Chapter 7

The Normal Probability Distribution

We look in detail at an important *continuous* probability distribution, the normal, when we can use it, and use it to approximate the binomial distribution. The uniform distribution is also discussed.

7.1 Properties of the Normal Distribution

Continuous *normal* distribution of random variable X, defined on interval $(-\infty, \infty)$, has density[1] with parameters μ and σ,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(x-\mu)/\sigma]^2}$$

and expected value (mean), variance and standard deviation,

$$E(X) = \mu, \quad V(X) = \sigma^2, \quad \sigma = \sqrt{V(X)}.$$

A normal random variable, X, may be transformed to a *standard* normal, Z,

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2},$$

where $\mu = 0$ and $\sigma = 1$ using following equation,

$$Z = \frac{X - \mu}{\sigma}.$$

We introduce normal distributions by first looking at another continuous probability distribution, the *uniform* distribution.

Exercise 7.1 (Properties of the Normal Distribution)

[1]Normal probability density function (pdf) is *not* directly used; StatCrunch is used instead.

1. *Uniform: temperatures.* Assume temperatures vary anywhere from 0^o to 1^o.

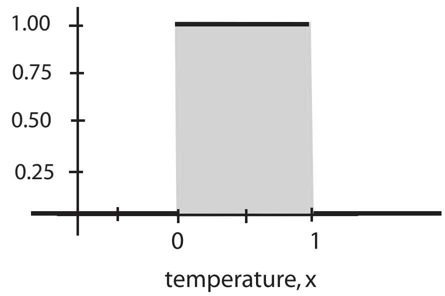


Figure 7.1 (Uniform: temperatures)

   (a) Chance temperature less than 0.7^o is **0.3** / **0.5** / **0.7** / **1**.
   (b) Chance temperature between 0.3^o and 0.7^o is **0.3** / **0.4** / **0.7** / **1**.
   (c) Chance temperature more than 0.4^o is **0.3** / **0.4** / **0.6** / **1**.

2. *Uniform: weight of potatoes.* An automated process fills one bag after another with Idaho potatoes. Although each filled bag should weigh 50 pounds, in fact, because of differing shapes and weights of each potato, each bag weighs anywhere from 50 pounds to 51 pounds.

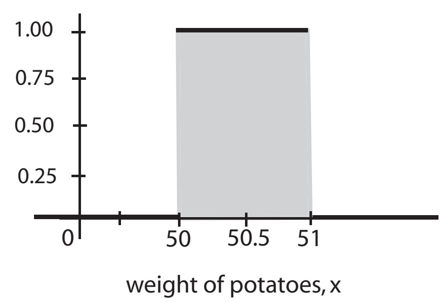


Figure 7.2 (Uniform: weights of potatoes)

   (a) Chance a bag chosen at random weighs between 50.5 and 51 pounds (choose one) **0.25** / **0.50** / **0.75** / **1**.
   Notice, *probability* of 0.50 equals *area* of 0.50.
   (b) Chance a bag chosen at random weighs between 49.5 and 50.5 pounds (choose one) **0.25** / **0.50** / **0.75** / **1**.
   (c) Chance a bag chosen at random weighs more than 49.5 pounds (choose one) **0.25** / **0.50** / **0.75** / **1**.
   (d) Chance a bag chosen at random weighs less than 49.5 pounds (choose one) **0** / **0.50** / **0.75** / **1**.

3. *Uniform: weight of potatoes again.*
An automated process fills one bag after another with Idaho potatoes. Although each filled bag should weigh 50 pounds, in fact, because of differing shapes and weights of each potato, each bag weighs anywhere from 49 pounds to 51 pounds, with following uniform density:

$$f(x) = \begin{cases} 0.5, & 49 < x \leq 51, \\ 0, & \text{elsewhere.} \end{cases}$$

Figure 7.3 (Continuous uniform: weights of potatoes)

(a) Chance a bag chosen at random weighs between 49.5 and 51 pounds

$$P(49.5 < X < 51) = \frac{51 - 49.5}{51 - 49} = \frac{1.5}{2} =$$

(choose one) **0.25** / **0.50** / **0.75** / **1**.

Notice, *probability* of 0.75 equals *area* of 0.75.

(b) Chance a bag chosen at random weighs between 49.5 and 50.5 pounds

$$P(49.5 < X < 50.5) = \frac{50.5 - 49.5}{51 - 49} = \frac{1}{2} =$$

(choose one) **0.25** / **0.50** / **0.75** / **1**.

(c) Chance a bag chosen at random weighs more than 49.5 pounds

$$P(X > 49.5) = \frac{51 - 49.5}{51 - 49} = \frac{1.5}{2} =$$

(choose one) **0.25** / **0.50** / **0.75** / **1**.

Notice, $P(X > 49.5) = P(49.5 < X < 51)$ because $P(X > 51) = 0$.

(d) Chance bag chosen has weight equal to or more than 49.5 pounds

$$P(X \geq 49.5) = P(X > 49.5) = \frac{51 - 49.5}{51 - 49} = \frac{1.5}{2} =$$

(choose one) **0.25** / **0.50** / **0.75** / **1**.

Notice, $P(X \geq 49.5) = P(X > 49.5)$ because[2] $P(X = 49.5) = 0$.

[2] $P(X = 49.5) = 0$ because this *continuous* distribution has an *infinity* of x values.

4. *Uniform: temperature again.*
   Temperature in La Porte has following uniform density:

$$f(x) = \begin{cases} \frac{1}{12}, & -4 < x \leq 8, \\ 0, & \text{elsewhere.} \end{cases}$$

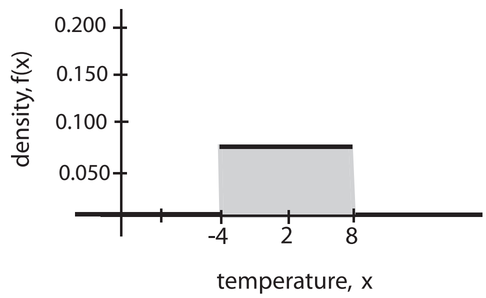


Figure 7.4 (Continuous uniform: temperature in La Porte)

   (a) Chance temperature is at most 0^o is
   $P(X \leq 0) =$ (choose one) $\mathbf{\frac{1}{12}}$ / $\mathbf{\frac{2}{12}}$ / $\mathbf{\frac{3}{12}}$ / $\mathbf{\frac{4}{12}}$.

