Chapter 9

Estimating the Value of a Parameter

We look at both point and *confidence interval (CI)* estimates of three parameters: proportion, p, mean, μ, and standard deviation, σ. A CI estimate provides a range of values, or an interval of values, that, together, act as an estimate for a parameter.

9.1 Estimating a Population Proportion

The confidence interval for proportion p from a binomial distribution is

$$\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where we assume a large simple random sample has been chosen and $n\hat{p}(1-\hat{p}) > 10$. Sometimes, notation $\hat{q} = 1-\hat{p}$ is used. If sampled from finite population, $n \leq 0.05N$.

Exercise 9.1 (Estimating a Population Proportion)

1. *Confidence interval (CI) for proportion, p, of purchase slips made with Visa.* It is found 54 of 180 (or $\hat{p} = \frac{54}{180} = 0.3$) randomly selected from 100,000 credit card purchase slips are made with Visa. Calculate a 95% CI of proportion p of purchase slips made with Visa.

   (a) *Point estimate.*
   Point estimate of *population* (actual, true) proportion of *all* credit card purchase slips made with Visa, p, is
   $\hat{p} =$ (choose one) **0.3 / 54 / 180**.
   Statistic $\hat{p} = 0.3$ probably does not exactly equal unknown parameter p.

   (b) *Check assumptions.*
   Since $n\hat{p}(1-\hat{p}) = 180(0.3)(0.7) = 37.8 > 10$,

and $n = 180 < 0.05(100000) = 5000$,
assumptions (choose one) **have** / **have not** been satisfied
and so it is appropriate $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ estimate parameter p.

(c) *95% Confidence Interval (CI) using StatCrunch.*
The 95% CI for proportion of all credit cards made with Visa, p, is
(0.251, 0.349) / **(0.273, 0.367)** / **(0.233, 0.367)**.
where this *interval* includes not only smallest possible proportion of 0.233 and largest possible proportion of 0.367, but also other proportions in between these two extremes such as point estimate, $\hat{p} = 0.3$.
Length of this CI is $L \approx 0.367 - 0.233 = 0.134$.
(Stat, Proportions, one sample, with summary, Number of successes: 54, Number of observations: 180, choose Confidence Interval, Calculate.)
So, 95% confident population parameter p in (0.233, 0.367).

(d) *90% CI using StatCrunch.*
The 90% CI for proportion of all credit cards made with Visa, p, is
(0.251, 0.349) / **(0.244, 0.356)** / **(0.233, 0.367)**.
(Options, Edit, Confidence Interval level: 0.90, Calculate.)
Length of this CI is $L \approx 0.356 - 0.244 = 0.112$.

(e) *85% CI using StatCrunch.*
The 85% CI for proportion of all credit cards made with Visa, p, is
(0.251, 0.349) / **(0.273, 0.367)** / **(0.233, 0.367)**.
(Options, Edit, Confidence Interval level: 0.85, Calculate.)
Length of this CI is $L \approx 0.349 - 0.251 = 0.098$.

(f) *Comparing CI lengths.*
Length of 95% CI for p, $L = 0.134$, is (choose one)
longer than / **same length as** / **shorter than**
length of 90% CI for p, $L = 0.112$, which is (choose one)
longer than / **same length as** / **shorter than**
length of 85% CI for p, $L = 0.098$.
Increasing confidence increases CI length.

