5.8 Test of Independence

We perform a test of independence using test statistic

$$\chi^2_{obs} = \sum_{i=1}^{a} \sum_{j=1}^{b} \frac{(O_{ij} - E_{ij})^2}{E_{ij}},$$

which is approximately chi-square, $(a-1)(b-1)$ df, provided expected frequencies $E_i \geq 5$ and where $E_{ij} = (\text{row total}, R_i) \times (\text{column total}, C_j) \div (\text{table total}, n)$ and O_{ij} is the value in the ith row and jth column of the table $i = 1, \ldots, a$, $j = 1, \ldots, b$, where a, b are number of rows, columns, respectively, in the table.

Exercise 5.8 (Test of Independence)

1. *Test of independence: brain cell growth.*
   Consider *observed* data from a random sample of 390 neurons in an investigation of effect of nutritional level on brain cell growth. Test if brain cell growth is independent of nutrition levels at $\alpha = 0.05$.

O_i	nutritional level $\rightarrow$	poor	adequate	excellent	row totals
neuron	slow	70	95	35	200
growth	normal	90	30	70	190
	column totals	160	125	105	390

```
O <- as.matrix(rbind(c(70, 95, 35), c(90, 30, 70))) # observed counts
nrow <- 2; row.tot <- c(200,190); ncol <- 3; col.tot <- c(160,125,105); n <- 390
dimnames(O) <- list(neuron.growth = c("slow", "normal"),
                    nutrition = c("poor","adequate","excellent"))
```

   (a) *Statement.*
      i. H_0 : proportion cell growth same for different nutrition
         versus H_1 : proportion cell growth different for different nutrition
      ii. H_0 : cell growth independent of nutrition
         versus H_1 : cell growth dependent of nutrition
      iii. H_0 : cell growth different for different nutrition
         versus H_1 : cell growth same for different nutrition

      No matter how this question is worded, null hypothesis for test is *always* independent and alternative hypothesis is *always* dependent.

   (b) *Test.*
      The p–value is (i) **0.00** (ii) **0.08** (iii) **0.10**.

```
independence.test <- function(O, nrow, row.tot, ncol, col.tot, n, signif.level) {
  E <- matrix(0L,nrow,ncol) # initialize expected and chi.contrib to zero
  chi2.test.statistic <- 0 # initialize test statistic to zero
  chi.contrib <- matrix(0L,nrow,ncol)
  for(i in 1:nrow) {
    for(j in 1:ncol) {
      E[i,j] <- row.tot[i]*col.tot[j]/n
      chi.contrib[i,j] <- (O[i,j]-E[i,j])^2/E[i,j]
    }
  }
  dimnames(E) <- dimnames(O); dimnames(chi.contrib) <- dimnames(E)
  print("observed"); print(O); cat("\n")
  print("expected"); print(E); cat("\n")
  print("chi-square contributions"); print(chi.contrib); cat("\n")
  k <- (nrow-1)*(ncol-1)
  chi2.test.statistic <- sum(chi.contrib)
  chi2.crit <- qchisq(signif.level,k,lower.tail=FALSE)
  p.value <- 1-pchisq(chi2.test.statistic,k)
  dat <- c(chi2.crit, chi2.test.statistic, p.value)
  names(dat) <- c("chi2 crit value", "chi2 test stat", "p value")
  print(dat)
}
independence.test(O, nrow, row.tot, ncol, col.tot, n, 0.05) # test of independence chi2-test
```

```
[1] "observed"
             nutrition
neuron.growth poor adequate excellent
       slow     70       95        35
       normal   90       30        70

[1] "expected"
             nutrition
neuron.growth poor adequate excellent
       slow   82.1     64.1      53.8
       normal 77.9     60.9      51.2

[1] "chi-square contributions"
             nutrition
neuron.growth poor adequate excellent
       slow   1.77     14.9      6.60
       normal 1.86     15.7      6.94

chi2 crit value  chi2 test stat         p value
       5.99e+00        4.77e+01        4.30e-11
```

Level of significance $\alpha =$ (i) **0.01** (ii) **0.05** (iii) **0.10**.

(c) *Conclusion.*
Since p–value $= 0.00 < \alpha = 0.05$,
(i) **do not reject** (ii) **reject** null H_0 : growth independent of nutrition.
Data indicates cell growth
(i) **independent of** (ii) **dependent on**
nutrition level.

(d) *How does neuron growth depend on nutrition?*

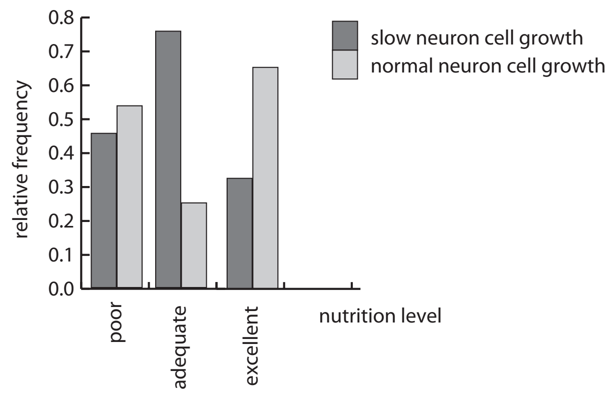

Figure 5.12: Bar graph: cell growth proportions, different nutrition

O_i	nutritional level $\rightarrow$	poor	adequate	excellent	row totals
neuron	slow	$\frac{70}{160} \approx 0.44$	$\frac{95}{125} = 0.76$	$\frac{35}{105} \approx 0.33$	200
growth	normal	$\frac{90}{160} \approx 0.56$	$\frac{30}{125} = 0.24$	$\frac{70}{105} \approx 0.67$	190
	column totals	160	125	105	390

Bar graph indicates below neuron cell growth
(i) **independent of** (ii) **dependent on**
nutrition level.

2. *Test of independence: brain cell growth again.*
Consider *observed* data from a random sample of 390 neurons in an investigation of effect of nutritional level on brain cell growth. Test if brain cell growth is independent of nutrition levels at $\alpha = 0.05$.

O_i	nutritional level $\rightarrow$	poor	adequate	excellent	row totals
neuron	slow	100	75	65	240
growth	normal	60	50	40	150
	column totals	160	125	105	390

```
O <- as.matrix(rbind(c(100, 75, 65), c(60, 50, 40))) # observed counts
nrow <- 2; row.tot <- c(240, 150); ncol <- 3; col.tot <- c(160, 125, 105); n <- 390
dimnames(O) <- list(neuron.growth = c("slow", "normal"),
                    nutrition = c("poor","adequate","excellent"))
```

(a) *Statement.*

i. H_0 : proportion cell growth same for different nutrition
versus H_1 : proportion cell growth different for different nutrition

ii. H_0 : cell growth independent of nutrition
versus H_1 : cell growth dependent of nutrition

iii. H_0 : cell growth different for different nutrition
versus H_1 : cell growth same for different nutrition

(b) *Test.*
The p–value is (i) **0.80** (ii) **0.91** (iii) **0.95**.

```
independence.test(O, nrow, row.tot, ncol, col.tot, n, 0.05) # test of independence chi2-test

[1] "observed"
             nutrition
neuron.growth poor adequate excellent
       slow    100       75        65
       normal   60       50        40

[1] "expected"
             nutrition
neuron.growth poor adequate excellent
       slow   98.5     76.9      64.6
       normal 61.5     48.1      40.4

[1] "chi-square contributions"
             nutrition
neuron.growth   poor adequate excellent
       slow   0.0240   0.0481   0.00229
       normal 0.0385   0.0769   0.00366

chi2 crit value  chi2 test stat         p value
          5.991           0.193           0.908
```

Level of significance $\alpha =$ (i) **0.01** (ii) **0.05** (iii) **0.10**.

