Chapter 12

Inference on Categorical Data

12.1 Goodness-of-Fit Test

We perform a goodness of fit test using test statistic

$$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}, \quad i = 1, 2, \ldots k,$$

where $E_i = np_i$, which is approximately chi-square, $k-1$ df, provided expected frequencies $E_i \geq 1$ and at least 80% of expected frequencies are more than 5.

Exercise 12.1 (Goodness-of-Fit Test)

1. *Goodness of fit test: age distribution.*
   Age distribution of a random sample of 463 people living in Uppsala, a city in Sweden, is compared to age distribution to *all* of Sweden. Test if age distribution in Uppsala is different from age distribution for all of Sweden at $\alpha = 0.05$.

age	Uppsala (out of 463)	Sweden (% total)
under 5	47	6.7%
5 to 16	75	14.1%
16 to 65	296	69.5%
over 65	45	9.7%

(Blank data table. Relabel var1 Uppsala, var2 percentage. Type data into these two columns. Data, Data expression, Expression: percentage*463, New column name: Sweden, Compute.)

   (a) *Hypothesis test*

      i. *Statement.* Choose *one or more.*

         A. H_0 : Uppsala age distribution same as Sweden
            versus H_1 : Uppsala age distribution different from Sweden

B. $H_0 : p_1 = 0.067, p_2 = 0.141, p_3 = 0.695, p_4 = 0.097$
versus H_1 : Uppsala age distribution different from null

C. $H_0 : p_1 = 0.067, p_2 = 0.141, p_3 = 0.695, p_4 = 0.097$
versus H_1 : at least one p_i not equal to null

No matter how this question is worded, null hypothesis for test is *always same* (as expected distribution) and alternative hypothesis is *always different* (from expected distribution)..

ii. *Test.*

age	O_i	$E_i = n \times p_i$	$\frac{(O_i - E_i)^2}{E_i}$
under 5	47	$463 \times 0.067 \approx 31$	8.2
5 to 16	75	$463 \times 0.141 \approx 65$	1.5
16 to 65	296	$463 \times 0.695 \approx 322$	2.1
over 65	45	$463 \times 0.097 \approx 45$	0

observed test statistic is $\chi^2 = \sum \frac{(O-E)^2}{E} = 8.2 + 1.5 + 2.1 + 0 =$
(circle one) **11.8** / **18.3** / **23.4**,
with degrees of freedom

$$\text{number of rows} - 1 = 4 - 1 =$$

(circle one) **1** / **3** / **4** df,
and so p–value is

$$\text{p–value} = P(\chi^2 \geq 11.8) =$$

(circle one) **0.01** / **0.08** / **0.10**.
(Stat, Goodness-of-fit, Chi-square test, Observed: Uppsala, Expected: Sweden, Calculate. There may be some round off error.)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

iii. *Conclusion.*
Since p–value $= 0.01 < \alpha = 0.05$,
do not reject / **reject** H_0 : Uppsala different from Sweden.
Sample statistic χ^2 indicates age distribution in Uppsala
same as / **different from**
age distribution for all of Sweden.

(b) *Related questions.*

i. *Understanding a goodness of fit test.*
Goodness of fit test involves comparing *observed* data with *expected* data. Expected data is data we derive *assuming* age distribution in Uppsala is *equal* to age distribution in all of Sweden. In other words, if observed data set and generated expected data sets are "close" to one another, this indicates age distribution in Uppsala
(circle one) **same as** / **different from**
age distribution for all of Sweden.

ii. *Table used to compare observed to expected.*

age	O	$E = n \times p_i$	$\frac{(O-E)^2}{E}$
under 5	47	$463 \times 0.067 \approx 31$	8.2
5 to 16	75	$463 \times 0.141 \approx 65$	1.5
16 to 65	296	$463 \times 0.695 \approx 322$	2.1
over 65	45	$463 \times 0.097 \approx 45$	0

Expected number under age 5 in Uppsala if age distribution in Uppsala same as Sweden.
If 6.7% of Sweden under age 5, then in a town of size 463 people, $463 \times 0.067 \approx$ (choose one) **24** / **31** / **49**
of these people should be age 5. In other words, in Uppsala, with 463 people, we would *expect* 31 people to be under age of 5 if age distribution in Uppsala same as age distribution in all of Sweden.

iii. *Large* χ^2 *test statistic indicates different distribution..*
The χ^2 test statistic is one possible way to measure how close observed and expected data sets are to one another. Since $\sum \frac{(O-E)^2}{E} = 11.8$ appears to be is *large*, indicates age distribution in Uppsala (choose one) **same as** / **different from** age distribution for all of Sweden.