   (b) Chance temperature is between -7^o and 0^o, inclusive, is
   $P(-7 \leq X \leq 0) =$ (choose one) $\mathbf{\frac{1}{12}}$ / $\mathbf{\frac{2}{12}}$ / $\mathbf{\frac{3}{12}}$ / $\mathbf{\frac{4}{12}}$.
   Notice $P(-7 \leq X \leq 0) = P(-4 \leq X \leq 0)$ because $P(X \leq -4) = 0$.

   (c) Chance temperature is between 1^o and 10^o is
   $P(1 < X < 10) =$ (choose one) $\mathbf{\frac{7}{12}}$ / $\mathbf{\frac{8}{12}}$ / $\mathbf{\frac{9}{12}}$ / $\mathbf{\frac{10}{12}}$.
   Notice $P(1 \leq X \leq 10) = P(1 \leq X \leq 8)$ because $P(X \geq 8) = 0$.

   (d) *Mean (average).* What is mean (average) temperature?
   $\mu_X =$ (choose one) **1** / **2** / **3** / **4**.
   Hint: Average temperature is $\frac{8+(-4)}{2} =$?

   (e) *Median.* What is *median* (50th percentile) temperature?
   (choose one) **1** / **2** / **3** / **4**.
   Hint: Median is temperature with 50% above and below this temperature.

   (f) *Mode.* What is *mode* (most probable) temperature?
   (choose one) **1** / **2** / **3** / **none**.
   Hint: If all temperatures equally probable, there is no mode.

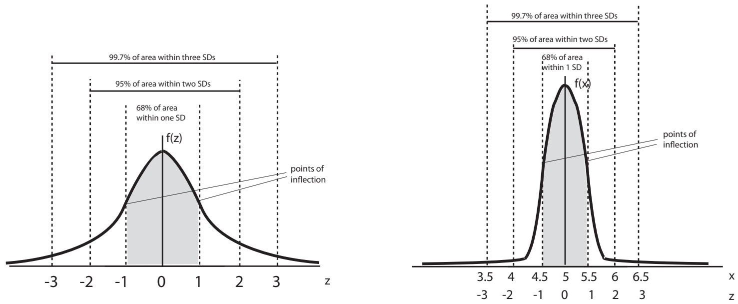
   (g) *Standard deviation.*

$$\sigma_X = \sqrt{\frac{(8-(-4))^2}{12}} \approx$$

   (choose one) **1.93** / **2.69** / **3.46** / **4.33**.

5. *Normal: temperature.*
   In Westville, in January, temperature, Z, is assumed *standard* normally distributed with mean $\mu = 0^o$ and standard deviation (SD) $\sigma = 1^o$, whereas in

February, temperature, X, is assumed normally distributed with mean $\mu = 5^o$ and standard deviation $\sigma = 0.5^o$



(a) January temperatures: standard normal, mean = 0, SD = 1

(b) February temperatures: nonstandard normal, mean = 5, SD = 0.5

Figure 7.5 (Graphs of normal probability distributions.)

(a) Both (any) normal distributions above are
(circle one) **skewed right** / **symmetric** / **skewed left**.

(b) Shape of both (any) normal are
(circle one) **triangular** / **bell–shaped** / **rectangular**.

(c) Total area (probability) under both (any) normal is
(circle one) **50%** / **75%** / **100%** / **150%**.

(d) *Standard* normal (January) centered at mean (average) temperature
(circle one) $\boldsymbol{\mu = 0^o}$ / $\boldsymbol{\mu = 5^o}$.

(e) Normal (February) centered at mean (average) temperature
(circle one) $\boldsymbol{\mu = 0^o}$ / $\boldsymbol{\mu = 5^o}$.

(f) **True** / **False** Mean = median = mode for both (any) normals.

(g) *Standard* normal (January) has SD in temperature of
(circle one) $\boldsymbol{\sigma = 0.5^o}$ / $\boldsymbol{\sigma = 1^o}$.

(h) Normal (February) has SD in temperature of
(circle one) $\boldsymbol{\sigma = 0.5^o}$ / $\boldsymbol{\sigma = 1^o}$.

(i) *Z-score* converts temperatures to standard ones; for example, February (normal) $x = 6^o$ is "equivalent" to January (standard normal)

$$z = \frac{x - \mu_X}{\sigma_X} = \frac{6 - 5}{0.5} =$$

(choose one) $\mathbf{1^o}$ / $\mathbf{2^o}$ / $\mathbf{3^o}$ in the sense both $x = 6^o$ and $z = 2^0$ are two SDs above average temperature. The z is a z-score.

(j) *Points of inflection* for both (any) normal occur at
(choose *two*!) $\boldsymbol{\mu - \sigma}$ / $\boldsymbol{\mu + \sigma}$ / $\boldsymbol{\mu + 2\sigma}$.

(k) **True** / **False** According to empirical rule, 68%, 95% and 99.7% of probability in both (all) normals is within 1, 2 and 3 SDs, respectively, of mean.

(l) **True** / **False** Since normal has *infinite* bounds, normal function approaches but never equals zero.

6. *Normal again: IQs.*
IQ scores for 16 year olds and 20 year olds described by normal distributions.

