + 21(-3x^2 + 5)^2](-6x)$
- **(iii)** $[48(-3x^2 + 5)^2 + 42(-3x^2 + 5)](-6x)$
- **(iv)** $[16(-3x^2 + 5)^2 + 21(-3x^2 + 5)^2 - 1]$
- **(v)** $[48(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 - 1](-6x)$

8. The number of people in a city of size 50,000 who will contract the flu within the first t days after the first reported case can be predicted using the formula, $N(t) = 45t^2 - 3.5t^3$, $0 \le t \le 12$.

(a) [1 point] What day after the outbreak (after $t = 0$) will the maximum number of people get the flu? Circle closest one.
$\frac{\mathbf{60}}{\mathbf{4}}$ / $\frac{\mathbf{60}}{\mathbf{5}}$ / $\frac{\mathbf{60}}{\mathbf{6}}$ / $\frac{\mathbf{60}}{\mathbf{7}}$ / $\frac{\mathbf{60}}{\mathbf{8}}$

(b) [1 point] When is the flu spreading most rapidly? Circle closest one.
$\frac{\mathbf{90}}{\mathbf{20}}$ / $\frac{\mathbf{90}}{\mathbf{21}}$ / $\frac{\mathbf{90}}{\mathbf{22}}$ / $\frac{\mathbf{90}}{\mathbf{23}}$ / $\frac{\mathbf{90}}{\mathbf{24}}$

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9. Consider the equation $3y - 3y^3 = x$.

(a) [1 point] $\frac{dy}{dx}$ = (circle one)

(i) $(3 - 9y^2)^2$
(ii) $(3 - 9y^2)$
(iii) $(3 - 9y^2)^{-1}$
(iv) $(3 - 9y^2)^{-2}$
(v) $(3 - 9y^2)^{-3}$

(b) [1 point] $\frac{d^2y}{dx^2}$ = (circle one)

(i) $18y(3 - 9y^2)^2$
(ii) $18y(3 - 9y^2)$
(iii) $18y(3 - 9y^2)^{-1}$
(iv) $18y(3 - 9y^2)^{-2}$
(v) $18y(3 - 9y^2)^{-3}$

10. Consider the functions $f(x) = -7(3)^{3x}$ and $g(x) = e^{2x^2-3}$

(a) [1 point] $f'(x) =$ (circle one)

- **(i)** $-7(3)^{3x}$
- **(ii)** $-7(3)^{3x}\ln 3$
- **(iii)** $-21(3)^{7x}\ln 3$
- **(iv)** $-21(7)^{3x}\ln 3$
- **(v)** $-21(3)^{3x}\ln 3$

(b) [1 point] $g'(x) =$ (circle one)

- **(i)** $(4x)e^{2x^2-3}$
- **(ii)** e^{2x^2-3}
- **(iii)** $(8x)e^{2x^2-3}$
- **(iv)** $(4x-3)e^{2x^2-3}$
- **(v)** $(4x)e^{2x^2-5}$

(c) [1 point] $\frac{d(fg)}{dx} =$ (circle one)

- **(i)** $e^{2x^2-3}(3)^{3x}\ln 3 + (-7)(3)^{3x}(4x)e^{2x^2-3}$
- **(ii)** $e^{2x^2-3}(-21)(3)^{3x}\ln 3 + (3)^{3x}(4x)e^{2x^2-3}$
- **(iii)** $e^{2x^2-3}(3)^{3x}\ln 3 + (3)^{3x}(4x)e^{2x^2-3}$
- **(iv)** $e^{2x^2-3}(-21)(3)^{3x}\ln 3 + (-7)(3)^{3x}(4x)e^{2x^2-3}$
- **(v)** $e^{2x^2-3}(-21)(3)^{3x}\ln 3 + (-7)(3)^{3x}e^{2x^2-3}$

(1) (a) **(iv)** $y = -7e^{\ 4t}$ (b) **(iv)** $y = \frac{8}{2+2e^{\ 20t}}$

(2) (a) **3** (b) **(i)** $\frac{sin^2(x) \quad 2x \sin x \cos x}{x^2}$

(3) (a) **(i)** $(1.60, 11.54)$
(b) **(ii) 0.50**
(c) **(v) has a limit (equal to** -18**),**
is continuous at $x = -1$ **(but is not continuous at** $x = 0, -2$**)**
and is differentiable (with derivative equal to -2 **at** $x = -1$**).**

(4) (a) -13 (b) $x = -3$, $x = 4$ (c) **16.7**

(5) (a) (ii) (b) (iv)

(6) (a) (i) $y = 2$
(b) $(-1, 2)$, $(0, 0)$, $(1, -2)$
(c) $x = -0.851$, $x = -0.526$, $x = 0.526$, $x = 0.851$

(7) (a) **(v)** $4x^4 + 7x^3 - x + 5$
(b) **(v)** $[16(-3x^2 + 5)^3 + 21(-3x^2 + 5)^2 - 1](-6x)$
(c) **(iii)** $[48(-3x^2 + 5)^2 + 42(-3x^2 + 5)](-6x)$

(8) (a) **(iv)** $\frac{60}{7}$
(b) **(ii)** $\frac{90}{21}$

(9) (a) **(iii)** $(3 - 9y^2)^{\ 1}$ (b) **(v)** $18y(3 - 9y^2)^{\ 3}$

(10) (a) **(v)** $-21(3)^{3x} \ln 3$
(b) **(i)** $(4x)e^{2x^2 \ 3}$
(c) **(iv)** $e^{2x^2 \ 3}(-2)(3)^{3x} \ln 3 + (-7)(3)^{3x}(4x)e^{2x^2 \ 3}$