2. *Goodness of fit test: flower colors.*
Observed color distribution of a random sample of 1000 roses is compared to expected (using genetic theory) color distribution of all roses. Test if observed color distribution of random sample of these 1000 roses contradicts expected (using genetic theory) color distribution of all roses at $\alpha = 0.01$

color	observed number	expected proportion
red	926	0.925
strong–pink	20	0.025
weak–pink	28	0.025
white	26	0.025

(Blank data table. Relabel var1 observed number, var2 expected proportion. Type data into these two columns. Data, Data expression, Expression: expected proportion*1000, New column name: expected number, Compute. Data, Save data, 12.1.2 flower color distribution.)

(a) *Statement.* Choose *one or more.*

i. H_0 : observed color distribution same as expected under genetic theory versus H_1 : observed color distribution different from expected

ii. $H_0 : p_1 = 0.925, p_2 = 0.025, p_3 = 0.025, p_4 = 0.025$ versus H_1 : observed color distribution different from null

iii. $H_0 : p_1 = 0.925, p_2 = 0.025, p_3 = 0.025, p_4 = 0.025$ versus H_1 : at least one p_i not equal to null

(b) *Test.*

color	O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
red	926	925	0.00
strong–pink	20	______	1.00
weak–pink	28	25	0.36
white	26	25	______

observed test statistic is
$\chi^2 = \sum \frac{(O-E)^2}{E} = 0.00 + 1.00 + 0.36 + 0.04 =$
(circle one) **1.40** / **3.23** / **4.11**,
with degrees of freedom

$$\text{number of rows} - 1 = 4 - 1 =$$

(circle one) **1** / **3** / **4** df,
and so p–value is

$$\text{p–value} = P(\chi^2 \geq 1.40) =$$

(circle one) **0.16** / **0.53** / **0.71**.

(Stat, Goodness-of-fit, Chi-square test, Observed: observed number, Expected: expected number (or expected proportion), Calculate.)

Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.
(Notice: $\alpha = 0.01$!)

(c) *Conclusion.*
Since p–value $= 0.71 > \alpha = 0.01$,
do not reject / **reject** null: H_0 : observed same as genetic theory.
Observed data indicates color distribution of roses
same as / **different from**
hypothesized (using genetic theory) color distribution of roses.

3. *Goodness of fit test: peas, equally likely.*
Given dataset of 556 *observed* frequencies of various types of peas, test whether or not proportion of round–yellow, wrinkled–yellow, round–green and wrinkled–green peas occurs with *equal* frequency or not at 5%.

type	round–yellow	wrinkled–yellow	round–green	wrinkled–green
frequency	315	101	108	32

,

(Blank data table. Relabel var1 observed pea, var2 expected pea proportion (all 0.25–why?). Type data into these two columns. Data, Data expression, Expression: expected pea proportion*556, New column name: expected pea, Compute.)

(a) *Statement.* Choose *one or more.*

i. H_0 : observed pea type distribution same as expected 9 : 3 : 3 : 1 distribution versus H_1 : observed pea type distribution different from expected

ii. $H_0 : p_1 = \frac{1}{4}, p_2 = \frac{1}{4}, p_3 = \frac{1}{4}, p_4 = \frac{1}{4}$
versus H_1 : observed pea type distribution different from null

iii. $H_0 : p_1 = \frac{1}{4}, p_2 = \frac{1}{4}, p_3 = \frac{1}{4}, p_4 = \frac{1}{4}$
versus H_1 : at least one p_i not equal to null

(b) *Test.*

pea type	O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
round–yellow	315	$\left(\frac{1}{4}\right) 556 = 139$	222.85
wrinkled–yellow	101	139	10.388
round–green	108	______	6.9137
wrinkled–green	32	139	______

observed test statistic is
$\chi^2 \approx \sum \frac{(O-E)^2}{E} = 222.85 + 10.388 + 6.9137 + 82.367 \approx$
(circle one) **311** / **322** / **345**,
with degrees of freedom

$$\text{number of rows} - 1 = 4 - 1 =$$

(circle one) **1** / **3** / **4** df,
and so p–value is

$$\text{p–value} = P(\chi^2 \geq 322) =$$

(circle one) **0.00** / **0.05** / **1.72**.

(Stat, Goodness-of-fit, Chi-square test, Observed: observed pea, Expected: expected pea, Calculate.)

Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(c) *Conclusion.*
Since p–value $= 0.00 < \alpha = 0.05$,
do not reject / **reject** null: H_0 : observed same as expected ratio.
Observed data indicates pea type distribution
same as / **different from**
the hypothesized equal pea type distribution.

4. *Goodness of fit test: peas again, classical approach.*
Given dataset of 556 *observed* frequencies of various types of peas, test whether or not proportion of round–yellow, wrinkled–yellow, round–green and wrinkled–green peas occurs in ratio 9 : 3 : 3 : 1 or not at 5%.

type	round–yellow	wrinkled–yellow	round–green	wrinkled–green
frequency	315	101	108	32

(Blank data table. Relabel var1 observed pea, var2 expected pea proportion. Type data into these two columns. Data, Data expression, Expression: expected pea proportion*556, New column name: expected pea, Compute. Data, Save data, 12.1.4 pea type distribution 2.)