(c) *Conclusion.*
Since p–value $= 0.91 > \alpha = 0.05$,
(i) **do not reject** (ii) **reject** null H_0 : same proportions.
Data indicates cell growth
(i) **independent of** (ii) **dependent on**
nutrition level.

(d) *Demonstrating neuron growth independent of nutrition*

O_i	nutritional level $\rightarrow$	poor	adequate	excellent	row totals
neuron	slow	$\frac{100}{160} \approx 0.63$	$\frac{75}{125} = 0.60$	$\frac{65}{105} \approx 0.62$	200
growth	normal	$\frac{60}{160} \approx 0.37$	$\frac{50}{125} = 0.40$	$\frac{40}{105} \approx 0.38$	190
	column totals	160	125	105	390

Bar graph indicates below neuron cell growth
(i) **independent of** (ii) **dependent on**
nutrition level; that is, bar graph indicates neuron cell growth
(i) **same proportion** (ii) **different proportions**
for different nutrition levels.

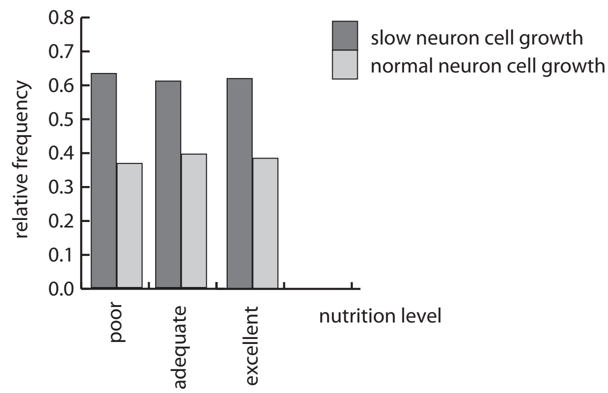

Figure 5.13: Bar graph: neuron cell growth proportions, different nutrition

5.9 One-Way ANOVA

A *one–way ANOVA* procedure tests if k means are the same

$$H_0 : \mu_1 = \cdots = \mu_k$$

or at least one mean is different using test statistic

$$F_{k-1,N-k} = \frac{MS(treatment)}{MS(error)} = \frac{\text{MSTr}}{\text{MSE}}$$

which has a F distribution, with $(N - k,\ k - 1)$ df, where N data points sampled randomly from k treatments (populations), and where k samples are independent of one another, each normally distributed and all with same variance, σ^2, and where $MS(treatemnt)$ is mean square due to treatment and $MS(error)$ is mean square due to error, and calculated as given in ANOVA table:

Source	Degrees of Freedom, DF	Sum Of Squares	Mean Squares	F-Ratio
Treatment (between)	$k-1$	$SSTr = \sum_{i=1}^{k} n_i(\bar{x}_i - \bar{\bar{x}})^2$	$MSTr = \frac{SSTr}{k-1}$	$F_{k-1,N-k} = \frac{MSTr}{MSE}$
Error (within)	$N-k$	$SSE = \sum_{i=1}^{k}(n_i-1)s_i^2$	$MSE = \frac{SSE}{N-k}$	
Total	$N-1$	$SSTO = \sum\sum(y_{ij} - \bar{\bar{x}})^2$		

The *Bonferroni multiple comparison confidence intervals (CIs)* for pairwise comparison of means, $\bar{x}_i - \bar{x}_j$, which guards against increased type I error (accidently rejecting the null), is given by

$$(\bar{x}_i - \bar{x}_j) \pm t_{(1-\alpha/(2c);N-k)}\sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

where c is the number of confidence intervals and MSE is the mean squared error from the associated one-way ANOVA table.

Exercise 5.9 (One-Way ANOVA)

1. *Test comparing multiple means, ANOVA: average drug responses A.*
Fifteen different patients, chosen at random, subjected to three drugs. Test if at least one of the three mean patient responses to drug is different at $\alpha = 0.05$.

drug 1	drug 2	drug 3
5.90	5.51	5.01
5.92	5.50	5.00
5.91	5.50	4.99
5.89	5.49	4.98
5.88	5.50	5.02
$\bar{x}_1 \approx 5.90$	$\bar{x}_2 \approx 5.50$	$\bar{x}_3 \approx 5.00$

(a) *Data Frames.*
A mixture of different types of data, numerical and categorical, is often used in data sets. In this example, response is numerical and the one factor "drug" is categorical. The data in the table above is typically arranged into a data frame in R as given below and which allows greater manipulation of the data. For example, the response which corresponds to drug 2, replication 3 is (i) **5.50** (ii) **5.51** (iii) **5.52**

```
drug <- c("1","1","1","1","1","2","2","2","2","2","3","3","3","3","3")
replication <- c(1,2,3,4,5,1,2,3,4,5,1,2,3,4,5)
response <- c(5.90, 5.92, 5.91, 5.89, 5.88, 5.51, 5.50, 5.50, 5.49, 5.50, 5.01, 5.00, 4.99, 4.98, 5.02)
data <- data.frame(drug,replication,response); data
drug <- factor(data$drug,c("1","2","3")) # convert integer to factor
```

```
   drug replication response
1     1           1     5.90
2     1           2     5.92
3     1           3     5.91
4     1           4     5.89
5     1           5     5.88
6     2           1     5.51
7     2           2     5.50
8     2           3     5.50
9     2           4     5.49
10    2           5     5.50
11    3           1     5.01
12    3           2     5.00
13    3           3     4.99
14    3           4     4.98
15    3           5     5.02
```

(b) *Statement.* Choose one.

i. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_1 \neq \mu_2, \mu_1 = \mu_3$.
ii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_1 \neq \mu_3, \mu_1 \neq \mu_2$.
iii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_i \neq \mu_j, i \neq j;\ i, j = 1, 2, 3$.
iv. H_0 : means same vs H_1 : at least one of the means different

No matter how the question is worded, null is *always* "means same" and alternative is *always* "at least one of the means different".

(c) *Test.*
p–value = (i) **0.00** (ii) **0.035** (iii) **0.043**.

```
# one-way ANOVA, dataframe data, c=3 dataframe columns, k=3 treatments, significant level 0.05
ANOVA.oneway(data, 3, 3, 0.05)

                          SS df                  MS                F
Treatment    2.03333333333333  2    1.01666666666667 5545.45454545465
Error     0.00219999999999996 12 0.000183333333333333
Total        2.03553333333333 14

F crit value  F test stat      p value
       3.885     5545.455        0.000
```

Level of significance $\alpha =$ (i) **0.01** (ii) **0.05** (iii) **0.10**.

(d) *Conclusion.*
Since p–value $= 0.00 < \alpha = 0.05$,
(i) **do not reject** (ii) **reject** null H_0 : means same.
Data indicates
(i) **average drug responses same**
(ii) **at least one of average drug responses different**

(e) *Related question.*
"H_1 : at least one of the means different" means: (choose *one or more*)

i. $\mu_1 \neq \mu_2$, but $\mu_2 = \mu_3$
ii. $\mu_1 \neq \mu_3$, but $\mu_2 = \mu_3$
iii. $\mu_2 \neq \mu_3$, but $\mu_1 = \mu_3$
iv. $\mu_1 \neq \mu_2$, $\mu_1 \neq \mu_3$ and $\mu_2 \neq \mu_3$

(f) *Bonferroni multiple comparison CIs for mean pairwise differences, $\bar{x}_i - \bar{x}_j$.*

```
# bonferroni comparisons, dataframe data, c=3 columns in dataframe, k=3 treatments, significance level 0.05
#   use MS.error from output of ANOVA.oneway function
bonferroni(data, 0.000183, 3, 3, 0.05)

       i      j mean.i mean.j                  E          ci.lower           ci.upper
1 Drug 1 Drug 2    5.9    5.5 0.0237803376124479 0.376219662387552 0.423780337612448
2 Drug 1 Drug 3    5.9      5 0.0237803376124479 0.876219662387552 0.923780337612448
3 Drug 2 Drug 3    5.5      5 0.0237803376124479 0.476219662387552 0.523780337612448
```

Since *none* of the three differences in treatment means include zero, the means of these three treatments (i) **are** (ii) **are not** significantly different from one another.

