Quiz 5 for Mathematics 223
Introductory Analysis I - Fall 1999
Material Covered: Sections 4.4,4.5 of workbook and text
For: 5th November

This is a 15 minute quiz, worth 6% and marked out of 6 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on one side of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): ______________________ . **ID Number:** __________.

last first

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1. [2] The equation of the tangent line to the curve defined by $x^2 - 3xy + 4y^2 = 7$ at $(\sqrt{\frac{7}{2}}, 0)$ is (circle one)

(a) $y = -\frac{2}{8}\left(x - \sqrt{\frac{7}{2}}\right)$ (b) $y = \frac{2}{3}\left(x - \sqrt{\frac{7}{2}}\right)$ (c) $y = \frac{4}{8}\left(x - \sqrt{\frac{7}{2}}\right)$
(d) $y = -\frac{3}{8}\left(x - \sqrt{\frac{7}{2}}\right)$ (e) $y = -\frac{6}{8}\left(x - \sqrt{\frac{7}{2}}\right)$

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2. [2] Suppose the radius r of a soup can (cylinder) is increasing at a rate of 1.5 centimeters per minute and the height h is increasing at 2.5 centimeters per minute. The volume of the soup can is given by $V = \pi r^2 h$.

(a) [1] If t is time, the rate of change of the radius is given by (circle one)

(i) $\frac{dV}{dt}$ (ii) $\frac{dh}{dt}$ (iii) $\frac{dt}{dr}$ (iv) $\frac{dr}{dt}$ (v) $\frac{dV}{dr}$

(b) [1] Find the rate of change in volume with respect to time at the instant when $r = 4$ centimeters and $h = 15$ centimeters. (Hint: Use the product rule.) (circle one)

(i) 180π (ii) 190π (iii) 200π (iv) 210π (v) 220π

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3. [2] Approximate $\sqrt{3(2.97)^4 - 9}$ using $f(x + \Delta x) \approx f(x) + f'(x)dx$.

1. [2] (b) $y = \frac{2}{3}\left(x - \sqrt{\frac{7}{2}}\right)$
using implicit differentiation: $2x - 3\left(x\frac{dy}{dx} + y\right) + 8y\frac{dy}{dx} = 0$
and so slope is $\frac{dy}{dx} = \frac{2x-3y}{3x-8y} = \frac{2\sqrt{\frac{7}{2}}-3(0)}{-3\sqrt{\frac{7}{2}}+8(0)} = \frac{2}{3}$
and using the point–slope form of a linear equation with slope $\frac{2}{3}$ and point $(\sqrt{\frac{7}{2}}, 0)$, the result follows

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2. [2]

(a) [1] (iv) $\frac{dr}{dt}$

(b) [1] (v) 220π
since $V = \pi r^2 h$, $\frac{dV}{dt} = \pi\left(r^2\frac{dh}{dt} + h2r\frac{dr}{dt}\right) = \pi(4^2(2.5) + 2(4)(15)(1.5))$

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3. [2] Approximate $\sqrt{3(2.97)^4 - 9}$ using $f(x + \Delta x) \approx f(x) + f'(x)dx$.
since $f(x) = \sqrt{3x^4 - 9}$, $x = 3$ and $\Delta x = dx = -0.03$,
and $f'(x) = \frac{1}{2}(3x^4 - 9)^{-\frac{1}{2}}(12x^3)$
$f(x + \Delta x) \approx f(x) + f'(x)dx = f(2.97) \approx f(3) + f'(3)dx$
$= \sqrt{3(3)^4 - 9} + \frac{1}{2}(3(3)^4 - 9)^{-\frac{1}{2}}(12(3)^3)(-0.03) \approx 14.9793504.$