TI–84+ Lab 12 For Mathematics 224

Topics: geometric sequence and series, annuities

Geometric Sequence and Series. If the first term is a, then the general term of a geometric sequence is

$$a_n = ar^{n-1}, \quad a_n = a_{n-1}r.$$

The *sum* (series) of the first n terms of a geometric sequence is

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1.$$

Determine both the geometric sequence and series when $a_1 = \frac{1}{2}, r = 3, n = 5$.

- sequence

  - by hand
    $\frac{1}{2}(3)^{1-1} = \frac{1}{2}(1) = \frac{1}{2},$
    $\frac{1}{2}(3)^{2-1} = \frac{1}{2}(3) = \frac{3}{2},$
    $\frac{1}{2}(3)^{3-1} = \frac{1}{2}(3)^2 = \frac{9}{2},$
    $\frac{1}{2}(3)^{4-1} = \frac{1}{2}(3)^3 = \frac{27}{2},$
    $\frac{1}{2}(3)^{5-1} = \frac{1}{2}(3)^4 = \frac{81}{2}$

  - by calculator
    2nd LIST OPS seq$\left(\frac{1}{2} * (3)^{X-1}, X, 1, 5, 1\right)$, MATH ENTER for fractions

- sum of first five terms in sequence

  - by hand
    $s_5 = \sum_{i=1}^{5} \frac{1}{2}(3)^{i-1} = \frac{1}{2}(3)^{1-1} + \frac{1}{2}(3)^{2-1} + \cdots + \frac{1}{2}(3)^{5-1} = \frac{\frac{1}{2}(3^5 - 1)}{3-1} = 60.5,$

  - by calculator
    2nd LIST OPS seq$\left(\frac{1}{2} * (3)^{X-1}, X, 1, 5, 1\right) \to$ 2nd L1,
    STAT CALC ENTER, read $\sum x = 60.5$

Annuities.

- Future value of 5 year term annuity, \$100 paid each quarter, earning interest at 8.5% annually, compounded quarterly, is
  $$S = R\left[\frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}}\right] = 100\left[\frac{\left(1 + \frac{0.085}{4}\right)^{4(5)} - 1}{\frac{0.085}{4}}\right] \approx 2460.21$$
  Calculator: $100 * ((1 + 0.085/4) \wedge (20) - 1)/(0.085/4)$

- Lab of computers replaced in 3 years time for anticipated (future) cost of $25,000 where $25,000 accumulated over 3 year period through equal installments made at end of each month. If yearly interest rate is 8.5%, size of each installment is
$$R = S\left[\frac{\left(\frac{r}{m}\right)}{\left(1+\frac{r}{m}\right)^{mt}-1}\right] = 25000\left[\frac{\left(\frac{0.085}{12}\right)}{\left(1+\frac{0.085}{12}\right)^{12(3)}-1}\right] \approx 612.11$$
Calculator: $25000 * (0.085/12)/((1 + 0.085/12) \wedge (36) - 1)$

- Present value of 5 year term annuity, $100 paid each quarter, earning 8.5% yearly interest, compounded quarterly, is
$$P = p\left[\frac{1-\left(1+\frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right] = 100\left[\frac{1-\left(1+\frac{0.085}{4}\right)^{-4(5)}}{\frac{0.085}{4}}\right] \approx 1615.59$$
Calculator: $100 * (1 - (1 + 0.085/4) \wedge (-20))/(0.085/4)$

- Car loan of $25,000 repaid monthly over 3 year period, yearly interest 8.5%. Amount of each installment
$$R = P\left[\frac{\left(\frac{r}{m}\right)}{1-\left(1+\frac{r}{m}\right)^{-mt}}\right] = 25000\left[\frac{\left(\frac{0.085}{12}\right)}{1-\left(1+\frac{0.085}{12}\right)^{-(12)3}}\right] \approx 789.19$$
Calculator: $25000 * (0.085/12)/(1 - (1 + 0.085/12) \wedge (-36))$