2. *Test comparing multiple means, ANOVA: average drug responses B.*
Fifteen different patients, chosen at random, subjected to three drugs. Test if at least one of the three mean patient responses to drug is different at $\alpha = 0.05$.

drug 1	drug 2	drug 3
5.90	6.31	4.52
4.42	3.54	6.93
7.51	4.73	4.48
7.89	7.20	5.55
3.78	5.72	3.52
$\bar{x}_1 \approx 5.90$	$\bar{x}_2 \approx 5.50$	$\bar{x}_3 \approx 5.00$

```
drug <- c("1","1","1","1","1","2","2","2","2","2","3","3","3","3","3")
replication <- c(1,2,3,4,5,1,2,3,4,5,1,2,3,4,5)
response <- c(5.90, 4.42, 7.51, 7.89, 3.78, 6.31, 3.54, 4.73, 7.20, 5.72, 4.52, 6.93, 4.48, 5.55, 3.52)
data <- data.frame(drug,replication,response); data
drug <- factor(data$drug,c("1","2","3")) # convert integer to factor
```

(a) *Statement.* Choose none, one or more.

i. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_1 \neq \mu_2, \mu_1 = \mu_3$.

ii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_1 \neq \mu_3, \mu_1 \neq \mu_2$.

iii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_i \neq \mu_j, i \neq j;\ i, j = 1, 2, 3$.

iv. H_0 : means same vs H_1 : at least one of the means different

(b) *Test.*

p–value = (i) **0.00** (ii) **0.35** (iii) **0.66**.

```
# one-way ANOVA, dataframe data, c=3 dataframe columns, k=3 treatments, significant level 0.05
ANOVA.oneway(data, 3, 3, 0.05)

                       SS df               MS                 F
Treatment 2.03333333333333  2 1.01666666666667 0.435954060447532
Error              27.9846 12          2.33205
Total     30.0179333333333 14

F crit value  F test stat      p value
      3.8853       0.4360       0.6565
```

Level of significance $\alpha =$ (i) **0.01** (ii) **0.05** (iii) **0.10**.

(c) *Conclusion.*

Since p–value $= 0.66 > \alpha = 0.05$,

(i) **do not reject** (ii) **reject** null H_0 : means same.

Data indicates

(i) **average drug responses same**

(ii) **at least one of average drug responses different**

(d) *Bonferroni multiple comparison CIs for mean pairwise differences, $\bar{x}_i - \bar{x}_j$.*

```
# bonferroni comparisons, dataframe data, c=3 dataframe columns, k=3 treatments, significance level 0.05
#   use MS.error from output of ANOVA.oneway function
bonferroni(data, 2.33205, 3, 3, 0.05)

       i      j mean.i mean.j                E          ci.lower         ci.upper
1 Drug 1 Drug 2    5.9    5.5 2.68448758617324 -2.28448758617324 3.08448758617325
2 Drug 1 Drug 3    5.9      5 2.68448758617324 -1.78448758617324 3.58448758617325
3 Drug 2 Drug 3    5.5      5 2.68448758617324 -2.18448758617324 3.18448758617324
```

Since *all* of the three differences in treatment means include zero, the means of these three treatments (i) **are** (ii) **are not** significantly different from one another.

(e) *Related questions.* ANOVA table is

```
                       SS df               MS                 F
Treatment 2.03333333333333  2 1.01666666666667 0.435954060447532
Error              27.9846 12          2.33205
Total     30.0179333333333 14

F crit value  F test stat      p value
      3.8853       0.4360       0.6565
```

$SSTr =$ (i) **1.017** (ii) **2.033** (iii) **2.332** (iv) **27.985**
$MSTr =$ (i) **1.017** (ii) **2.033** (iii) **2.332** (iv) **27.985**
$SSE =$ (i) **1.017** (ii) **2.033** (iii) **2.332** (iv) **27.985**
$MSE =$ (i) **1.017** (ii) **2.033** (iii) **2.332** (iv) **27.985**
so test statistic is $F = \frac{MST}{MSE} = \frac{1.0167}{2.3321} =$ (i) **0.345** (ii) **0.436** (iii) **0.767**,
with $k-1 = 3-1 = 2$ and $n-k = 15-3 = 12$ DF,
so p–value $= P(F \geq 0.436) =$ (i) **0.00** (ii) **0.35** (iii) **0.66**

```
1 - pf(0.436,2,12)
```

and critical value of F at 5% is (i) **3.89** (ii) **4.36** (iii) **7.67**

```
qf(0.95,2,12)
```

3. *Comparing drug response data set A and data set B.*

   (a) *Comparing averages, $\bar{x}_1$, $\bar{x}_2$ and $\bar{x}_3$, in two drug response data sets.*
   Of following two data sets, data set A,

drug 1	drug 2	drug 3
5.90	5.51	5.01
5.92	5.50	5.00
5.91	5.50	4.99
5.89	5.49	4.98
5.88	5.50	5.02
$\bar{x}_1 \approx 5.90$	$\bar{x}_2 \approx 5.50$	$\bar{x}_3 \approx 5.00$
$s_1 \approx 0.12$	$s_2 \approx 0.004$	$s_3 \approx 0.01$

   and data set B,

drug 1	drug 2	drug 3
5.90	6.31	4.52
4.42	3.54	6.93
7.51	4.73	4.48
7.89	7.20	5.55
3.78	5.72	3.52
$\bar{x}_1 \approx 5.90$	$\bar{x}_2 \approx 5.50$	$\bar{x}_3 \approx 5.00$
$s_1 \approx 1.82$	$s_2 \approx 1.27$	$s_3 \approx 1.30$

   three *average* patient responses within drugs in data set A are
   smaller than (ii) **same as** (ii) **larger than**
   three *average* patient responses within drugs in data set B.

   (b) *Comparing SDs, s_1, s_2 and s_3, in two drug response data sets.*
   The *standard deviations* in patient responses within drugs in data A are **smaller than** (ii) **the same as** (ii) **larger than** the *standard deviations* in patient responses within drugs in data B.

(c) *Comparing averages, taking into account SDs.*
As shown in figure below, since standard deviations within drugs in data set A are smaller than they are for data set B, we are "more certain" about where averages are in data set A, than we are about where averages are in data set B. Consequently, it is "easiest" to tell if average patient responses are different from one another in data set (i) **A** (ii) **B**. This is why p–value is smaller for data set A, than it is for data set B even though averages are same in both sets.

