Final for Mathematics 223
Introductory Analysis I - Fall 1999
Material Covered: Chapters 1–6 of Workbook and text
15th December

This is a 2 hour final, worth 22% and marked out of 22 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): _____ . **ID Number:** _____.
last first

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1. A couple of chapter 1 questions.

(a) [2] The line through the intersection of $4x + 3y = 2$ and $x + 5y = -8$ which has slope $-\frac{2}{5}$ is given by (circle one)

(i) $y - 2 = -\frac{2}{5}(x + 2)$ (ii) $y + 2 = -\frac{2}{5}(x + 2)$

(iii) $y - 2 = -\frac{2}{5}(x - 2)$ (iv) $y + 2 = -\frac{2}{5}(x - 2)$

(v) $y - 2 = \frac{2}{5}(x + 2)$

(b) [1] The value of k for which the line $4x + ky = 5$

passes through the point $(1, -1)$ is _____.

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2. More chapter 1 questions.

(a) [1] If $g(x) = 2\sqrt{x} - 4$, $\frac{g(x+h)-g(x)}{h}$ is (circle one)

(i) $\frac{2\sqrt{x+h}-2\sqrt{x}}{x+h}$ (ii) $\frac{2\sqrt{x+h}-2\sqrt{x}-4}{h}$ (iii) $\frac{2\sqrt{x+h}-2\sqrt{x}-8}{h}$

(iv) $\frac{2\sqrt{x+h}-2\sqrt{x}}{h}$ (v) $\frac{2\sqrt{x+h}+2\sqrt{x}}{h-4}$

(b) [1] The function $f(x) = \sqrt{|x| + 1}$ is (circle one) **odd / even / cubic / a polynomial / composite**.

3. A couple of chapter 2 questions.

(a) [1] $\lim_{x\to 0} \frac{x^3-x^2}{x^2+2x} =$ _____.

(b) [1] If $y = \sqrt[4]{x}x^5x^{-\frac{5}{4}}$, then $\frac{dy}{dx}$ = (circle one) $\mathbf{5x^4}$ / $\mathbf{4x^{-\frac{3}{4}}}$ / $\mathbf{3\sqrt[3]{x}}$ / $\mathbf{16x^3}$ / $\mathbf{4x^3}$

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4. More chapter 2 questions.

(a) [1] If $f(x) = \frac{3x^2-5x}{(x+5)^3}$, then $f'(x)$ = (circle one)

(i) $\frac{(x+5)^3(6x-5)-2(3x^2-5x)(x+5)}{(3x^2-5x)^2}$ (ii) $\frac{(x+5)(6x-5)-3(3x^2-5x)}{(x+5)^4}$

(iii) $\frac{(x+5)^3(6x-5)-2(3x^2-5x)(x+5)^2}{(x+5)^4}$ (iv) $\frac{(x+5)^2(6x-5)-2(3x^2-5x)(x+5)^2}{(x+5)^6}$

(v) $\frac{(x+5)^2(6x-5)-2(3x^2-5x)(x+5)}{(x+5)^4}$

(b) [1] If $y = (x^2+1)^3$, then $f'''(1)$ = (circle one) **92** / **118** / **157** / **173** / **192**

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5. A couple of chapter 3 questions.

(a) [1] Let

$$f(x) = \begin{cases} \frac{x^3-9x}{2x-6} & \text{if } x \neq 3 \\ 3 & \text{if } x = 3, \end{cases}$$

then, at $x = 3$, $f(x)$ (circle one)

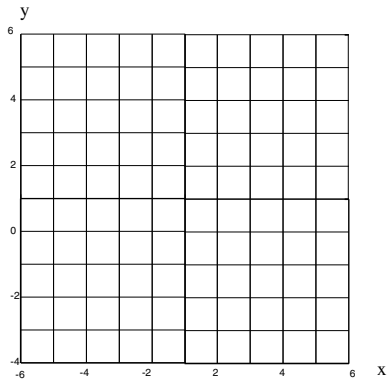
(i) does not have a limit, is discontinuous and nondifferentiable.
(ii) does not have a limit, is continuous and nondifferentiable.
(iii) has a limit, is discontinuous and differentiable.
(iv) has a limit, is continuous and nondifferentiable.
(v) has a limit, is discontinuous and nondifferentiable.