(a) *Statement.* Choose *one or more.*

i. H_0 : observed pea type distribution same as expected $9:3:3:1$ distribution versus H_1 : observed pea type distribution different from expected

ii. $H_0 : p_1 = \frac{9}{16}, p_2 = \frac{3}{16}, p_3 = \frac{3}{16}, p_4 = \frac{1}{16}$ versus H_1 : observed pea type distribution different from null

iii. $H_0 : p_1 = \frac{9}{16}, p_2 = \frac{3}{16}, p_3 = \frac{3}{16}, p_4 = \frac{1}{16}$ versus H_1 : at least one p_i not equal to null

(b) *Test.*

pea type	O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
round–yellow	315	$\left(\frac{9}{16}\right) 556 = 312.75$	0.02
wrinkled–yellow	101	104.25	0.10
round–green	108	______	0.13
wrinkled–green	32	34.75	______

(Stat, Goodness-of-fit, Chi-square test, Observed: observed pea, Expected: expected pea, Calculate.)

observed test statistic is
$\chi_0^2 = \sum \frac{(O-E)^2}{E} = 0.02 + 0.10 + 0.13 + 0.22 =$
(circle one) **0.31** / **0.47** / **1.23**,
with degrees of freedom

$$\text{number of rows} - 1 = 4 - 1 =$$

(circle one) **1** / **3** / **4** df,
and so critical value at $\alpha = 0.05$ is

$$\chi^2 =$$

(circle one) **5.81** / **6.81** / **7.81**.
(Stat, Calculators, Chi-square, DF: 3, Prob(X $\geq$ [?]) = 0.05 Compute.)

(c) *Conclusion.*
Since $\chi_0^2 = 0.47 < \chi^2 = 7.81$,
do not reject / **reject** null: H_0 : observed same as expected ratio.
Observed data indicates pea type distribution
same as / **different from**
the hypothesized $9:3:3:1$ pea type distribution.

(d) *Check assumptions.*

i. All E_i (choose one) **are** / **are not** greater than 1.

ii. More than 80% of E_i should be more than 5. In fact, in this case, (choose one) **0%** / **50%** / **100%** of $E_i > 5$.

12.2 Tests for Independence and the Homogeneity of Proportions

We perform a test of independence and a test of homogeneity of proportions using *same* test statistic

$$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i},$$

where E_i = (row total) × (column total) ÷ (table total), which is approximately chi-square, $(r-1)(c-1)$ df, provided expected frequencies $E_i \geq 1$ and at least 80% of expected frequencies are greater than 5.

Exercise 12.2 (Tests for Independence and Homogeneity of Proportions)

1. *Test of independence: fathers, sons and college.*
   Random sample of college attendance by fathers and their oldest sons in a midwestern city recorded in table below. Test whether or not a son attends college is *dependent* on whether or not father attends college at $\alpha = 0.01$.

observed, O_i	son attended college	son did not attend college	
father attended college	18	12	30
father did not attend college	22	33	55
	40	45	85

(Blank data table. Relabel var1 father, var2 son attends, var3 son does not attend. Type data into these three columns. Data, Save data, 12.2.1 father and son data.)

(a) *Hypothesis test.*

i. *Statement.* Choose one.

A. H_0 : son attends equals father attending
versus H_1 : son attends does not equal to father attending

B. H_0 : son attends independent of father attending
versus H_1 : son attends dependent on father attending

C. H_0 : son attends dependent of father attending
versus H_1 : son attends independent of father attending

No matter how this question is worded, null hypothesis for test is *always* independent and alternative hypothesis is *always* dependent.

ii. *Test.*

attendance	observed, O_i	expected, E_i, if independent	$\frac{(O_i-E_i)^2}{E_i}$
both father and son	18	$\frac{30 \cdot 40}{85} \approx 14.1$	1.08
not father, son does	22	$\frac{55 \cdot 40}{85} \approx 25.9$	0.59
father does, not son	12	$\frac{30 \cdot 45}{85} \approx 15.9$	0.96
neither father nor son	33	$\frac{55 \cdot 45}{85} \approx 29.1$	0.52

(Options, Edit, Display: Expected Counts, Compute (for E_i),
and Options, Edit, Display: Contributions to Chi-Square, Compute (for $\frac{(O_i-E_i)^2}{E_i}$).)

The observed test statistic is

$$\sum \frac{(O_i - E_i)^2}{E_i} = 1.08 + 0.59 + 0.96 + 0.52 = 3.15$$

with degrees of freedom

$$(\text{number of rows} - 1) \times (\text{number of columns} - 1) = (2-1) \times (2-1) =$$

(circle one) **1** / **2** / **3** df,
and so p–value is

$$\text{p–value} = P(\chi^2 \geq 3.15) =$$

(circle one) **0.01** / **0.08** / **0.10**.

(Stat, Tables, Contingency, with summary, Select columns for table: son attends, son does not attend, Row labels in: father, Next, choose Expected number, Chi-Square, Calculate.)

Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.
(Notice: $\alpha = 0.01$!)

iii. *Conclusion.*
Since p–value $= 0.08 > \alpha = 0.01$,
do not reject / **reject** null H_0 : independence.
Observed data indicates whether or not a son attends college
independent of / **dependents on**
whether or not father attends college.

(b) *Related questions*

i. *Understanding a test of independence.*
A test of independence involves comparing *observed* data with *expected* data. Expected data is data we derive *assuming* a son's attendance in college is *in*dependent of a father's attendance in college. If observed data set and generated expected data sets are "close" to one another, whether or not a son attends college
(circle one) **is independent of** / **dependents**
on whether or not father attends college.

ii. *Table used to compare observed to expected under independence.*

attendance	observed, O_i	expected, E_i, if independent	$\frac{(O_i-E_i)^2}{E_i}$
both father and son	18	$\frac{30\cdot 40}{85} \approx 14.1$	1.08
not father, son does	22	$\frac{55\cdot 40}{85} \approx 25.9$	0.59
father does, not son	12	$\frac{30\cdot 45}{85} \approx 15.9$	0.96
neither father nor son	33	$\frac{55\cdot 45}{85} \approx 29.1$	0.52

For example, expected number of families in which both father and son attends college, *if* a son attending college is assumed to be *independent* of a father attending college, is

$$\text{proportion fathers who attend} \times \text{proportion sons who attend} \times \text{sample size} = \frac{30}{85}\cdot\frac{40}{85}\cdot 85 = \frac{30\cdot 40}{85} \approx$$

(circle one) **14.1** / **27.5** / **153**.

iii. *Small χ^2 test statistic indicates independence.*
The χ^2 test statistic is one possible way to measure how close observed and expected data sets are to one another.
Since $\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = 1.08 + 0.59 + 0.96 + 0.52 = 3.15$ *appears* to be *small*, this indicates whether or not a son attends college is (circle one) **independent of** / **dependent on** whether or not a father attends college.

2. *Test of independence: flu symptoms.*
Consider *observed* data from a random sample of 354 patients in an investigation of effect of a new drug on reducing flu symptoms. Test whether or not reduction of flu symptoms is *dependent* on whether or not drug is administered at $\alpha = 0.01$.

observed, O_i	drug	no drug	subtotals
flu symptoms reduced	120	81	201
flu symptoms not reduced	50	103	153
subtotals	170	184	354

(Blank data table. Relabel var1 flu, var2 flu symptoms reduced, var3 flu symptoms not reduced. Type data into these three columns.)

(a) *Hypothesis test.*

i. *Statement.* Choose one.

A. H_0 : flu symptoms equals of drug
versus H_1 : flu symptoms does not equal drug

B. H_0 : flu symptoms independent of drug
versus H_1 : flu symptoms dependent on drug

C. H_0 : flu symptoms *not* independent of drug
versus H_1 : flu symptoms independent of drug

ii. *Test.*
The p–value is (circle one) **0.00** / **0.08** / **0.10**.
(Stat, Tables, Contingency, with summary, Select columns for table: symptoms reduced, symptoms not reduced, Row labels in: drug, Next, choose Expected number, Chi-Square, Calculate.)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.
(Notice: $\alpha = 0.01$!)

iii. *Conclusion.*
Since p–value $= 0.00 < \alpha = 0.01$,
(circle one) **do not reject** / **reject** null H_0 : independence.
Data indicates flu symptoms are
(circle one) **independent of** / **dependent on** drug.

iv. *Check assumptions.*
A. All E_i (choose one) **are** / **are not** greater than 1.
B. At least 80% of E_i should be greater than 5. In fact, in this case, (choose one) **0%** / **50%** / **100%** of $E_i > 5$.

(b) *Detailed calculations for p–value of test statistic.*

flu study	observed, O_i	expected, E_i	$\frac{(O_i-E_i)^2}{E_i}$
drug given, flu reduced	120	96.5	5.72
drug not given, flu reduced	50	73.5	7.51
drug given, flu not reduced	81	104.5	5.28
drug not given, flu not reduced	103	79.5	6.95

(Given on StatCrunch output, second numbers in each element of contingency table.)
Observed test statistic is

$$\sum \frac{(O_i - E_i)^2}{E_i} = 5.72 + 7.51 + 5.28 + 6.95 =$$

(circle one) **11.23** / **21.33** / **25.46**,
with degrees of freedom

$$(\text{number of rows} - 1) \times (\text{number of columns} - 1)$$
$$= (2 - 1) \times (2 - 1) =$$

(circle one) **1** / **2** / **3** df,
and so p–value is

$$\text{p–value} = P(\chi^2 \geq 25.46) =$$

(circle one) **0.00** / **0.08** / **0.10**.
(Given on StatCrunch output.)