(g) *Margin of error.*
Half of length, L, is margin of error, $E = \frac{L}{2}$.
Consequently, for 95% CI for p,
$E = \frac{L}{2} = \frac{0.134}{2} =$ (circle one) **0.067** / **0.056** / **0.049**,
and for 90% CI for p,
$E = \frac{L}{2} = \frac{0.112}{2} =$ (circle one) **0.067** / **0.056** / **0.049**,
and for 85% CI for p,
$E = \frac{L}{2} = \frac{0.098}{2} =$ (circle one) **0.067** / **0.056** / **0.049**,

(h) *Other ways of writing confidence intervals.*
Different possible ways of writing 95% CI include (choose *one or more*!)

i. **(0.233, 0.367)**
ii. $\mathbf{(0.3 - 0.067, 0.3 + 0.067)}$
iii. $\mathbf{0.3 \pm 0.067}$

where $\hat{p} = 0.3$ is *point estimate* and $E = 0.067$ is margin of error.
In a similar way,
90% CI of parameter p is (choose one)
$\mathbf{0.3 \pm 0.067}$ / $\mathbf{0.3 \pm 0.056}$ / $\mathbf{0.3 \pm 0.049}$
and 85% CI of parameter p is (choose one)
$\mathbf{0.3 \pm 0.067}$ / $\mathbf{0.3 \pm 0.056}$ / $\mathbf{0.3 \pm 0.049}$

(i) *CI using formula, a first look.*
Since $\hat{p} = 0.3$ and $n = 180$, the 95% CI for p is

$$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$$

the *incomplete* answer (choose one)

i. $\mathbf{180 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$,
ii. $\mathbf{0.3 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$,
iii. $\mathbf{0.7 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.7(1-0.3)}{180}}}$.

In a similar way,
90% CI is (choose one)
$\mathbf{0.3 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$ / $\mathbf{0.7 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$ / $\mathbf{180 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$
and 85% CI is (choose one)
$\mathbf{0.3 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$ / $\mathbf{0.7 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$ / $\mathbf{180 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{0.3(1-0.3)}{180}}}$
All three CIs same except three critical values, $z_{\frac{\alpha}{2}}$, are different.

(j) *CI using formula: calculating critical value,* $z_{\frac{\alpha}{2}}$.

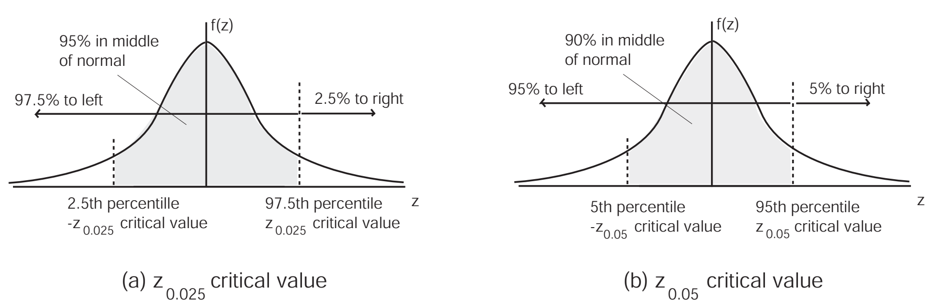

Figure 9.1 (Critical values.)

Critical value for 95% $= (1 - \alpha) \cdot 100\% = (1 - 0.05) \cdot 100\%$ CI is
$z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} =$ (circle one) **1.96** / **1.645** / **1.44**.

(Calculate critical value at 0.025[1]:
Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ ?) = 0.025 Compute.)
Critical value for 90% = $(1-\alpha)\cdot 100\% = (1-0.10)\cdot 100\%$ CI is
$z_{\frac{\alpha}{2}} = z_{\frac{0.10}{2}} = z_{0.05} =$ (circle one) **1.96 / 1.645 / 1.44**.
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ ?) = 0.05 Compute.)
Critical value for 85% = $(1-\alpha)\cdot 100\% = (1-0.15)\cdot 100\%$ CI is
$z_{\frac{\alpha}{2}} = z_{\frac{0.15}{2}} = z_{0.075} =$ (circle one) **1.96 / 1.645 / 1.44**.
(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ ?) = 0.075 Compute.)

