StatCrunch Lab 14 for Statistics 301

Topics: Test of slope β_1 of linear regression, CI and PIs

Test of slope β_1.
Consider data on sales (y, in \$1000s) of pizzas versus student population (x, in 1000s). Based on $n = 10$ data points, $b_1 = 5$. Test if population slope, β_1, is *different* than zero at a level of significance of 5%. Also, calculate 95% CI.

number students, x	2	6	8	8	12	16	20	20	22	26
pizza sales, y	58	105	88	118	117	137	157	169	149	202

1. *Hypothesis test, two–sided.*
   (a) *Statement.*
   $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$
   (b) *Test.*
   $$\text{p–value} = 2\times P(b_1 \geq 5) = 2\times P\left(\frac{b_1 - \beta_1}{\frac{s_e}{\sqrt{\sum(x_i-\bar{x})^2}}} \geq \frac{5-0}{\frac{13.83}{\sqrt{568}}}\right) \approx 2\times P\left(t \geq 8.62\right) \approx 0.00$$
   Stat, Regression, Simple Linear, X-Variable: number, Y-Variable: pizza sales, Next Null Slope = 0 Alternative: $\neq$, Calculate.
   Level of significance $\alpha = 0.05$
   (c) *Conclusion.*
   Since p–value = $0.00 < \alpha = 0.05$, reject null $H_0 : \beta_1 = 0$.
2. *Confidence interval for β_1.*
   $$b_1 \pm t_{\frac{\alpha}{2}} \frac{s_e}{\sqrt{\sum(x_i - \bar{x})^2}} = 5 \pm 2.31\left(\frac{13.829}{\sqrt{568}}\right) \approx (3.7, 6.3)$$
   Stat, Regression, Simple Linear, X-Variable: number, Y-Variable: pizza sales, Next, check Confidence Intervals Level: 0.95, Calculate. Notice 95% L. Limit and 95% U. Limit for Slope.

Confidence and Prediction Intervals.

brightness, x	1	2	3	4	5	6	7	8	9	10
ability to read, y	70	70	75	88	91	94	100	92	90	85

95% CI for $\hat{y}$ at $x^* = 3.5$ is

$$\hat{y} \pm t_{\frac{\alpha}{2}} s_e \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}} \approx (73.71, 87.62)$$

Stat, Regression, Simple Linear, X-Variable: brightness, Y-Variable: reading ability, Next, Next, check Predict Y for X = 3.5, Level 0.95, Calculate. Notice 95% C.I. for mean.

95% PI for $\hat{y}$ at $x^* = 3.5$ is

$$\hat{y} \pm t_{\frac{\alpha}{2}} s_e \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}} \approx (61.32, 100.01)$$

Notice 95% P.I. for mean.