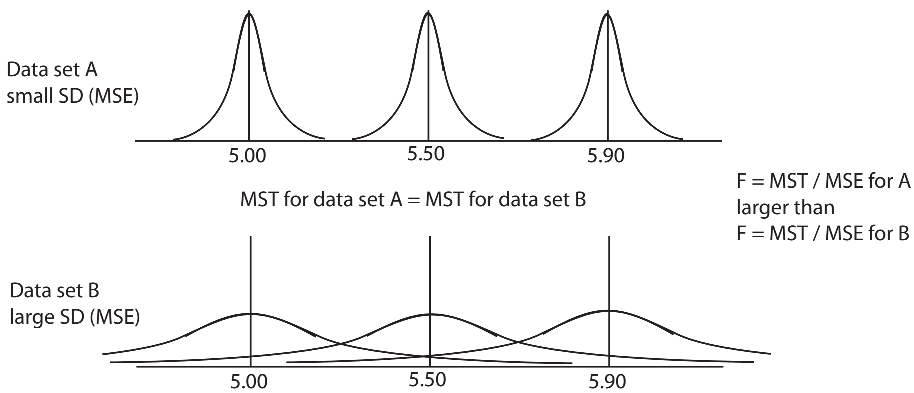

Figure 5.14: Comparing average drug responses

(d) *Comparing F test statistics.*
Since mean square treatment (MSTr) same for data sets A and B, but mean square error (MSE) smaller for data set A than for data set B,

$$F = \frac{MSTr}{MSE} = \frac{\text{distance between means}}{\text{variability in data}}$$

is (i) **smaller** (ii) **same as** (iii) **larger** for A than for B, so greater chance of rejecting null (means same) for A than for B.

5.10 Two-Way ANOVA

Two two-way (two–factor) analyses of variance (ANOVA) procedures, assuming equal sample sizes constant variance and normality, are demonstrated in this section. The first, the *factorial design*, uses data to test whether the

- mean main effect due to factor A is zero or not,
- mean main effect due to factor B is zero or not,
- mean interaction effect due to factor AB is zero or not.

The ANOVA table corresponding to this design is given below.

Source	df	Sum Of Squares, SS	Mean Squares, MS	F-ratio
Factor A	$a-1$	$SSA = nb\sum_{i=1}^{a}(\bar{x}_i - \bar{\bar{x}})^2$	$MSTA = \frac{SSA}{a-1}$	$F_{a-1,ab(r-1)} = \frac{MSA}{MSE}$
Factor B	$b-1$	$SSB = na\sum_{j=1}^{b}(\bar{x}_j - \bar{\bar{x}})^2$	$MSTB = \frac{SSB}{b-1}$	$F_{b-1,ab(r-1)} = \frac{MSB}{MSE}$
Interaction AB	$(a-1)(b-1)$	$SSAB = n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{x}_{ij} - \bar{x}_i - \bar{x}_j - \bar{\bar{x}})^2$	$MSTAB = \frac{SSAB}{(a-1)(b-1)}$	$F_{(a-1)(b-1),ab(r-1)} = \frac{MSAB}{MSE}$
Error	$ab(r-1)$	$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}(x_{ijk} - \bar{x}_{ij})^2$	$MSE = \frac{SSE}{ab(n-1)}$	
Total	$rab-1$	$SSTO = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}(x_{ijk} - \bar{\bar{x}})^2$		

The second, the *randomized block design*, uses data to test whether

- mean effect due to the treatment is zero or not,
- mean effect due to the block is zero or not.

The ANOVA table corresponding to this design is given below.

Source	df	Sum Of Squares, SS	Mean Squares, MS	F-ratio
Treatment	$k-1$	$SSTr = b\sum_{i=1}^{k}(\bar{x}_i - \bar{\bar{x}})^2$	$MSTr = \frac{SSTr}{k-1}$	$F_{k-1,(k-1)(b-1)} = \frac{MSTr}{MSE}$
Block	$b-1$	$SSB = k\sum_{j=1}^{b}(\bar{x}_j - \bar{\bar{x}})^2$	$MSB = \frac{SSB}{b-1}$	$F_{b-1,(k-1)(b-1)} = \frac{MSB}{MSE}$
Error (Interaction)	$(k-1)(b-1)$	$SSE = \sum_{i=1}^{k}\sum_{j=1}^{b}(\bar{x}_{ij} - \bar{x}_i - \bar{x}_j - \bar{\bar{x}})^2$	$MSTE = \frac{SSE}{(k-1)(b-1)}$	
Total	$kb-1$	$SSTO = \sum_{i=1}^{k}\sum_{j=1}^{b}(x_{ij} - \bar{\bar{x}})^2$		

Exercise 5.10 (Two-Way ANOVA)

1. *Data Frame.*
   Consider the effect of air temperature *and* noise on the ROC of deer mice.

	Factor B, noise $\rightarrow$	$j=1$, low	$j=2$, medium	$j=3$, high	row ave
	$i=1$, 0^o F	10.3 7.2	9.1 5.4	6.1 2.1	$\bar{x}_1 = 6.7$
Factor A, temperature	$i=2$, 10^o F	1.8 9.8	12.1 4.2	5.1 6.2	$\bar{x}_2 = 6.5$
	$i=3$, 20^o F	1.2 8.1	6.5 4.1	1.2 2.1	$\bar{x}_3 = 3.9$
	$i=4$, 30^o F	12.4 15.1	16.1 17.2	18.1 19.1	$\bar{x}_4 = 16.3$
	column ave	$\bar{x}_1 = 8.2$	$\bar{x}_2 = 9.3$	$\bar{x}_3 = 7.5$	$\bar{\bar{x}} = 8.4$

   Construct a dataframe for this data. The ROC which corresponds to temperature 20^o, noise level "medium" and replication 1 is (i) **4.1** (ii) **6.5** (iii) **8.1**

```
temperature <- c(0,0,0,0,0,0,10,10,10,10,10,10,20,20,20,20,20,20,30,30,30,30,30,30)
noise <- c("low","low","medium","medium","high","high",and so on "low","low","medium","medium","high","high")
replications <- c(1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2)
ROC <- c(10.3, 7.2, 9.1, 5.4, and so on 17.2, 18.1, 19.1);
data <- data.frame(temperature,noise,replications,ROC); data
temperature <- factor(data$temperature,c("0","10","20","30")) # convert temperature to factor
noise <- factor(data$noise,c("low","medium","high")) # convert noise to factor
```

```
   temperature  noise replications  ROC
1            0     low           1 10.3
2            0     low           2  7.2
3            0  medium           1  9.1
4            0  medium           2  5.4
5            0    high           1  6.1
6            0    high           2  2.1
7           10     low           1  1.8
8           10     low           2  9.8
9           10  medium           1 12.1
10          10  medium           2  4.2
11          10    high           1  5.1
12          10    high           2  6.2
13          20     low           1  1.2
14          20     low           2  8.1
15          20  medium           1  6.5
16          20  medium           2  4.1
17          20    high           1  1.2
18          20    high           2  2.1
19          30     low           1 12.4
20          30     low           2 15.1
21          30  medium           1 16.1
22          30  medium           2 17.2
23          30    high           1 18.1
24          30    high           2 19.1
```

2. *Review: one–way ANOVA of temperature factor A alone.*
Consider the effect of air temperature *and* noise on the ROC of deer mice but test if temperature alone is significant at 5%.