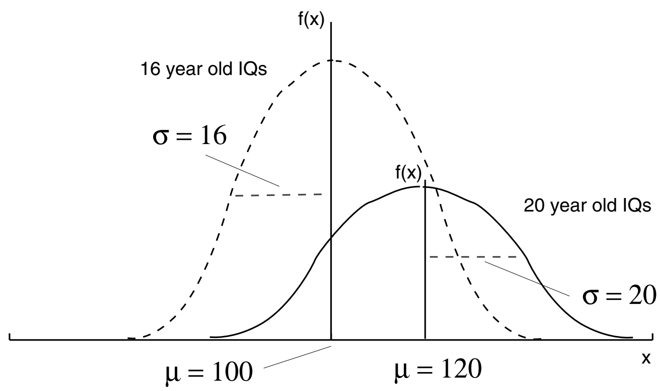


Figure 7.6 (Normal distributions of IQ scores)

(a) Mean IQ score for 20 year olds
$\mu =$ (circle one) **100** / **120** / **124** / **136**.

(b) Average (or mean) IQ score for 16 year olds
$\mu =$ (circle one) **100** / **120** / **124** / **136**.

(c) Standard deviation in IQ score for 20 year olds
$\sigma =$ (circle one) **16** / **20** / **24** / **36**.

(d) Standard deviation in IQ score for 16 year olds
$\sigma =$ (circle one) **16** / **20** / **24** / **36**.

(e) Normal for 20 year old IQ scores is
(circle one) **broader than** / **as wide as** / **narrower than**
normal for 16 year old IQ scores.

(f) Normal for 20 year old IQ scores is
(circle one) **shorter than** / **as tall as** / **taller than**
normal for 16 year old IQ scores.

(g) Total area (probability) under normal for 20 year old IQ scores is
(circle one) **smaller than** / **same as** / **larger than**
area (probability) under normal for 16 year old IQ scores.

(h) Normal distributions for IQ scores for 20 year old IQ scores and for 16 year old IQ scores are (choose one) **both** / **neither** standard normals.

(i) A 20 year old IQ score of $x = 140$ is

$$z = \frac{x - \mu_{20}}{\sigma_{20}} = \frac{140 - 120}{20} =$$

(choose one) **1** / **2** / **3** SDs above average 20 year old IQ score.

(j) A 16 year old IQ score of $x = 84$ is

$$z = \frac{x - \mu_{16}}{\sigma_{16}} = \frac{84 - 100}{16} =$$

(choose one) $\mathbf{-1}$ / $\mathbf{-2}$ / $\mathbf{-3}$ SDs *below* average 16 year old IQ score.

(k) Normal distributions are (choose one) **model** / **actual** IQ scores.

(l) Random variable with normal distribution is **continuous** / **discrete**.

7. *Percentage, standard normal: temperatures.*
Temperature, Z, modeled as a *standard* normal, mean $\mu = 0^o$ and SD $\sigma = 1^o$.

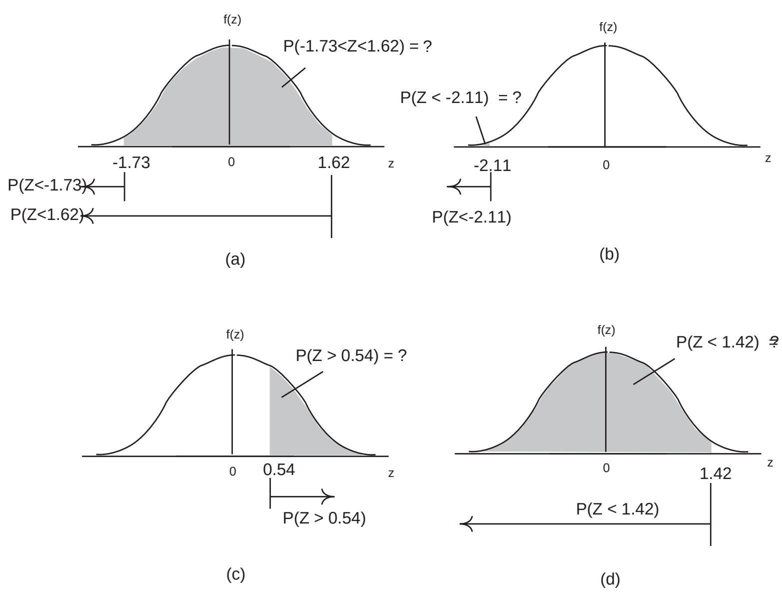


Figure 7.7 (Probabilities for standard normal)

(a) Probability temperature is between -1.73^o and 1.62^o ((a) above),
$P(-1.73 < Z < 1.62) =$ (circle one) **0.0174** / **0.2946** / **0.9222** / **0.9056**.
(Stat, Calculators, Normal, Between, Mean: 0, Std. Dev.: 1, Prob(-1.73 ≤ X ≤ 1.62) = ? Compute.)

(b) Probability temperature is less than -2.11^o ((b) above),
$P(Z < -2.11) =$ (circle one) **0.0174** / **0.2946** / **0.9222** / **0.9056**.
(Stat, Calculators, Normal, Standard, Mean: 0, Std. Dev.: 1, Prob(X ≤ -2.11) = ? Compute.)

(c) Probability temperature is greater than 0.54^o ((c) above),
$P(Z > 0.54) =$ (circle one) **0.0174** / **0.2946** / **0.9222** / **0.9056**.
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X $\geq$ 0.54) = $\boxed{?}$ Compute.)

(d) Probability temperature is less than 1.42^o ((d) above),
$P(Z < 1.42) =$ (circle one) **0.0174** / **0.2946** / **0.9222** / **0.9056**.
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X $\leq$ 1.42) = $\boxed{?}$ Compute.)

(e) **True** / **False** Probability temperature at *exactly* 1.42^o is *zero*.
(Stat, Calculators, Normal, Between, Mean: 0, Std. Dev.: 1, Prob(1.42 $\leq$ X $\leq$ 1.42) = $\boxed{?}$ Compute.)

(f) **True** / **False** $P(Z < 1.42^o) = P(Z \leq 1.42^o)$.
Hint: Think about previous question.