(c) *How are they dependent (by drug/no drug)?*

observed, O_i	drug	no drug	subtotals
flu symptoms reduced	$\frac{120}{170} \approx 0.71$	$\frac{81}{184} \approx 0.44$	201
flu symptoms not reduced	$\frac{50}{170} \approx 0.29$	$\frac{103}{184} \approx 0.56$	153
subtotals	170	184	354

(Click options in dialog box StatCrunch, choose column percents in Display then Compute)

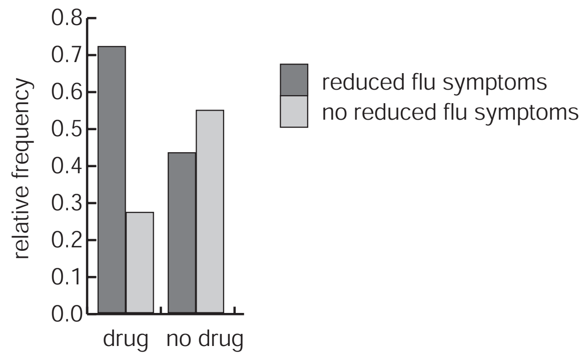

Figure 12.1 (Bar graph: flu symptoms dependent on drug.)

(Blank data table. Relabel var1 flu, var2 drug, var3 no drug, type flu symptoms reduced and flu symptoms not reduced under flu column, type 0.71 and 0.29 under drug column and 0.44 and 0.56 under no drug column. Chart, Columns, choose flu symptoms, flu symptoms not reduced, Row labels in: drug, Plot: vertical bars (split).)

Flu symptoms **reduced** / **not reduced** more likely if given drug, less likely if not given drug.

3. *Test of independence: plant growth.*
Consider *observed* data from a random sample of 1000 plants in an investigation of effect of nutritional level on plant growth. Test whether plant growth *dependents* on nutrition at $\alpha = 0.05$.

O_i	nutritional level →	poor	adequate	excellent	row totals
plant growth	below average	70	95	35	200
	average	130	450	30	610
	above average	90	30	70	190
	column totals	290	575	135	1000

(Blank data table. Relabel var1 plant growth, var2 poor, var3 adequate, var4 excellent. Type data into these four columns. Data, Save data, 12.2.4 plant and nutrition data.)

(a) *Statement.* Choose one.

i. H_0 : plant growth is equal to nutrition
versus H_1 : plant growth is greater than nutrition

ii. H_0 : plant growth independent of nutrition
versus H_1 : plant growth dependent of nutrition

iii. H_0 : plant growth *not* independent of nutrition
versus H_1 : plant growth independent of nutrition

(b) *Test.*
The p–value is (circle one) **0.00** / **0.08** / **0.10**.
(Stat, Tables, Contingency, with summary, Select columns for table: poor, adequate, excellent, Row labels in: plant growth, Next, choose Expected number, Chi-Square, Calculate.)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(c) *Conclusion.*
Since p–value $= 0.00 < \alpha = 0.05$,
do not reject / **reject** null H_0 : independence.
Data indicates plant growth
is independent of / **dependents on**
nutrition.

(d) *Detailed calculations for p–value of test statistic.*

flu study	O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
below plant, poor nutrition	70	58.0	2.48
average plant, poor nutrition	130	$\frac{(290)(610)}{1000} =$ ______	12.43
above plant, poor nutrition	90	55.1	22.11
below plant, adequate nutrition	95	115	3.48
average plant, adequate nutrition	450	350.8	28.08
above plant, adequate nutrition	30	$\frac{(575)(190)}{1000} =$ ______	57.49
below plant, excellent nutrition	35	27	2.37
average plant, excellent nutrition	30	82.4	33.28
above plant, excellent nutrition	70	25.7	76.68

where *expected* values can also be written as

E_i	nutritional level →	poor	adequate	excellent	row totals
plant	below average	58.0	115	27	200
growth	average	176.9	350.8	82.4	610
	above average	55.1	109.3	25.7	190
	column totals	290	575	135	1000

(Click options in dialog box Statcrunch, choose expected count in Display then Compute. Things may not add up exactly because of round-off error.)

Observed test statistic is

$$\sum \frac{(O_i - E_i)^2}{E_i} = 2.48+12.43+22.11+3.48+28.08+57.49+2.37+33.28+76.68 =$$

(circle one) **123.4** / **198.3** / **238.4**,
with degrees of freedom

$$(\text{number of rows} - 1) \times (\text{number of columns} - 1)$$
$$= (3 - 1) \times (3 - 1) =$$

(circle one) **1** / **3** / **4** df,
and so p–value is

$$\text{p–value} = P(\chi^2 \geq 238.4) =$$

(circle one) **0.00** / **0.08** / **0.10**.

(e) *Classical approach.*
Since critical value at $\alpha = 0.05$ is
$\chi^2 =$ (circle one) **5.81** / **6.81** / **9.49**.
(Stat, Calculators, Chi-square, DF: 4, Prob(X $\geq$ [?]) = 0.05 Compute.)
then $\chi^2_0 = 238.4 > \chi^2 = 9.49$,
do not reject / **reject** null: H_0 : independence
which **agrees** / **disagrees** with p-value approach.