	Factor B, noise $\rightarrow$	$j = 1$, low	$j = 2$, medium	$j = 3$, high	row ave
	$i = 1$, 0^o F	10.3	9.1	6.1	$\overline{x}_1 = 6.7$
		7.2	5.4	2.1	
Factor A, temperature	$i = 2$, 10^o F	1.8	12.1	5.1	$\overline{x}_2 = 6.5$
		9.8	4.2	6.2	
	$i = 3$, 20^o F	1.2	6.5	1.2	$\overline{x}_3 = 3.9$
		8.1	4.1	2.1	
	$i = 4$, 30^o F	12.4	16.1	18.1	$\overline{x}_4 = 16.3$
		15.1	17.2	19.1	
	column ave	$\overline{x}_1 = 8.2$	$\overline{x}_2 = 9.3$	$\overline{x}_3 = 7.5$	$\overline{\overline{x}} = 8.4$

(a) *Statement.*

i. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 :\ \mu_1 \neq \mu_2, \mu_1 = \mu_3$.

ii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 :\ \mu_1 \neq \mu_3, \mu_1 \neq \mu_2$.

iii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 :\ \mu_i \neq \mu_j, i \neq j;\ i, j = 1, 2, 3$.

iv. H_0 : means same vs H_1 : at least one of the means different

(b) *Test.*
p–value = (i) **0.00** (ii) **0.035** (iii) **0.043**.

```
# one-way ANOVA, dataframe data, c=4 dataframe columns, 4 temperatures, sign level 0.05
# order data frame by temperature; that is, group ROCs by different temperatures
data <- data[order(data$temperature),]; data
ANOVA.oneway(data, 4, 4, 0.05)

                       SS df               MS                F
Treatment 539.138333333334  3 179.712777777778 19.4620725338724
Error               184.68 20            9.234
Total     723.818333333334 23

F crit value  F test stat      p value
   3.098e+00    1.946e+01    3.791e-06
```

Level of significance $\alpha =$ (i) **0.01** (ii) **0.05** (iii) **0.10**.

(c) *Conclusion.*
Since p–value $= 0.00 < \alpha = 0.05$,
(i) **do not reject** (ii) **reject** null H_0 : means same.
Data indicates
(i) **mean ROC same for different temperatures**
(ii) **at least one mean ROC different**

3. *Review: one–way ANOVA of noise factor B alone.*
Again consider the effect of air temperature and noise on the ROC of deer mice but test if noise alone is significant at 5%.

	Factor B, noise $\rightarrow$	$j = 1$, low	$j = 2$, medium	$j = 3$, high	row ave
	$i = 1$, 0^o F	10.3	9.1	6.1	$\overline{x}_1 = 6.7$
		7.2	5.4	2.1	
Factor A,	$i = 2$, 10^o F	1.8	12.1	5.1	$\overline{x}_2 = 6.5$
temperature		9.8	4.2	6.2	
	$i = 3$, 20^o F	1.2	6.5	1.2	$\overline{x}_3 = 3.9$
		8.1	4.1	2.1	
	$i = 4$, 30^o F	12.4	16.1	18.1	$\overline{x}_4 = 16.3$
		15.1	17.2	19.1	
	column ave	$\overline{x}_1 = 8.2$	$\overline{x}_2 = 9.3$	$\overline{x}_3 = 7.5$	$\overline{\overline{x}} = 8.4$

(a) *Statement.*

i. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_1 \neq \mu_2, \mu_1 = \mu_3$.
ii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_1 \neq \mu_3, \mu_1 \neq \mu_2$.
iii. $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_i \neq \mu_j, i \neq j;\ i, j = 1, 2, 3$.
iv. H_0 : means same vs H_1 : at least one of the means different

(b) *Test.*
p–value = (i) **0.007** (ii) **0.035** (iii) **0.818**.

```
# one-way ANOVA, dataframe data, c=4 dataframe columns, 3 noise levels, sign level 0.05
# order data frame by noise; that is, group ROCs by different noise levels
data <- data[order(data$noise),]; data
ANOVA.oneway(data, 4, 3, 0.05)
```

```
                        SS df               MS                 F
Treatment 13.6808333333333  2 6.84041666666667 0.202283008572284
Error             710.1375 21 33.8160714285714
Total     723.818333333333 23

F crit value  F test stat      p value
      3.4668       0.2023       0.8184
```

Level of significance $\alpha =$ (i) **0.01** (ii) **0.05** (iii) **0.10**.

(c) *Conclusion.*
Since p–value $= 0.818 > \alpha = 0.050$,
(i) **do not reject** (ii) **reject** null H_0 : means same.
Data indicates
(i) **mean ROC same for different noises**
(ii) **at least one mean ROC different**

4. *Two-Way ANOVA factorial: test main factor A, temperature.*
Again consider the effect of air temperature and noise on the ROC of deer mice and test if temperature is significant at 5% using two–way ANOVA.

	Factor B, noise $\rightarrow$	$j = 1$, low	$j = 2$, medium	$j = 3$, high	row ave
	$i = 1$, 0^o F	10.3	9.1	6.1	$\overline{x}_1 = 6.7$
		7.2	5.4	2.1	
Factor A,	$i = 2$, 10^o F	1.8	12.1	5.1	$\overline{x}_2 = 6.5$
temperature		9.8	4.2	6.2	
	$i = 3$, 20^o F	1.2	6.5	1.2	$\overline{x}_3 = 3.9$
		8.1	4.1	2.1	
	$i = 4$, 30^o F	12.4	16.1	18.1	$\overline{x}_4 = 16.3$
		15.1	17.2	19.1	
	column ave	$\overline{x}_1 = 8.2$	$\overline{x}_2 = 9.3$	$\overline{x}_3 = 7.5$	$\overline{\overline{x}} = 8.4$

(a) *Statement*

i. $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ versus $H_a :\ \alpha_1 \neq \alpha_2, \alpha_1 = \alpha_3$.
ii. $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ versus $H_a :\ \alpha_1 \neq \alpha_3, \alpha_1 \neq \alpha_2$.
iii. $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ versus
$H_a :$ at least one $\alpha_i \neq 0,\ i = 1, 2, 3, 4$.
iv. $H_0 : \mu_{1\cdot} = \mu_{2\cdot} = \mu_{3\cdot} = \mu_{4\cdot}$ versus
$H_a :$ at least one $\mu_{i\cdot} \neq \mu_{j\cdot},\ i, j = 1, 2, 3, 4$.

(b) *Test*
Since the test statistic is $F = \frac{179.71}{9.61} = 18.70$, the p–value, with
$a - 1 = 4 - 1 =$ (i) **2** (ii) **3** (iii) **6** (iv) **12** and
$ab(n-1) = 4(3)(2-1) =$ (i) **2** (ii) **3** (iii) **6** (iv) **12**
degrees of freedom, is given by
p–value $= P(F \geq 18.70) =$ (i) **0** (ii) **0.26** (iii) **0.43**.

```
# two-way ANOVA, data: 4 dataframe columns, 4 temp (a) x 3 noises (b), 2 repl (r), sign 0.05
# order data frame by temperature, then by noise
data <- data[order(data$temperature, data$noise), ]; data
ANOVA.twoway.factorial(data, 4, 4, 3, 2, 0.05) # one-way ANOVA

                    SS df                MS                  F
A     539.138333333334  3 179.712777777778  18.7038450419196
B     13.6808333333336  2 6.84041666666678 0.711925411968788
AB    55.6991666666661  6 9.28319444444435 0.966160740098283
Error            115.3 12 9.60833333333335
Total 723.818333333333 23

 F crit A  F test A p value A
 3.49e+00  1.87e+01  8.08e-05
```

The level of significance is 0.05.

(c) *Conclusion*
Since the p–value, 0, is smaller than the level of significance, 0.05,
(i) **fail to reject** (ii) **reject** the null hypothesis that the average mice ROC responses to the **temperatures** (ii) **noises** are the same.