8. *Percentiles, standard normal: temperatures.*

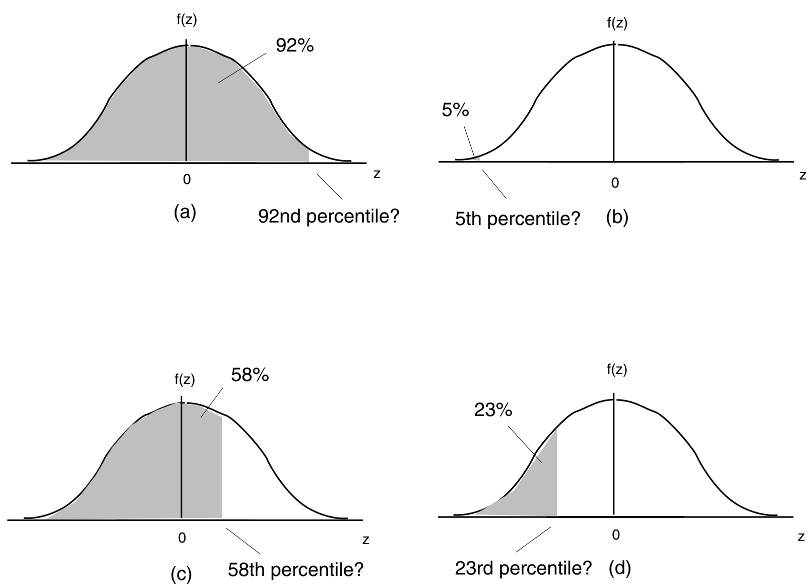


Figure 7.8 (Percentiles for standard normal)

(a) The 92nd percentile ((a) above) is
(circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$.
(Stat, Calculators, Normal, Standard, Mean: 0, Std. Dev.: 1, Prob(X $\leq$ $\boxed{?}$) = 0.92 Compute.
Gives $\boxed{?}$ = 1.41.)

(b) The z-score where area to left is 0.92 ((a) above) is
(circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$.
Hint: same as 92nd percentile.

(c) If $P(Z < a) = 0.92$, $a =$ (circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$.
Hint: same as 92nd percentile.

(d) The 5th percentile ((b) above) is
(circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$.
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≤ ?) = 0.05 Compute.)

(e) The z-score where area to left is 0.05 ((b) above) is
(circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$.
Hint: same as 5th percentile.

(f) The 58th percentile ((c) above) is
(circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$.
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≤ ?) = 0.58 Compute.)

(g) The 23rd percentile ((d) above) is
(circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$.
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≤ ?) = 0.23 Compute.)

(h) Temperature, where 77% of temperatures are *above* this temperature is $z_{0.77}$ = (circle one) $\mathbf{-1.65^o}$ / $\mathbf{-0.74^o}$ / $\mathbf{0.20^o}$ / $\mathbf{1.41^o}$ /
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ ?) = 0.77 Compute.
Remember 23rd percentile temperature has 23% of temperatures below it and 77% above it.

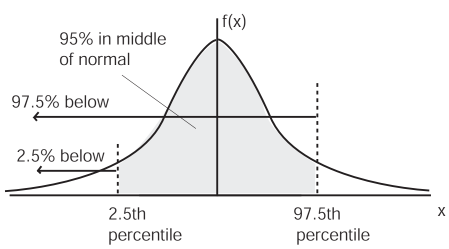
(i) Temperature, where 1% of temperatures are *above* this temperature is $z_{0.01}$ = (circle one) $\mathbf{0.15^o}$ / $\mathbf{0.74^o}$ / $\mathbf{1.20^o}$ / $\mathbf{2.33^o}$ /
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ ?) = 0.01 Compute.)

(j) Since standard normal distribution is symmetric, centered at 0^o, and contains "100%" of the probability, 50th percentile (median)
(circle one) **below** 0^o / **equal to** 0^o / **above** 0^o.

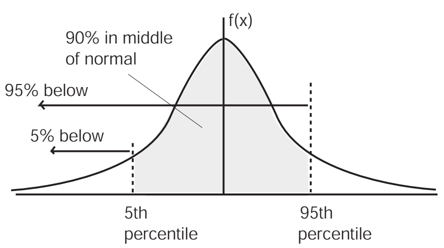
(k) **True** / **False** The 75th percentile is temperature with 75% of temperatures below it and 25% above it.

(l) Third quartile (75th percentile) is **below** 0^o / **equal to** 0^o / **above** 0^o.

9. *Middle percentages and percentiles.*



(a) 95% in middle, caught between 2.5th and 97.5th percentiles



(b) 90% in middle, caught between 5th and 95th percentiles

Figure 7.9 (Percentages and percentiles)

(a) If 95% of probability is located in *middle* of normal, then 2.5% is below lower bound, the (choose one) **2.5*th*** / **5*th*** / **10*th*** percentile.
Furthermore, 97.5% is below upper bound, the (choose one) **90*th*** / **95*th*** / **97.5*th*** percentile.
Look at figure (a) above.

(b) When 90% of probability is located in middle of normal, lower bound is (choose one) **2.5*th*** / **5*th*** / **10*th*** percentile and upper bound is (choose one) **92.5*th*** / **95*th*** / **97.5*th*** percentile.
Look at figure (b) above.

(c) When 80% of probability is located in middle of normal, lower bound is (choose one) **2.5*th*** / **5*th*** / **10*th*** percentile. and upper bound is (choose one) **90*th*** / **95*th*** / **97.5*th*** percentile.

7.2 Applications of the Normal Distribution

Exercise 7.2 (Applications of the Normal Distribution)

1. *Percentages, normal: IQ scores.*

(a) For 16 year old IQ scores, where $\mu_{16} = 100$ and $\sigma_{16} = 16$,
$P(X < 84) =$ (circle one) **0.8413** / **0.1587** / **−0.1587**
(Stat, Calculators, Normal, Mean: 100, Std. Dev.: 16, Prob(X ≤ 84) = [?], Compute.)