4. *Test of homogeneity of proportions: brain cell growth.*
Consider *observed* data from a random sample of 390 neurons in an investigation of effect of nutritional level on brain cell growth. Test if proportion of brain cell growth is same or different for different nutrition levels at $\alpha = 0.05$.

O_i	nutritional level $\rightarrow$	poor	adequate	excellent	row totals
neuron	slow	70	95	35	200
growth	normal	90	30	70	190
	column totals	160	125	105	390

(Choose second row (average cell growth) then Edit, Rows, Delete. Delete rows!)

(a) *Statement.* Choose one.

i. H_0 : proportion cell growth same for different nutrition
versus H_1 : proportion cell growth different for different nutrition

ii. H_0 : cell growth independent of nutrition
versus H_1 : cell growth dependent of nutrition

iii. H_0 : cell growth different for different nutrition
versus H_1 : cell growth same for different nutrition

No matter how this question is worded, null hypothesis for test is *always* homogeneous (same) proportions and alternative hypothesis is *always* different proportions.

(b) *Test.*
The p–value is (circle one) **0.00** / **0.08** / **0.10**.
(Stat, Tables, Contingency, with summary, Select columns for table: A poor, A adequate, A excellent, Row labels in: A cell growth, Next, choose Expected number, Chi-Square, Calculate.)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(c) *Conclusion.*
Since p–value = 0.00 < $\alpha = 0.05$,
do not reject / **reject** null H_0 : same proportions.
Data indicates cell growth
same proportion / **different proportions**
for different nutrition levels.

(d) *How are proportions different (by nutrition level)?*

O_i	nutritional level →	poor	adequate	excellent	row totals
neuron	slow	$\frac{70}{160} \approx 0.44$	$\frac{95}{125} = 0.76$	$\frac{35}{105} \approx 0.33$	200
growth	normal	$\frac{90}{160} \approx 0.56$	$\frac{30}{125} = 0.24$	$\frac{70}{105} \approx 0.67$	190
	column totals	160	125	105	390

(Click options in dialog box StatCrunch, choose column percents in Display then Compute)

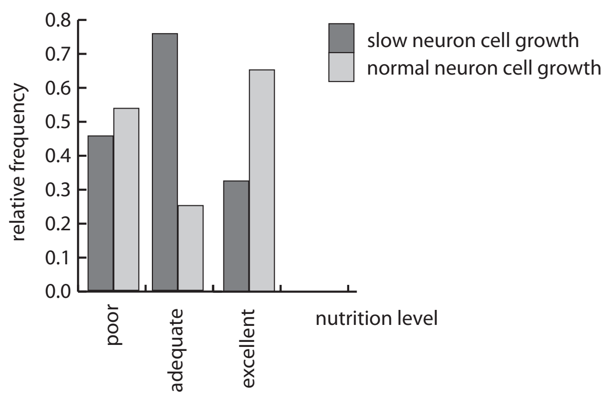

Figure 12.2 (Bar graph: cell growth proportions, different nutrition.)

Bar graph indicates cell growth
same proportion / **different proportions**
for different nutrition levels.

5. *Test of homogeneity of proportions: brain cell growth again.*
Consider *observed* data from a random sample of 390 neurons in an investigation of effect of nutritional level on brain cell growth. Test if proportion of brain cell growth is same or different for different nutrition levels at $\alpha = 0.05$.

O_i	nutritional level →	poor	adequate	excellent	row totals
neuron	slow	100	75	65	240
growth	normal	60	50	40	150
	column totals	160	125	105	390

(Blank data table. Relabel var1 neuron growth, var2 poor, var3 adequate, var4 excellent. Type data into these four columns.)

(a) *Statement.* Choose one.

i. H_0 : proportion cell growth same for different nutrition
versus H_1 : proportion cell growth different for different nutrition

ii. H_0 : cell growth independent of nutrition
versus H_1 : cell growth dependent of nutrition

iii. H_0 : cell growth different for different nutrition
versus H_1 : cell growth same for different nutrition

(b) *Test.*
The p–value is (circle one) **0.80** / **0.91** / **0.95**.
(Stat, Tables, Contingency, with summary, Select columns for table: B poor, B adequate, B excellent, Row labels in: B neuron growth, Next, choose Expected number, Chi-Square, Calculate.)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(c) *Conclusion.*
Since p–value $= 0.91 > \alpha = 0.05$,
do not reject / **reject** null H_0 : same proportions.
Data indicates cell growth
same proportion / **different proportions**
for different nutrition levels.