(d) *Comment: Factor A, Temperature, Effect.*
Temperature effects are differences between mean ROC at each temperature level and grand mean ROC,
$\hat{\alpha}_1 = \bar{x}_1 - \bar{\bar{x}} = 6.7 - 8.4 =$ (i) $\mathbf{-4.5}$ (ii) $\mathbf{-1.9}$ (iii) $\mathbf{-1.7}$ (iv) $\mathbf{7.9}$
$\hat{\alpha}_2 = \bar{x}_2 - \bar{\bar{x}} = 6.5 - 8.4 =$ (i) $\mathbf{-4.5}$ (ii) $\mathbf{-1.9}$ (iii) $\mathbf{-1.7}$ (iv) $\mathbf{7.9}$
$\hat{\alpha}_3 = \bar{x}_3 - \bar{\bar{x}} = 3.9 - 8.4 =$ (i) $\mathbf{-4.5}$ (ii) $\mathbf{-1.9}$ (iii) $\mathbf{-1.7}$ (iv) $\mathbf{7.9}$
$\hat{\alpha}_4 = \bar{x}_4 - \bar{\bar{x}} = 16.3 - 8.4 =$ (i) $\mathbf{-4.5}$ (ii) $\mathbf{-1.9}$ (iii) $\mathbf{-1.7}$ (iv) $\mathbf{7.9}$
where, notice, (i) **none** (ii) **some** (ii) **all** of the temperature effects are "close" to *zero* confirming the mean ROC responses are different for different temperatures

(e) *Comment: one–way versus two–way*: The temperature factor is

i. significant for both one–way ANOVA and two–way ANOVA
ii. significant for one–way ANOVA but not two–way ANOVA
iii. *in*significant for one–way ANOVA but significant for two–way ANOVA
iv. *in*significant for both one–way ANOVA and two–way ANOVA

5. *Two–way ANOVA factorial: test main factor B, noise.*
Again consider the effect of air temperature and noise on the ROC of deer mice and test if noise is significant at 5% using two–way ANOVA.

	Factor B, noise $\rightarrow$	$j=1$, low	$j=2$, medium	$j=3$, high	row ave
	$i=1$, 0^o F	10.3	9.1	6.1	$\overline{x}_1 = 6.7$
		7.2	5.4	2.1	
Factor A,	$i=2$, 10^o F	1.8	12.1	5.1	$\overline{x}_2 = 6.5$
temperature		9.8	4.2	6.2	
	$i=3$, 20^o F	1.2	6.5	1.2	$\overline{x}_3 = 3.9$
		8.1	4.1	2.1	
	$i=4$, 30^o F	12.4	16.1	18.1	$\overline{x}_4 = 16.3$
		15.1	17.2	19.1	
	column ave	$\overline{x}_1 = 8.2$	$\overline{x}_2 = 9.3$	$\overline{x}_3 = 7.5$	$\overline{\overline{x}} = 8.4$

(a) *Statement*

i. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus $H_a : \beta_1 \neq \beta_2, \beta_1 = \beta_3$.

ii. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus $H_a : \beta_1 \neq \beta_3, \beta_1 \neq \beta_2$.

iii. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus
$H_a :$ at least one $\beta_i \neq 0, \ i = 1,2,3$.

iv. $H_0 : \mu_{\cdot 1} = \mu_{\cdot 2} = \mu_{\cdot 3}$ versus
$H_a :$ at least one $\mu_{i\cdot} \neq \mu_{j\cdot}, \ i,j = 1,2,3$.

(b) *Test*

Since the test statistic is $F = \frac{6.84}{9.61} = 0.71$, the p–value, with
$b - 1 = 3 - 1 =$ (i) **2** (ii) **3** (iii) **6** (iv) **12** and
$ab(n-1) = 4(3)(2-1) =$ (i) **2** (ii) **3** (iii) **6** (iv) **12**
degrees of freedom, is given by
p–value $= P(F \geq 0.71) =$ (i) **0.00** (ii) **0.35** (iii) **0.51**.

```
# two-way ANOVA, data: 4 dataframe columns, 4 temp (a) x 3 noises (b), 2 repl (r), sign 0.05
# order data frame by temperature, then by noise
data <- data[order(data$temperature, data$noise), ]; data
ANOVA.twoway.factorial(data, 4, 4, 3, 2, 0.05) # one-way ANOVA
```

```
                    SS df               MS                 F
A     539.138333333334  3 179.712777777778  18.7038450419196
B     13.6808333333336  2 6.84041666666678 0.711925411968788
AB    55.6991666666661  6 9.28319444444435 0.966160740098283
Error            115.3 12 9.60833333333335
Total 723.818333333333 23


 F crit B  F test B p value B
   3.8853    0.7119    0.5103
```

The level of significance is 0.05.

(c) *Conclusion.*

Since the p–value, 0.51, is larger than the level of significance, 0.05,
(i) **fail to reject** (ii) **reject** the null hypothesis that the average mice ROC responses to the (i) **temperatures** (ii) **noises** are the same.

(d) *Comment: Factor B, Noise, Effect.*

Noise effects are differences between mean ROC at each noise level and

grand mean ROC,
$\hat{\beta}_1 = \bar{x}_1 - \bar{\bar{x}} = 8.2 - 8.4 =$ (i) $\mathbf{-0.2}$ (ii) **0.9** (iii) $\mathbf{-0.9}$
$\hat{\beta}_2 = \bar{x}_2 - \bar{\bar{x}} = 9.3 - 8.4 =$ (i) $\mathbf{-0.2}$ (ii) **0.9** (iii) $\mathbf{-0.9}$
$\hat{\beta}_3 = \bar{x}_3 - \bar{\bar{x}} = 7.5 - 8.4 =$ (i) $\mathbf{-0.2}$ (ii) **0.9** (iii) $\mathbf{-0.9}$
where, notice, (i) **none** (ii) **some** (ii) **all** noise effects are "close", within random error, to *zero* confirming mean ROC responses are the same for different noise levels

(e) *Comment: one–way versus two–way*: The noise factor is

i. significant for both one–way ANOVA and two–way ANOVA
ii. significant for one–way ANOVA but not two–way ANOVA
iii. *in*significant for one–way ANOVA but significant for two–way ANOVA
iv. *in*significant for both one–way ANOVA and two–way ANOVA

which indicates noise is not as an important factor as temperature when influencing ROC mean responses

6. *Two–way ANOVA Factorial: test interaction factor AB, temp × noise.*
Again consider the effect of air temperature and noise on the ROC of deer mice and test if temperature × noise is significant at 5% using two–way ANOVA.

	Factor B, noise →	$j = 1$, low	$j = 2$, medium	$j = 3$, high	row ave
	$i = 1$, 0^o F	10.3	9.1	6.1	$\overline{x}_1 = 6.7$
		7.2	5.4	2.1	
Factor A,	$i = 2$, 10^o F	1.8	12.1	5.1	$\overline{x}_2 = 6.5$
temperature		9.8	4.2	6.2	
	$i = 3$, 20^o F	1.2	6.5	1.2	$\overline{x}_3 = 3.9$
		8.1	4.1	2.1	
	$i = 4$, 30^o F	12.4	16.1	18.1	$\overline{x}_4 = 16.3$
		15.1	17.2	19.1	
	column ave	$\overline{x}_1 = 8.2$	$\overline{x}_2 = 9.3$	$\overline{x}_3 = 7.5$	$\overline{\overline{x}} = 8.4$

(a) *Statement*

i. $H_0 : \alpha\beta_{11} = \alpha\beta_{12} = \alpha_4 = 0$ versus $H_a :\ \alpha_1 \neq \alpha_2, \alpha_1 = \alpha_3$.
ii. $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ versus $H_a :\ \alpha_1 \neq \alpha_3, \alpha_1 \neq \alpha_2$.
iii. $H_0 : \alpha\beta_{11} = \alpha\beta_{12} = \cdots = \alpha\beta_{43} = 0$ versus
$H_a :$ at least one $\alpha\beta_{ij} \neq 0,\ i = 1, 2, 3, 4; j = 1, 2, 3$.
iv. $H_0 : \mu_{1\cdot} = \mu_{2\cdot} = \mu_{3\cdot} = \mu_{4\cdot}$ versus
$H_a :$ at least one $\mu_{i\cdot} \neq \mu_{j\cdot},\ i, j = 1, 2, 3, 4$.