(b) For 16 year old IQ scores, where $\mu_{16} = 100$ and $\sigma_{16} = 16$,
$P(X < 100) =$ (circle one) **0.4413** / **0.5000** / **0.6587**
(Stat, Calculators, Normal, Mean: 100, Std. Dev.: 16, Prob(X ≤ 100) = [?], Compute.)

(c) For 16 year old IQ scores, where $\mu_{16} = 100$ and $\sigma_{16} = 16$,
$P(84 < X < 100) =$ (circle one) **0.3413** / **0.4901** / **0.5587**
(Stat, Calculators, Normal, Between, Mean: 100, Std. Dev.: 16, Prob(84 ≤ X ≤ 100) = [?] Compute.)

(d) For 20 year old IQ scores, where $\mu_{20} = 120$ and $\sigma_{20} = 20$,
$P(84 < X < 100) =$ (circle one) **0.0413** / **0.1227** / **0.3597**
(Stat, Calculators, Normal, Between, Mean: 120, Std. Dev.: 20, Prob(84 ≤ X ≤ 100) = [?] Compute.)

(e) *More probability questions.*

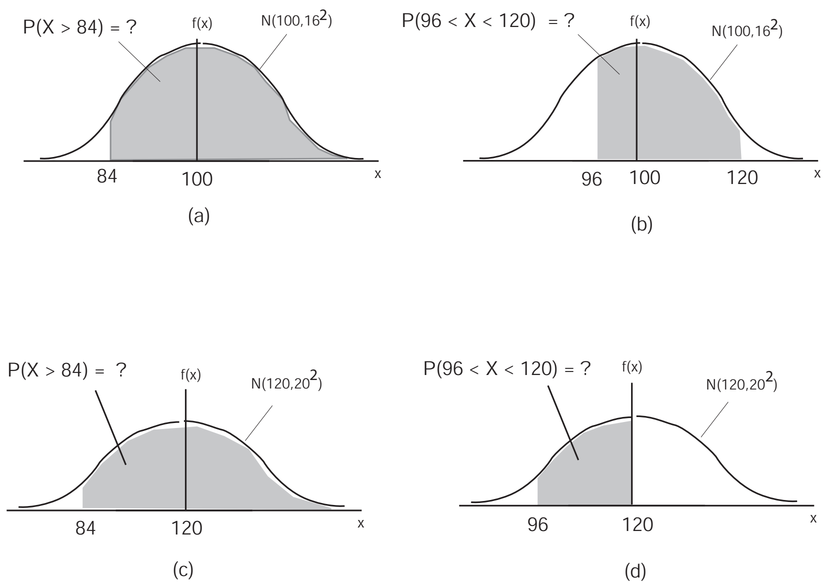


Figure 7.10 (Probabilities for normal)

Use calculator and figure above to match columns in following table.

Column I	Column II
(a) $P(X > 84)$, 16 year olds (Mean: 100, Std. Dev.: 16 Prob(X $\geq$ 84) = ?)	**(a)** 0.4931
(b) $P(96 < X < 120)$, 16 year olds (Mean: 100, Std. Dev.: 16, Prob(96 $\leq$ X $\leq$ 120) = ?)	**(b)** 0.9641
(c) $P(X > 84)$, 20 year olds (Mean: 120, Std. Dev.: 20 Prob(X $\geq$ 84) = ?)	**(c)** 0.8413
(d) $P(96 < X < 120)$, 20 year olds (Mean: 120, Std. Dev.: 20, Prob(96 $\leq$ X $\leq$ 120) = ?)	**(d)** 0.3849

Column I	(a)	(b)	(c)	(d)
Column II				

(f) Empirical rule says 68% of 16 year old IQ scores are inside interval $(\mu_{16} - \sigma_{16},\ \mu_{16} + \sigma_{16}) = (100 - 16, 100 + 16) =$ (circle one) **(84, 116)** / **(68, 132)** / **(52, 148)**.

(g) Empirical rule says 95% of 16 year old IQ scores are inside interval $(\mu_{16} - 2\sigma_{16},\ \mu_{16} + 2\sigma_{16}) =$ (circle one) **(84, 116)** / **(68, 132)** / **(52, 148)**.

(h) A 16 year old with IQ score 138 is (choose one) **typical** / **unusual** because this score is greater than two SDs above average IQ score.

(i) Empirical rule says 95% of 20 year old IQ scores are inside interval $(\mu_{20} - 2\sigma_{20},\ \mu_{20} + 2\sigma_{20}) =$ (circle one) **(100, 140)** / **(80, 160)** / **(60, 160)**, so IQ score 138 is (choose one) **typical** / **unusual** in this case.

2. *Percentiles, normal: IQ scores again.*

   (a) 75th percentile for 16 year olds, where $\mu_{16} = 100$ and $\sigma_{20} = 16$, is:
   (circle one) **103.5** / **106.7** / **110.8** / **112.3**.
   (Stat, Calculators, Normal, Mean: 100, Std. Dev.: 16, Prob(X $\leq \boxed{?}$) = 0.75 Compute.)

   (b) 32th percentile for 16 year olds, where $\mu_{16} = 100$ and $\sigma_{20} = 16$, is:
   (circle one) **83.5** / **92.5** / **98.8** / **100.3**.
   (Stat, Calculators, Normal, Mean: 100, Std. Dev.: 16, Prob(X $\leq \boxed{?}$) = 0.32 Compute.)

   (c) 75th percentile for 20 year olds, where $\mu_{20} = 120$ and $\sigma_{20} = 20$, is:
   (circle one) **133.5** / **106.5** / **125.4** / **142.3**.
   (Stat, Calculators, Normal, Mean: 120, Std. Dev.: 20, Prob(X $\leq \boxed{?}$) = 0.75 Compute.)