(d) *What proportion neuron cell growth for different nutritional levels?*

O_i	nutritional level →	poor	adequate	excellent	row totals
neuron	slow	$\frac{100}{160} \approx 0.63$	$\frac{75}{125} = 0.60$	$\frac{65}{105} \approx 0.62$	200
growth	normal	$\frac{60}{160} \approx 0.37$	$\frac{50}{125} = 0.40$	$\frac{40}{105} \approx 0.38$	190
	column totals	160	125	105	390

(Click options in dialog box StatCrunch, choose column percents in Display then Compute)

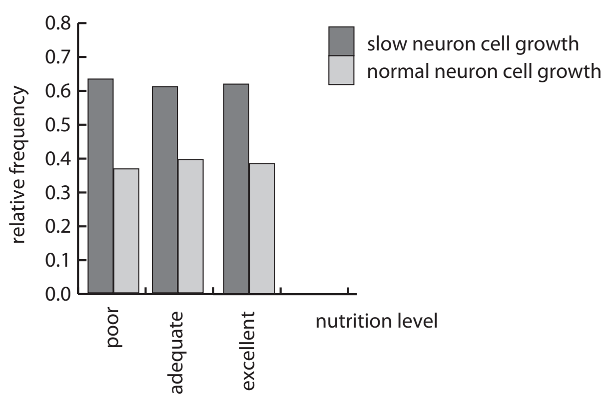

Figure 12.3 (Bar graph: neuron cell growth proportions, different nutrition.)

Bar graph indicates below neuron cell growth
same proportion / **different proportions** of about 60%
for different nutrition levels.

(e) *Comparing test of homogeneity with test of independence.*
Since test of homogeneity indicates same proportion of neuron cell growth for different nutrition levels, this implies brain cell growth
independent / **dependent**
of nutrition levels.

6. *Test of homogeneity of proportions: company stocks.*
Consider types of stocks (A, B or C) for small and large companies. Test if proportion of stock types is same or different for different companies at $\alpha = 0.05$.

company	stock	company	stock
small	A	large	C
small	B	small	C
small	C	large	B
large	B	small	A
small	B	small	A
small	B	small	B
large	B	small	B
large	A	large	C
large	C	large	B
large	C	large	A

(a) *Contingency table.* Fill in blanks.

O_i	stock type →	A	B	C	row totals
company	large	______	______	______	10
	small	______	______	______	10
	column totals	5	9	6	20

(Stat, Tables, Contingency, with data, Row variable: company, Column variable: stock, Calculate.)

(b) *Check assumptions.*

i. All E_i (choose one) **are** / **are not** greater than 1.

ii. More than 80% of E_i should be more than 5.
In fact, (choose one) **0%** / **17%** / **57%** of $E_i \geq 5$ (although one $O_i = 1$).

Oh well, continue anyway.

(c) *Statement.* Choose one.

i. H_0 : stock type different for different companies
versus H_1 : stock type same for different companies

ii. H_0 : proportion stock types same for different companies
versus H_1 : proportion stock types different for different companies

iii. H_0 : stock types independent of companies
versus H_1 : stock types dependent on companies

(d) *Test.*
The p–value is (circle one) **0.61** / **0.71** / **0.81**.
(Check output from above.)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(e) *Conclusion.*
Since p–value $= 0.61 > \alpha = 0.05$,
do not reject / **reject** null H_0 : same proportions.
Data indicates stock types
same proportion / **different proportions**
for different companies.

7. *Test of homogeneity of proportions: plant types.*
Of 504 wheat plant types documented prior to 1850, 30 exist today; of 496 corn plant types documented prior to 1850, only 37 exist today. Test if proportion of surviving plants today is same or different for different plants at $\alpha = 0.05$.

(a) *Contingency table.* Fill in blanks.

O_i	plant →	wheat	corn	row totals
exist today?	yes	______	______	67
	no	______	______	933
	column totals	504	496	1000

(b) *Statement.* Choose one.

i. H_0 : number surviving different for different plant types
versus H_1 : number surviving same for different plants

ii. H_0 : proportion surviving same for different plant types
versus H_1 : proportion surviving different for different plant types

iii. H_0 : number surviving independent of plant types
versus H_1 : number surviving dependent on plant types

(c) *Test.*
The p–value is (circle one) **0.11** / **0.31** / **0.34**.
(Stat, Tables, Contingency, with summary, Select columns for table: wheat, corn, Row labels in: exist?, Next, choose Expected number, Chi-Square, Calculate.)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(d) *Conclusion.*
Since p–value $= 0.34 > \alpha = 0.05$,
do not reject / **reject** null H_0 : same proportions.
Data indicates proportion surviving
same proportion / **different proportions**
for different plant types.

(e) *Test statistic.*
$\chi_0^2 \approx$ (circle one) **0.31** / **0.77** / **0.91**.