(b) [1] All of the inflection points for $f(x) = x^3 - 6x^2 + 9x - 3$

are (x, y) = _____.

6. [2] Sketch a function on the graph below which satisfies the following conditions:

- f is continuous everywhere except at $x = 5$
- $f(0) = 4$
- $\lim_{x\to 5^-} f(x) = 3$, $\lim_{x\to 5^+} f(x) = -2$
- $f'(x) = 0$ on $(-\infty, 3)$
- $f''(x) > 0$ on $(3, 5)$ and $f''(x) < 0$ on $(5, \infty)$



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7. More chapter 3 questions.

(a) [1] The vertical asymptote(s) for $f(x) = \frac{3x^2}{-3x^2+4}$, is/are (circle none, one or more)

$\boldsymbol{x = \sqrt{\frac{3}{4}}}$ / $\boldsymbol{x = \sqrt{\frac{4}{3}}}$ / $\boldsymbol{x = -\sqrt{\frac{4}{3}}}$ / $\boldsymbol{y = \sqrt{\frac{4}{3}}}$ / $\boldsymbol{y = -1}$.

(b) [1] The

$\lim_{x\to-\infty} \frac{2\sqrt[3]{x}}{4+\sqrt{x}} =$ _____.

8. A couple of chapter 4 questions.

(a) [2] An agile circus dog runs along a wire which has the shape of a parabola with equation $y = 15 + 0.01x^2$, $-20 \leq x \leq 20$, where x and y are in meters. Suppose that the horizontal component of the speed is $\frac{dx}{dt} = 4$ meters per second. The vertical component of the speed $\frac{dy}{dt}$ at the instant when the dog is at the point $(10, 16)$ is (circle one) **0.8 / 1.0 / 1.2 / 1.4 / 1.6**

(b) [1] If $3t - t^2 + x^2 + 4xt = 0$, then $\frac{dx}{dt} =$

(i) $\frac{2t+4x-3}{2x+4t}$ (ii) $\frac{2t-4x-3}{2x-4t}$ (iii) $\frac{2t-4x-3}{2x+4t}$ (iv) $\frac{2t-4x+3}{2x+4t}$ (v) $\frac{2t-4x-3}{2x-4t}$

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9. A couple of chapter 5 questions.

(a) [1] Consider the equation $y = 5^{3e^{2x}}$. Then $\frac{dy}{dx} =$ (circle one)

(i) $6e^{2x}5^{3e^{2x}} \ln 3e^{2x}$

(ii) $5e^{2x} \ln 6e^{2x}$

(iii) $6e^{2x}6e^{2x} \ln 5$

(iv) $6e^{2x}5^{3e^{2x}} \ln 5$

(v) $5e^{2x} \ln 5$

(b) [1] Consider the equation $y = \log_3(2x)$.

Then $\frac{dy}{dx} =$ _____.

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10. [2] Welsh Pembroke Corgis can only grow to a maximum height of 1.5 feet long. If Obie (a Welsh Pembroke Corgi) is now 1.1 feet tall, how long will she be 4 years from now, if $k = 0.2$? (Circle closest one)

(i) 1.23 (ii) 1.29 (iii) 1.32 (iv) 1.39 (v) 1.43

[Hint: Use the simple bounded model, $f'(t) = k(L - y)$, where $f(0) = y_0$.]

(1) (a) iv; (b) -1.

(2) (a) iv; (b) even.

(3) (a) 0; (b) $4x^3$

(4) (a) ii; (b) 192.

(5) (a) v; (b) (2,-1).

(6) sketch.

(7) (a) $x = \sqrt{\frac{4}{3}}$, $x = -\sqrt{\frac{4}{3}}$; (b) undefined.

(8) (a) 0.8; (b) iii.

(9) (a) iv; (b) $\frac{2}{2x \ln 3}$

(10) iii.