   (d) *More percentile questions.*

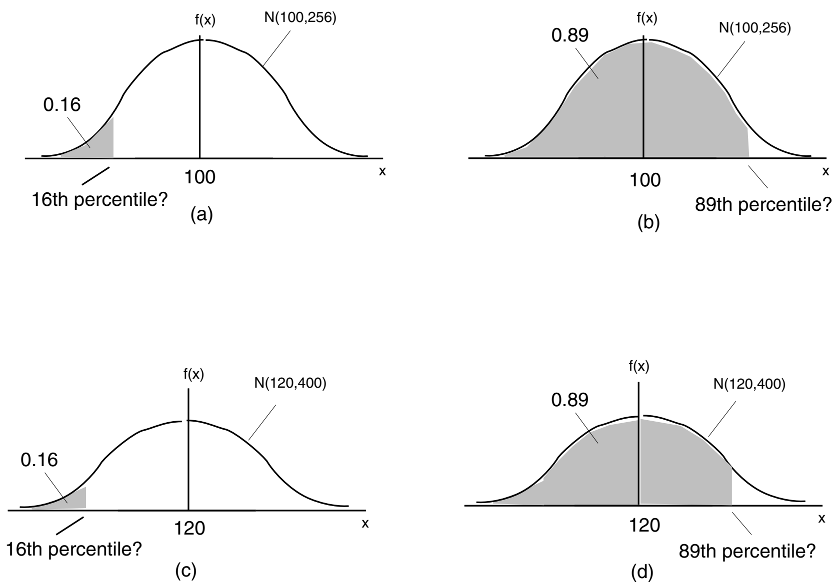


Figure 7.11 (Percentiles for normal.)

Use calculator and figure above to match columns in following table.

Column I	Column II
(a) 16th percentile, 16 year olds (Mean: 100, Std. Dev.: 16, Prob(X $\leq \boxed{?}$) = 0.16)	**(a)** 119.6
(b) 89th percentile, 16 year olds (Mean: 100, Std. Dev.: 16, Prob(X $\leq \boxed{?}$) = 0.89)	**(b)** 84.1
(c) 16th percentile, 20 year olds (Mean: 120, Std. Dev.: 20, Prob(X $\leq \boxed{?}$) = 0.16)	**(c)** 144.5
(d) 89th percentile, 20 year olds (Mean: 120, Std. Dev.: 20, Prob(X $\leq \boxed{?}$) = 0.89)	**(d)** 100.1

Column I	(a)	(b)	(c)	(d)
Column II				

(e) Since normal distribution of 16 year old scores symmetric, centered at 100, 50th percentile is (choose one) **below** 100 / **equal to** 100 / **above** 100.

(f) 90th percentile, 16 year old scores:
(choose one) **below** 100 / **equal to** 100 / **above** 100.

(g) 90th percentile, 20 year old scores:
(circle one) **below** 120 / **equal to** 120 / **above** 120.

(h) **True** / **False**
90th percentile is IQ score with 90% of IQ scores below and so 10% above.

3. *More percentages and percentiles: chimp brain weights*
It was found in 1979 the brain weights of a certain population of adult chimps follow approximately a normal, mean 270 gm and standard deviation 40 gm.

(a) Percentage of adult chimps brains weighing between 250 gm and 300 gm:
(choose closest one) **0.383** / **0.465** / **0.633** / **0.547** / **0.318**.
(Stat, Calculators, Normal, Between, Mean: 270, Std. Dev.: 40, Prob($250 \leq X \leq 300$) = $\boxed{?}$ Compute.)

(b) *Number* of 750 adult chimps brains weighing between 250 gm and 300 gm:
(choose closest one) **249** / **301** / **349** / **397** / **403**.
Hint: $0.465 \times 750 = ?$

(c) The 80th percentile brain weight is:
(choose closest one) **304** / **308** / **310** / **312** / **314**.
(Stat, Calculators, Normal, Mean: 270, Std. Dev.: 40, Prob($X \leq \boxed{?}$) = 0.80 Compute.)

(d) A brain weight of 240 gm, expressed as a percentile, is
(choose closest one) ***12th*** / ***23rd*** / ***32nd*** / ***45th*** / ***55th***.
(Stat, Calculators, Normal, Mean: 270, Std. Dev.: 40, Prob($X \leq 240$) = $\boxed{?}$, Compute.)

7.3 Assessing Normality

If a *normal probability plot* of ordered data versus normal quantiles is *linear*, this indicates sampled data belongs to a normal probability distribution. For example, data in plot (a) is normal because points are "close enough" to linear since they are within dotted bounds. Data in plot (b), though, is not normal because points fall outside bounds.

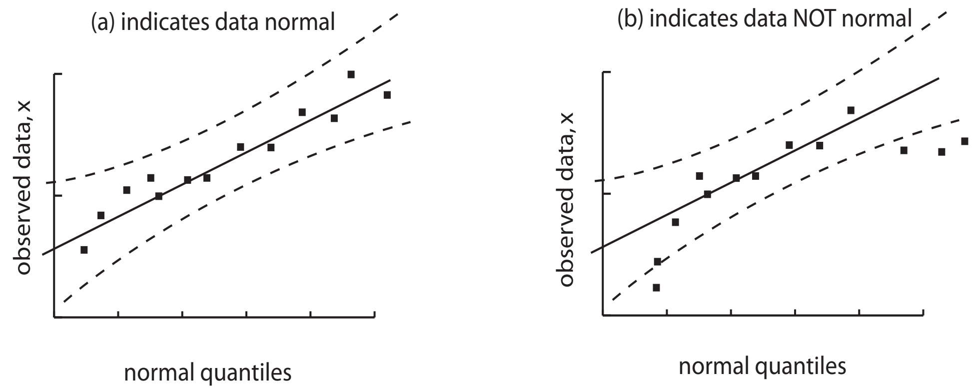


Figure 7.12 (Normal probability plots.)