(b) *Test*
Since the test statistic is $F = \frac{9.28}{9.61} = 0.97$, the p–value, with

$(a-1)(b-1) = (4-1)(3-1) =$ (i) **2** (ii) **3** (iii) **6** (iv) **12** and
$ab(n-1) = 4(3)(2-1) =$ (i) **2** (ii) **3** (iii) **6** (iv) **12**
degrees of freedom, is given by
p–value $= P(F \geq 0.97) =$ (i) **0.00** (ii) **0.49** (iii) **0.43**.

```
# two-way ANOVA, data: 4 dataframe columns, 4 temp (a) x 3 noises (b), 2 repl (r), sign 0.05
# order data frame by temperature, then by noise
data <- data[order(data$temperature, data$noise), ]; data
ANOVA.twoway.factorial(data, 4, 4, 3, 2, 0.05) # one-way ANOVA

                    SS df               MS                 F
A     539.138333333334  3 179.712777777778  18.7038450419196
B     13.6808333333336  2 6.84041666666678 0.711925411968788
AB    55.6991666666661  6 9.28319444444435 0.966160740098283
Error            115.3 12 9.60833333333335
Total 723.818333333333 23

 F crit AB  F test AB p value AB
    2.9961     0.9662     0.4870
```

The level of significance is 0.05.

(c) *Conclusion*
Since p–value, 0.49, is larger than the level of significance, 0.05, we
(i) **fail to reject** (ii) **reject** null hypothesis there is no interaction effect.

(d) *Comment: Interaction AB, Temperature × Noise, Effect.*
Interaction effects are differences between mean ROC at each temperature × noise treatment and the *sum* of the grand mean ROC and the corresponding temperature and noise effects,
$\hat{\alpha\beta}_{11} = \bar{x}_{11} - (\bar{\bar{x}} + \alpha_1 + \beta_1) = 8.75 - (8.4 - 1.7 - 0.2) =$
(i) **−0.55** (ii) **−0.35** (iii) **2.25** (iv) **3.2**
$\hat{\alpha\beta}_{11} = \bar{x}_{12} - (\bar{\bar{x}} + \alpha_1 + \beta_2) = 7.25 - (8.4 - 1.7 + 0.9) =$
(i) **−0.55** (ii) **−0.35** (iii) **2.25** (iv) **3.2**
$\vdots$
$\hat{\alpha\beta}_{42} = \bar{x}_{32} - (\bar{\bar{x}} + \alpha_4 + \beta_2) = 16.65 - (8.4 + 7.9 + 0.9) =$
(i) **−0.55** (ii) **−0.35** (iii) **2.25** (iv) **3.2**
$\hat{\alpha\beta}_{43} = \bar{x}_{11} - (\bar{\bar{x}} + \alpha_4 + \beta_2) = 18.60 - (8.4 + 7.9 - 0.9) =$
(i) **−0.55** (ii) **−0.35** (iii) **2.25** (iv) **3.2**

where, notice, (i) **none** (ii) **some** (iii) **all** noise effects are "close", within random error, to *zero* confirming mean ROC responses are the same for different temperature x noise interactions levels; which implies temperature and noise are acting (i) **independent of** (ii) **dependent on** one another

7. *Two–way ANOVA randomized block: test noise level treatment effect.*
Consider the effect of noise level on ROC of deer mice, blocking over temperature levels; in particular, test if noise is significant at 5% using two–way ANOVA randomized block design.

	Treatment, noise →	$j = 1$, low	$j = 2$, medium	$j = 3$, high	row ave
	$i = 1$, 0^o F	10.3	9.1	6.1	$\overline{x}_1 = 8.5$
Block,	$i = 2$, 10^o F	1.8	12.1	5.1	$\overline{x}_2 = 6.3$
Temperature	$i = 3$, 20^o F	1.2	6.5	1.2	$\overline{x}_3 = 3.0$
	$i = 4$, 30^o F	12.4	16.1	18.1	$\overline{x}_4 = 15.5$
	column ave	$\overline{x}_1 = 6.4$	$\overline{x}_2 = 11.0$	$\overline{x}_3 = 7.6$	$\overline{\overline{x}} = 8.3$

```
temperature <- c(0,0,0,10,10,10,20,20,20,30,30,30)
noise <- c("low","medium","high","low","medium","high","low","medium","high","low","medium","high")
ROC <- c(10.3, 9.1, 6.1, 1.8, 12.1, 5.1, 1.2, 6.5, 1.2, 12.4, 16.1, 18.1)
data <- data.frame(temperature,noise,ROC); data
temperature <- factor(data$temperature,c("0","10","20","30")) # convert temperature to factor
noise <- factor(data$noise,c("low","medium","high")) # convert noise to factor
```

(a) *Statement*

i. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus $H_a : \beta_1 \neq \beta_2, \beta_1 = \beta_3$.

ii. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus $H_a : \beta_1 \neq \beta_3, \beta_1 \neq \beta_2$.

iii. $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus
$H_a :$ at least one $\beta_i \neq 0$, $i = 1, 2, 3$.

iv. $H_0 : \mu_{\cdot 1} = \mu_{\cdot 2} = \mu_{\cdot 3}$ versus
$H_a :$ at least one $\mu_{i\cdot} \neq \mu_{j\cdot}$, $i, j = 1, 2, 3$.

(b) *Test*

Since the test statistic is $F = \frac{21.98}{9.36} = 2.35$, the p–value, with
$k - 1 = 3 - 1 =$ (i) **2** (ii) **3** (iii) **6** (iv) **12** and
$(k-1)(b-1) = (3-1)(4-1) =$ (i) **2** (ii) **3** (iii) **6** (iv) **12**
degrees of freedom, is given by
p–value $= P(F \geq 2.35) =$ (i) **0** (ii) **0.18** (iii) **0.43**.

```
# two-way ANOVA for randomized block design, dataframe columns (c), noise (k), temp (b), signif 0.05
# order data frame by treatment, noise:
data <- data[order(data$noise), ]; data
ANOVA.twoway.block(data, 3, 4, 3, 0.05)
```

```
                        SS df               MS                F
Treatment 43.9616666666667  2 21.9808333333334  2.3476132554069
Block     254.006666666667  3 84.6688888888889 9.04286943364883
Error     56.1783333333332  6 9.36305555555553
Total     354.146666666667 11

 F crit treat  F test treat p value treat
       5.1433        2.3476        0.1766

 F crit block  F test block p value block
      4.75706       9.04287       0.01208
```

The level of significance is 0.05.

(c) *Conclusion*

Since the p–value, 0.18, is larger than level of significance, 0.05,
(i) **fail to reject** (ii) **reject** the null hypothesis the average mice ROC responses to (i) **temperatures** (ii) **noises** are the same.

(d) *Comment: randomized block versus factorial design*
Factorial designs (i) **always** (ii) **sometimes** (iii) **never** have at least two replications per treatment (combination), whereas, randomized block designs have (only) one replication per treatment $\times$ block combination. Whereas each factor is considered "equal" to one another in a factorial design, blocks are intended to "serve" the treatment in a randomized block design.

(e) *Comment: block effect should be independent of treatment effect*
Using temperature as a block is a poor choice because the temperature $\times$ noise interaction could be (i) **significant** (ii) **insignificant**. A better choice of block is (i) **mouse** (ii) **pressure** because each mouse would be used for all three noise levels, so mouse $\times$ noise would be random, mouse and noise would be independent of one another.

8. *Two–way ANOVA factorial: aov R function*
Again consider the effect of air temperature and noise on the ROC of deer mice and test if noise is significant at 5% using two–way ANOVA.