12.3 Inference about Two Population Proportions: Dependent Samples

If samples are dependent (matched), obtained at random and in the form of

A ↓ B →	success	failure
success	f_{11}	f_{12}
failure	f_{21}	f_{22}

where

$$f_{12} + f_{21} \geq 10,$$

then McNemar's test statistic for marginal homogeneity of proportions, $p_{1.} = p_{.1}$, is

$$\chi_0^2 = \frac{(f_{12} - f_{21})^2}{f_{12} + f_{21}}.$$

Exercise 12.3 (Inference about Two Population Proportions: Dependent Samples)

1. *Test $p_1 - p_2$, dependent samples: flu symptoms.*
   Observed data from a random sample of patients in an investigation of the effect of a new drug on reducing flu symptoms is given in the table below. All of 354 patients were given *both* drug and placebo (no drug). Both treatments lasted 12 weeks. Each patient was in study for a total of 24 weeks. Does this data support claim proportion of patients with reduced flu symptoms who took drug, $p_{1.}$, is *different* than patients with reduced flu symptoms who took placebo, $p_{.1}$, at $\alpha = 0.05$?

drug ↓ placebo →	reduced	not reduced	subtotals
flu symptoms reduced	120	81	201
flu symptoms not reduced	50	103	153
subtotals	170	184	354

   (a) *Check assumptions.*
   Since

$$f_{12} + f_{21} = 81 + 50 = 131 \geq 10$$

   assumptions **are** / **are not** satisfied, so continue.

   (b) *Statement.* Is *always* (circle one)

   i. $H_0 : p_{1.} = p_{.1}$ versus $H_1 : p_{1.} \neq p_{.1}$
   ii. $H_0 : p_{1.} = p_{.1}$ versus $H_1 : p_{1.} < p_{.1}$
   iii. $H_0 : p_{1.} = p_{.1}$ versus $H_1 : p_{1.} > p_{.1}$

(c) *Test.*
The p–value is (circle one) **0.003** / **0.007** / **0.009**.
(Stat, Tables, Contingency, with summary, Select columns for table: placebo reduced, placebo not reduced, Row labels in: drug 1, Hypothesis test: McNemar's test ..., Compute!)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(d) *Conclusion.*
Since p–value $= 0.007 < \alpha = 0.05$,
do not reject / **reject** null $H_0 : p_{1\cdot} = p_{\cdot 1}$.
In other words, population proportion reduced flu symptoms by drug
is less than / **equals** / **is greater than** / **is different from**
population proportion reduced flu symptoms by placebo.

(e) *Test statistic.*
$\chi_0^2 = \frac{(81-50)^2}{81+50} \approx$ (circle one) **0.007** / **7.34** / **354**.

2. *Test $p_1 - p_2$, dependent samples: flu symptoms again.*
Same test as before, but with different numbers.

drug ↓ placebo →	reduced	not reduced	subtotals
flu symptoms reduced	130	71	201
flu symptoms not reduced	40	113	153
subtotals	170	184	354

(a) *Check assumptions.*
Since

$$f_{12} + f_{21} = 71 + 40 = 111 \geq 10,$$

assumptions **are** / **are not** satisfied, so continue.

(b) *Statement.* Is *always* (circle one)

i. $H_0 : p_{1\cdot} = p_{\cdot 1}$ versus $H_1 : p_{1\cdot} \neq p_{\cdot 1}$
ii. $H_0 : p_{1\cdot} = p_{\cdot 1}$ versus $H_1 : p_{1\cdot} < p_{\cdot 1}$
iii. $H_0 : p_{1\cdot} = p_{\cdot 1}$ versus $H_1 : p_{1\cdot} > p_{\cdot 1}$

(c) *Test.*
The p–value is (circle one) **0.003** / **0.008** / **0.009**.
(Stat, Tables, Contingency, with summary, Select columns for table: placebo reduced, placebo not reduced, Row labels in: drug 2, Hypothesis test: McNemar's test ..., Compute!)
Level of significance $\alpha =$ (choose one) **0.01** / **0.05** / **0.10**.

(d) *Conclusion.*
Since p–value $= 0.003 < \alpha = 0.05$,
do not reject / **reject** null $H_0 : p_{1\cdot} = p_{\cdot 1}$.
In other words, population proportion reduced flu symptoms by drug
is less than / **equals** / **is greater than** / **is different from**
population proportion reduced flu symptoms by placebo.

(e) *Classical approach.*
Since critical value at $\alpha = 0.05$ is
$\chi^2 =$ (circle one) **3.84** / **8.66** / **9.49**.
(Stat, Calculators, Chi-square, DF: 1, Prob(X $\geq$ $\boxed{?}$) = 0.05 Compute.)
then $\chi_0^2 = 8.66 > \chi^2 = 3.84$,
do not reject / **reject** null: $H_0 : p_{1\cdot} = p_{\cdot 1}$
which **agrees** / **disagrees** with p-value approach.

Summary Section

HYPOTHESIS TESTS		mean μ	variance σ^2	proportion p
	one	chapter 10	chapter 10	chapter 10
sample	two	chapter 11	chapter 11	chapter 11
	multiple	chapter 13	not covered	$\chi_0^2 = \sum \frac{(O_i - E_i)^2}{E_i}$