Exercise 7.3 (Assessing Normality)

1. *Normal probability plot*[3]*: pH levels of soil samples.*

4.3	5	5.9	6.5	7.6	7.7	7.7	8.2	8.3	9.5
10.4	10.4	10.5	10.8	11.5	12	12	12.3	12.6	12.6
13	13.1	13.2	13.5	13.6	14.1	14.1	15.1		

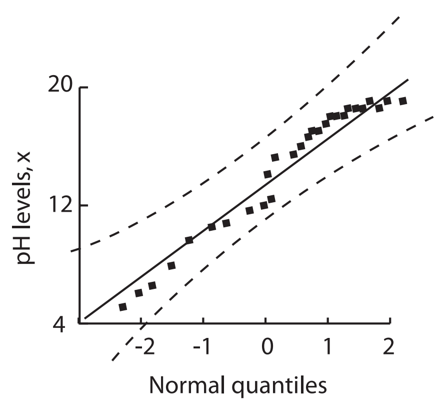


Figure 7.13 (Normal probability plot for pH levels.)

(a) *Data normal?*
Normal probability plot[4] indicates pH data **normal** / **not normal** because data within dotted bounds.
(Blank data table. StatCrunch, My Data, choose 3.2 pH Chebyshev. Graphics, QQ Plot, Select Columns: pH, Create Graph!)

[3]Although plots not exactly same as StatCrunch plots, treated in same way.
[4]StatCrunch does NOT give bounds, so hard to tell if plot linear enough to indicate normality.

(b) *Mean and SD.*
sample average pH level $\bar{x} \approx$ (choose one) **10.55 / 11.55 / 12.55**,
sample SD in pH levels $\sigma \approx s_X \approx$ (choose one) **2.01 / 3.01 / 4.01**.
(Stat, Summary Stats, Columns, Select Columns: pH, Calculate)

(c) *Probability question.*
Assume data normal, $\mu \approx \bar{x} \approx 10.55$ and $\sigma \approx s_X \approx 3.01$,
so, chance pH level is between 9 and 12, inclusive:
$P(9 \leq X \leq 12) =$ (choose one) **0.28 / 0.32 / 0.38**.
(Stat, Calculators, Normal, Between, Mean: 10.55, Std. Dev.: 3.01, Prob($9 \leq X \leq 12$) = $\boxed{?}$ Compute.)

2. *Normal probability plot: patient ages.*

32, 37, 39, 40, 41, 41, 41, 42, 42, 43,
44, 45, 45, 45, 46, 47, 47, 49, 50, 51

(a) *Data normal?*
Normal probability plot indicates ages **normal / not normal**.
(StatCrunch, My Data, 2.2 Age Histogram. Graphics, QQ Plot, Select Columns: age, Create Graph!)

(b) *Mean and SD.*
sample average age $\bar{x} \approx$ (choose one) **42.35 / 43.35 / 44.35**,
sample SD in ages $\sigma \approx s_X \approx$ (choose one) **4.57 / 4.78 / 5.01**.
(Stat, Summary Stats, Columns, Select Columns: age, Calculate)

(c) *Probability question.* (Even though *not* normal, continue anyway.)
Assume data normal, $\mu \approx \bar{x} \approx 43.35$ and $\sigma \approx s_X \approx 4.57$,
so, chance patient ages less than 40:
$P(X < 40) =$ (choose one) **0.19 / 0.23 / 0.28**.
(Stat, Calculators, Normal, Mean: 43.35, Std. Dev.: 4.57, Prob($X \leq 40$) = $\boxed{?}$, Compute.)

7.4 The Normal Approximation to Binomial Probability Distribution

Normal approximation, with continuity correction, to binomial, where $\mu = np$ and $\sigma = \sqrt{np(1-p)}$ is fairly good if $np(1-p) \geq 10$. This is an example of the Central Limit Theorem, which will be discussed more in the next chapter.

Exercise 7.4 (The Normal Approximation to Binomial Probability Distribution)

1. *Normal approximation to binomial: defective widgets.*
Each of fourteen ($n = 14$) widgets are defective 55% ($p = 0.55$) of the time. Assume this problem obeys conditions of a binomial experiment.

(a) Since $\mu = np = 14(0.55) =$ (choose one) **5.4** / **7.7** / **8.3**
(Data, Compute expression, Expression: 14*0.55, Compute.)
and $\sigma = \sqrt{np(1-p)} = \sqrt{14(0.55)(1-0.55)} = \sqrt{14(0.55)(0.45)} \approx$ (choose one) **1.861451** / **2.172334** / **2.3333333**
(Data, Compute expression, Expression: sqrt(14*0.55*0.45), Compute.)
chance at most 9 defective widgets is
(approximate) normal:
$P(X \leq 9) \approx P(X \leq 9.5) \approx$ (circle one) **0.75** / **0.82** / **0.83**.
(Stat, Calculators, Normal, Mean: 7.7, Std. Dev.: 1.861451, Prob(X ≤ 9.5) = [?], Compute.)
(exact) binomial:
$P(X \leq 9) =$ (circle one) **0.75** / **0.82** / **0.83**.
(Stat, Calculators, Binomial, n: 14, p: 0.55, Prob(X ≤ 9) = [?], Compute.)

(b) Chance between 3 and 7 defective widgets, inclusive, is
(approximate) normal:
$P(3 \leq X \leq 7) \approx P(2.5 \leq X \leq 7.5) \approx$ (circle one) **0.45** / **0.52** / **0.67**.
(Stat, Calculators, Normal, Between, Mean: 7.7, Std. Dev.: 1.86, Prob(2.5 ≤ X ≤ 7.5) = [?] Compute.)
(exact) binomial:
$P(3 \leq X \leq 7) =$ (circle one) **0.45** / **0.52** / **0.67**.
(Stat, Calculators, Binomial, Between, n: 14, p: 0.55, Prob(3 ≤ X ≤ 7) = [?] Compute.)

2. *Normal approximation to binomial: lawyer and ten trials.*
Lawyer estimates she has a 40% ($p = 0.4$) of winning each of her next 10 ($n = 10$) cases. Assume this problem obeys conditions of a binomial experiment.