(k) *CI using formula.*
A 95% CI for proportion of Visa credit card purchase slips, p, is $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$

i. $\mathbf{0.3 \pm 1.96 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$
ii. $\mathbf{0.3 \pm 1.645 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$
iii. $\mathbf{0.3 \pm 1.44 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$

and a 90% CI for proportion of Visa credit card purchase slips, p, is

i. $\mathbf{0.3 \pm 1.96 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$
ii. $\mathbf{0.3 \pm 1.645 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$
iii. $\mathbf{0.3 \pm 1.44 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$

and an 85% CI for proportion of Visa credit card purchase slips, p, is

i. $\mathbf{0.3 \pm 1.96 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$
ii. $\mathbf{0.3 \pm 1.645 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$
iii. $\mathbf{0.3 \pm 1.44 \times \sqrt{\frac{0.3(1-0.3)}{180}}}$

(l) *Population, Sample, Statistic*[2] *and Parameter.* Match columns.

terms	credit card example
(a) population	**(a)** Visa or not, all purchase slips
(b) sample	**(b)** proportion of all slips made with Visa, p
(c) statistic	**(c)** Visa or not, 180 purchase slips
(d) parameter	**(d)** proportion of 180 slips made with Visa, $\hat{p}$

terms	(a)	(b)	(c)	(d)
credit card example				

[1]There is 2.5% to *right* of $z_{0.025}$ critical value. This is because 95% CI occupies *middle* portion of normal curve and so 2.5% above and below this CI.
[2]Anything with a "hat" on it is a statistic; for example, $\hat{p}$.

2. *95% CI, proportion of student heights over 6 feet tall.*
   37 of 102 PNW students, chosen at random from 10,000, over 6 feet tall.

   (a) *Point estimate*
   Point estimate of proportion, p, of student heights over 6 feet tall is
   $\hat{p} = \frac{37}{102} \approx$ (choose one) **0.363** / **0.378** / **0.391**.

   (b) *Check assumptions.*
   Since $n\hat{p}(1-\hat{p}) = 102\left(\frac{37}{102}\right)\left(1-\frac{37}{102}\right) \approx 23.6 > 10$,
   and $n = 102 < 0.05(10000) = 500$,
   assumptions (choose one) **have** / **have not** been satisfied
   and so it is appropriate $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ estimate parameter p.

   (c) *95% CI for population parameter p of PNW students over 6 feet tall.*

   i. *Using StatCrunch.* The 95% CI for p is (choose one)
   $\mathbf{(0.269, 0.456)}$ / $\mathbf{(0.273, 0.367)}$ / $\mathbf{(0.233, 0.367)}$.
   (Stat, Proportions, one sample, with summary, Number of successes: 37, Number of observations: 102, choose Confidence Interval, Calculate.)

   ii. *Using formula: critical value.*
   Critical value for $95\% = (1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI for p is
   $z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} =$ (circle one) **1.28** / **1.96** / **2.58**.
   (Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X $\geq$ [?]) = 0.025 Compute.)

   iii. *Using formula.*
   Since $\hat{p} = \frac{37}{102}$ and $n = 102$, the 95% CI for p is
   $\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$ (circle one)
   $\mathbf{0.36 \pm 1.28 \times \sqrt{\frac{0.36(1-0.36)}{102}}}$
   $\mathbf{0.36 \pm 1.96 \times \sqrt{\frac{0.36(1-0.36)}{102}}}$
   $\mathbf{0.36 \pm 2.58 \times \sqrt{\frac{0.36(1-0.36)}{102}}}$
   $\approx (0.269, 0.456)$

   iv. *Length*, L, of 95% CI is
   $L = 0.456 - 0.269 =$ (circle one) **0.176** / **0.187** / **0.354**.
   Half of length, margin of error,
   $E = \frac{L}{2} =$ (circle one) **0.088** / **0.0935** / **0.177**.
   Notice, margin of error[3] also equals

$$E = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \times \sqrt{\frac{\frac{37}{102}(1-\frac{37}{102})}{102}} \approx 0.0935.$$

   (d) *92% CI for parameter p*

[3] Answer here might be a little bit different from previous one due to round off error.

i. *Using StatCrunch.* The 92% CI for p is (choose one) $\mathbf{(0.269, 0.456)}$ / $\mathbf{(0.279, 0.446)}$ / $\mathbf{(0.233, 0.367)}$.

