Test For Proportion $p$.

Of $n = 600$ batteries chosen at random, $\frac{54}{600}$ of them are found to be defective. Does data support hypotheses of an increase in defective batteries (from $0.08$) at $\alpha = 0.05$ in this case? Solve using both p-value and classical approaches to hypothesis testing.

1. **P-value approach.**

   (a) **Statement.** $H_0 : p = 0.08$ versus $H_1 : p > 0.08$

   (b) **Test.**

   $$p\text{-value} = P(\hat{p} \geq 0.09) = P \left( \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \geq \frac{0.09 - 0.08}{\sqrt{\frac{0.08(1-0.08)}{600}}} \right) \approx P(Z \geq 0.903) \approx 0.18$$

   Stat, Proportions, One sample, with summary, Number of successes: 54, Number of observations: 600, Next, Null: prop. = 0.08 Alternative: > Calculate.

   Level of significance $\alpha = 0.05$

   (c) **Conclusion.**

   Since $p\text{-value} = 0.18 > \alpha = 0.05$, do not reject null guess: $H_0 : p = 0.08$.

2. **Classical approach.**

   (a) **Statement.** $H_0 : p = 0.08$ versus $H_1 : p > 0.08$

   (b) **Test.**

   $$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \approx \frac{0.09 - 0.08}{\sqrt{\frac{0.08(1-0.08)}{600}}} \approx 0.903$$

   $$z_\alpha = z_{0.05} \approx 1.645$$

   Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob($X \leq ?$) = 0.95 Calculate.

   (c) **Conclusion.**

   Since $z_0 = 0.903 < z_{0.05} = 1.645$, do not reject null guess: $H_0 : p = 0.08$. 
Test For Mean \( \mu \)

Average hourly wage in US is assumed to be $10.05 in 1985. Midwest big business, however, claims average hourly wage to be larger than this. A random sample of size \( n = 15 \) of midwest workers determines average hourly wage \( \bar{x} = $10.83 \) and standard deviation in wages \( s = 3.25 \). Does data support big business’s claim at \( \alpha = 0.05 \)? Assume normality.

1. **Statement.** \( H_0 : \mu = $10.05 \) versus \( H_1 : \mu > $10.05 \)

2. **Test.**

\[
p\text{-value} = P(\bar{X} \geq 10.83) = P \left( \frac{\bar{X} - \mu_0}{\frac{s}{\sqrt{n}}} \geq \frac{10.83 - 10.05}{\frac{3.25}{\sqrt{15}}} \right) \approx P(t \geq 0.93) \approx 0.18
\]

Stat, T Statistics, with summary, Sample mean: 10.83, Sample std. dev.: 3.25, Sample size: 15, Next, Null: mean = 10.05, Alternative: > Calculate.

Level of significance \( \alpha = 0.05 \)

3. **Conclusion.**

Since \( p\text{-value} = 0.18 > \alpha = 0.05 \), do not reject null guess: \( H_0 : \mu = 10.05 \).

Test For Variance \( \sigma^2 \).

In a simple random sample of 28 cars, SD in gap between door and jamb is \( s = 0.7 \) mm. Test if SD is greater than 0.6 mm at \( \alpha = 0.05 \). Assume normality with no outliers.

1. **Statement.** \( H_0 : \sigma = 0.6 \) versus \( H_1 : \sigma > 0.6 \)

2. **Test.**

\[
p\text{-value} = P(s \geq 0.7) = P \left( \frac{(n-1)s^2}{\sigma^2} \geq \frac{(28-1)(0.7^2)}{0.6^2} \right) \approx P \left( \chi^2 \geq 36.75 \right) \approx 0.10
\]

Stat, Variance, with summary, Sample variance: 0.49, Sample size: 28, Next, Null: mean = 0.36, Alternative: > Calculate. Notice, SDs must be squared to variances to use StatCrunch.

Level of significance \( \alpha = 0.05 \)

3. **Conclusion.**

Since \( p\text{-value} = 0.10 > \alpha = 0.05 \), do not reject null guess: \( H_0 : \sigma = 0.6 \).