(a) Chance she has at most 7 wins is,
since $\mu = np = 10(0.4) =$ (choose one) **4** / **5** / **6**
and $\sigma = \sqrt{np(1-p)} = \sqrt{10(0.4)(1-0.4)} = \sqrt{10(0.4)(0.6)} \approx$ (choose one) **1.2322345** / **1.5491934** / **1.9943345**,
(Data, Compute expression, Expression: sqrt(10*0.4*0.6), Compute.)
then
(approximate) normal:
$P(X \leq 7) \approx P(X \leq 7.5) =$ (circle one) **0.85** / **0.92** / **0.99**.
(Stat, Calculators, Normal, Mean: 4, Std. Dev.: 1.5491934, Prob(X ≤ 7.5) = [?], Compute.)
(exact) binomial:
$P(X \leq 7) =$ (circle one) **0.85** / **0.92** / **0.99**.
(Stat, Calculators, Binomial, n: 10, p: 0.4, Prob(X ≤ 7) = [?], Compute.)

(b) Chance she has at least 5 wins is
(approximate) normal:
$P(X \geq 5) \approx P(X \geq 4.5) =$ (circle one) **0.37** / **0.42** / **0.57**.
(Stat, Calculators, Normal, Mean: 4, Std. Dev.: 1.5491934, Prob(X ≥ 4.5) = [?], Compute.)
(exact) binomial:

$P(X \geq 5) =$ (circle one) **0.37** / **0.42** / **0.57**.
(Stat, Calculators, Binomial, n: 10, p: 0.4, Prob(X $\geq$ 5) = $\boxed{?}$, Compute.)

(c) Chance she has *more than* 5 wins is,
(approximate) normal:
$P(X > 5) = P(X \geq 6) \approx P(X \geq 5.5) =$ (circle one) **0.17** / **0.21** / **0.24**.
(Stat, Calculators, Normal, Mean: 4, Std. Dev.: 1.5491934, Prob(X $\geq$ 5.5) = $\boxed{?}$, Compute.)
(exact) binomial:
$P(X > 5) = P(X \geq 6) =$ (circle one) **0.17** / **0.21** / **0.24**.
(Stat, Calculators, Binomial, n: 10, p: 0.4, Prob(X $\geq$ 6) = $\boxed{?}$, Compute.)

(d) Chance she has *exactly* 5 wins is,
(approximate) normal:
$P(X = 5) \approx P(4.5 \leq X \leq 5.5) =$ (circle one) **0.17** / **0.21** / **0.24**.
(Calculators, Between, Normal, Mean: 4, Std. Dev.: 1.5491934, Prob(4.5 $\leq$ X $\leq$ 5.5) = $\boxed{?}$, Compute.)
(exact) binomial:
$P(X = 5) =$ (circle one) **0.17** / **0.20** / **0.24**.
(Stat, Calculators, Binomial, n: 10, p: 0.4, Prob(X = 5) = $\boxed{?}$, Compute.)

3. *Understanding normal approximation to binomial: lawyer revisited.*
Compare binomial and normal approximation chance lawyer has at least 5 wins.

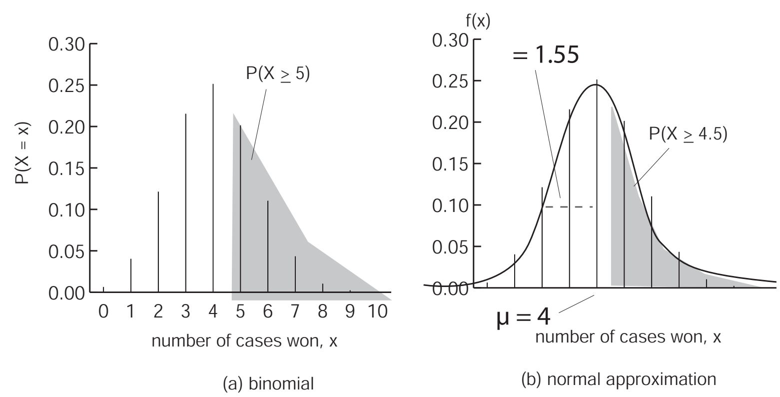


Figure 7.14 (Binomial and normal approximation)

(a) *Exact binomial.*
Recall, (exact) chance lawyer has at least 5 wins is
$P(X \geq 5) =$ (circle one) **0.367** / **0.422** / **0.572**.
(Stat, Calculators, Binomial, n: 10, p: 0.4, Prob(X $\geq$ 5) = $\boxed{?}$, Compute.)

(b) *Approximate normal approximation to binomial.*

i. *Symmetric?* Since

$$np(1-p) = 10(0.4)(0.6) = 2.4 < 10,$$

binomial (choose one) **is** / **is not** symmetric enough to be approximated by normal in this case.

ii. *Replace normal mean, SD with binomial mean, SD.*
Since binomial mean and SD are

$$\mu = np = 10(0.4) = 4, \quad \sigma = \sqrt{np(1-p)} = \sqrt{10(0.4)(0.6)} \approx 1.55$$

approximate with normal X where $\mu = 4$ and $\sigma = 1.55$ for
$P(X \geq 5) =$ (circle one) **0.374** / **0.259**.
(Stat, Calculators, Normal, Mean: 4, Std. Dev.: 1.5491934, Prob(X $\geq$ 5) = $\boxed{?}$, Compute.)
Notice $P(X \geq 5) = 0.259$ is not close to exact binomial value, 0.367.

iii. *Continuity correction in normal approximation.*
To improve *continuous* normal approximation to *discrete* binomial, a *continuity correction* factor is introduced. In this case, 0.5 is subtracted from 5 and revised normal approximation becomes
$P(X \geq 5 - 0.5) = P(X \geq 4.5) =$ (circle one) **0.374** / **0.259**.
(Stat, Calculators, Normal, Mean: 4, Std. Dev.: 1.5491934, Prob(X $\geq$ 4.5) = $\boxed{?}$, Compute.)
Notice $P(X \geq 4.5) = 0.374$ close to exact binomial value, 0.367, even though symmetry condition violated.