(Stat, Proportions, one sample, with summary, Number of successes: 37, Number of observations: 102, choose Confidence Interval 0.92, Calculate.)

ii. *Using formula: critical value.*
Critical value for 92% $= (1-\alpha)\cdot 100\% = (1-0.08)\cdot 100\%$ CI for p is $z_{\frac{\alpha}{2}} = z_{\frac{0.08}{2}} = z_{0.04} =$ (circle one) **1.75** / **1.96** / **2.58**.

(Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X $\geq$?) = 0.04 Compute.)

iii. *Using formula.*
Since $\hat{p} = \frac{37}{102}$ and $n = 102$, the 95% CI for p is
$\hat{p} \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} =$ (circle one)
$\mathbf{0.36 \pm 1.75 \times \sqrt{\frac{0.36(1-0.36)}{102}}}$
$\mathbf{0.36 \pm 1.96 \times \sqrt{\frac{0.36(1-0.36)}{102}}}$
$\mathbf{0.36 \pm 2.58 \times \sqrt{\frac{0.36(1-0.36)}{102}}}$
$\approx (0.279, 0.446)$

iv. *Margin of error.*
$E = z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.75 \times \sqrt{\frac{\frac{37}{102}(1-\frac{37}{102})}{102}} \approx$ **0.074** / **0.084** / **0.098**

v. *Confidence Level and Sample Size.*
The larger the confidence level (critical value, $z_{\frac{\alpha}{2}}$) the **larger** / **smaller** the margin of error.
The larger the sample size, n, the **larger** / **smaller** the margin of error.

3. *Sample size given margin of error and level of confidence.*
Sample size necessary to achieve a required margin of error, E, with a given level of confidence in a confidence interval determined using formula, if *prior* $\hat{p}$ available,

$$n = \hat{p}(1-\hat{p})\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2,$$

and if prior $\hat{p}$ unavailable,

$$n = \frac{1}{4}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2.$$

(a) *Sample size for proportion p with prior $\hat{p}$: purchase slips.*
In an initial simple random sample, twenty–five (25) of 100 purchase slips chosen are Visa. What is sample size, n, required to estimate proportion Visa purchase slips, p, to within margin of error of $E = 0.01$ with 85% confidence? Here

$$n = \hat{p}(1-\hat{p})\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = \left(\frac{25}{100}\right)\left(\frac{75}{100}\right)\left(\frac{1.44}{0.01}\right)^2 \approx$$

(circle one) **3888** / **5184** / **5470**.

Prior $\hat{p} = \frac{25}{100}$. Since 85% = $(1-\alpha)\cdot 100\% = (1-0.15)\cdot 100\%$, then $z_{\frac{\alpha}{2}} = z_{\frac{0.15}{2}} = z_{0.075}$ and so Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ [?]) = 0.075 Compute gives $z_{\frac{\alpha}{2}} \approx 1.44$ OR (more accurately): Stat, Proportion Stats, One Sample, Width/Sample Size, Confidence level: 0.85, Target Population: 0.25, Width: 0.02 (notice: $2 \times 0.01 = 0.02$) Compute gives Sample size: 3886

(b) *Sample size for proportion p without prior $\hat{p}$: purchase slips.*
What is sample size, n, required to estimate proportion Visa purchase slips, p, to within margin of error of $E = 0.01$ with 85% confidence? Here

$$n = \frac{1}{4}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = \frac{1}{4}\left(\frac{1.44}{0.01}\right)^2 \approx$$

(circle one) **4409** / **5184** / **5470**.