	Factor B, noise $\rightarrow$	$j = 1$, low	$j = 2$, medium	$j = 3$, high	row ave
	$i = 1$, 0^o F	10.3	9.1	6.1	$\overline{x}_1 = 6.7$
		7.2	5.4	2.1	
Factor A,	$i = 2$, 10^o F	1.8	12.1	5.1	$\overline{x}_2 = 6.5$
temperature		9.8	4.2	6.2	
	$i = 3$, 20^o F	1.2	6.5	1.2	$\overline{x}_3 = 3.9$
		8.1	4.1	2.1	
	$i = 4$, 30^o F	12.4	16.1	18.1	$\overline{x}_4 = 16.3$
		15.1	17.2	19.1	
	column ave	$\overline{x}_1 = 8.2$	$\overline{x}_2 = 9.3$	$\overline{x}_3 = 7.5$	$\overline{\overline{x}} = 8.4$

```
replications <- c(1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2)
temperature <- c(0,0,0,0,0,0,10,10,10,10,10,10,20,20,20,20,20,20,30,30,30,30,30,30)
noise <- c("low","low","medium","medium","high","high","low","low","medium","medium","high","high","low","low","medium","
ROC <- c(10.3, 7.2, 9.1, 5.4, 6.1, 2.1, 1.8, 9.8, 12.1, 4.2, 5.1, 6.2, 1.2, 8.1, 6.5, 4.1, 1.2, 2.1, 12.4, 15.1, 16.1, 17
data <- data.frame(replications,temperature,noise,ROC); data
temperature <- factor(data$temperature,c("0","10","20","30")) # convert temperature to factor
noise <- factor(data$noise,c("low","medium","high")) # convert noise to factor
```

(a) *ANOVA table and p-values.*
The following two–way factorial ANOVA tells us the temperature factor is significant at level $\alpha =$ (i) **0.1** (ii) **0.01** (iii) **0.001**.

```
summary(aov(ROC ~ temperature + noise + temperature * noise))
```

```
                  Df Sum Sq Mean Sq F value   Pr(>F)
temperature        3  539.1  179.71  18.704 8.08e-05 ***
noise              2   13.7    6.84   0.712    0.510
temperature:noise  6   55.7    9.28   0.966    0.487
Residuals         12  115.3    9.61
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(b) *Summary interaction mean statistics.*
The average ROC for 0^o temperature $\times$ low noise level is
(i) **5.80** (ii) **7.25** (iii) **8.75**.

```
tapply(ROC,list(temperature,noise), mean) # mean responses for temperature x noise treatments
```

```
     low medium  high
0   8.75   7.25  4.10
10  5.80   8.15  5.65
20  4.65   5.30  1.65
30 13.75  16.65 18.60
```

(c) *Interaction Plots.*

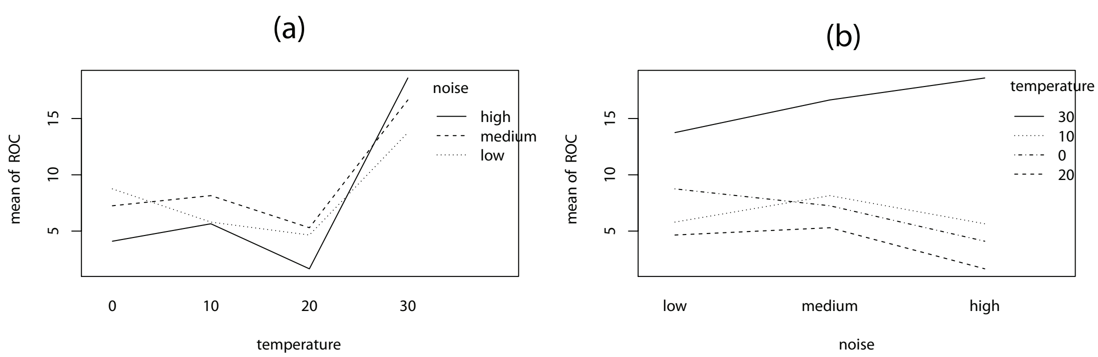

Figure 5.15: Interaction Plots

```
par(mfrow=c(1,2))
interaction.plot(temperature,noise,ROC) # interaction plots
interaction.plot(noise,temperature,ROC)
par(mfrow=c(1,1))
```

i. *Main Factor A, Temperature, Effect*
Interaction plot (b) indicates there (i) **is** (ii) **is not** significant differences in mean ROC for different temperature levels.

ii. *Main Factor B, Noise, Effect*
Interaction plot (a) indicates there (i) **is** (ii) **is not** significant differences in mean ROC for different noise levels.

iii. *Interaction Factor AB, Temperature $\times$ Noise, Effect*
Interaction plots (a) and (b) both indicate there (i) **is** (ii) **is not** significant interaction effects. Although the plots are *not exactly* parallel, indicating interaction, they are not "badly" non-parallel.

9. *Two–way ANOVA factorial: unbalanced data, aov R function*
Again consider effect of air temperature and noise on ROC of deer mice and test if noise is significant at 5% using two–way ANOVA but for unbalanced data.

	Factor B, noise →	$j = 1$, low	$j = 2$, medium	$j = 3$, high	row ave
	$i = 1$, 0^o F	10.3 7.2	9.1 5.4	6.1 2.1	$\overline{x}_1 = 6.7$
Factor A, temperature	$i = 2$, 10^o F	1.8 9.8	12.1 4.2	5.1 6.2	$\overline{x}_2 = 6.5$
	$i = 3$, 20^o F	1.2 8.1	6.5 4.1	1.2 2.1	$\overline{x}_3 = 3.9$
	$i = 4$, 30^o F	12.4 15.1	16.1 17.2		$\overline{x}_4 = 15.2$
	column ave	$\overline{x}_1 = 8.2$	$\overline{x}_2 = 9.3$	$\overline{x}_3 = 3.8$	$\overline{\overline{x}} = 7.4$

```
replications <- c(1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2)
temperature <- c(0,0,0,0,0,0,10,10,10,10,10,10,20,20,20,20,20,20,30,30,30,30)
noise <- c("low","low","medium","medium","high","high",and so on "low","low","medium","medium","high","high")
ROC <- c(10.3, 7.2, 9.1, 5.4, and so on 17.2);
data <- data.frame(replications,temperature,noise,ROC); data
temperature <- factor(data$temperature,c("0","10","20","30")) # convert integer to factor
noise <- factor(data$noise,c("low","medium","high")) # arrange levels this way
```

(a) *How is the data unbalanced?*
Number of observations for different temperature levels
(i) **same** (ii) **different** (iii) **missing**
Number of observations for different noise levels
(i) **same** (ii) **different** (iii) **missing**
Observations for some treatment combinations (cells)
(i) **same** (ii) **different** (iii) **missing**

(b) *ANOVA table and p-values.*
p-value (0.0000) for previous balanced main effect temperature
(i) **same as** (ii) **different from**
p-value (0.0015) for unbalanced main effect temperature

p-value (0.510) for previous balanced main effect noise
(i) **same as** (ii) **different from**
p-value (0.2126) for unbalanced main effect noise

In general, balanced two–way factorial ANOVA
(i) **same as** (ii) **different from**
unbalanced two-way ANOVA

```
summary(aov(ROC ~ temperature + noise + temperature * noise))
```

```
                  Df Sum Sq Mean Sq F value Pr(>F)
temperature        3    326   108.6   10.40 0.0015 **
noise              2     37    18.7    1.79 0.2126
temperature:noise  5     17     3.3    0.32 0.8914
Residuals         11    115    10.4
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```