OR (more accurately): Stat, Prop Stats, One Sample, Width/Sample Size, Conf level: 0.85, Target Pop: 0.5 (because if $\hat{p} = (1-\hat{p}) = 0.5$ in formula with prior, then $\hat{p}(1-\hat{p}) = 0.25 = \frac{1}{4}$ in formula without prior), Width: 0.02 (notice: $2 \times 0.01 = 0.02$) Compute gives Sample size: 5182

Without prior $\hat{p} = 0.25$, sample size (choose one)
decreases / **remains same** / **increases** from $n \approx 3888$ to $n \approx 5184$.

(c) *Sample size for proportion p without $\hat{p}$: purchase slips.*
What is sample size, n, required to estimate proportion of Visa credit cards, p, to within margin of error of $E = 0.02$ with 90% confidence? Here

$$n = \frac{1}{4}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = \frac{1}{4}\left(\frac{1.65}{0.02}\right)^2 \approx$$

(circle one) **1702** / **1884** / **2470**.

Prior $\hat{p} = \frac{25}{100}$. Since 90% = $(1-\alpha)\cdot 100\% = (1-0.10)\cdot 100\%$, then $z_{\frac{\alpha}{2}} = z_{\frac{0.10}{2}} = z_{0.05}$ and so Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ [?]) = 0.05 Compute gives $z_{\frac{\alpha}{2}} \approx 1.465$ OR (more accurately): Stat, Prop Stats, One Sample, Width/Sample Size, Conf level: 0.90, Target Pop: 0.5, Width: 0.04 (notice: $2 \times 0.02 = 0.04$) Compute gives Sample size: 1691

9.2 Estimating a Population Mean

The $(1-\alpha)\cdot 100\%$ confidence interval for μ with *un*known σ is called a *t-interval*:

$$\bar{x} \pm t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right).$$

and used when either underlying distribution is normal with no outliers or if simple random sample size large ($n \geq 30$).

Exercise 9.2 (Estimating a Population Mean)

1. *Estimates for population average weight of PNW students.*
   Average weight of simple random sample of 11 PNW students is $\bar{x} = 167$ pounds with sample SD $s = 20.1$ pounds. Weights normally distributed, no outliers.

   (a) *Point estimate.*
   Point estimate of population weight of *all* students, μ, is
   $\bar{x} =$ (choose one) **11** / **20.1** / **167**.
   Also notice σ is *un*known and *estimated* by $s = 20.1$.

   (b) *95% CI*

   i. *Using StatCrunch.* The 95% CI for μ is (circle one)
   (143.5, 182.5) / **(151.5, 180.5)** / **(153.5, 180.5)**.
   (Stat, T statistics, One sample, with summary, Sample mean: 167 Sample std. dev.: 20.1 sample size: 11, Next, choose Confidence Interval 0.95, Calculate.)
   So, 95% confident population parameter μ in (153.5, 180.5).

   ii. *Using formula: degrees of freedom (df).*
   df $= n - 1 = 11 - 1 =$ (circle one) **10** / **11**.

   iii. *Using formula: critical value.*
   Critical value 95% $= (1 - \alpha) \cdot 100\% = (1 - 0.05) \cdot 100\%$ CI, 10 df
   $t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} \approx$ (circle one) **1.28** / **2.23** / **2.58**.
   (Stat, Calculators, T (Not Normal!), DF: 10, Prob(X $\geq$ [?]) = 0.025 Compute.)

   iv. *Using formula.*
   The 95% CI for μ is
   $\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (circle one)
   $\mathbf{20.1 \pm 167 \times \frac{2.23}{\sqrt{11}}}$ / $\mathbf{2.23 \pm 167 \times \frac{20.1}{\sqrt{11}}}$ / $\mathbf{167 \pm 2.23 \times \frac{20.1}{\sqrt{11}}}$
   which equals (circle one)
   20.1 ± 12.51 / **2.23 ± 13.51** / **167 ± 13.51** $\approx (153.5, 180.5)$.

   (c) *99% CI*

   i. *Using StatCrunch.* The 99% CI for μ is (circle one)
   (147.8, 186.2) / **(151.5, 180.5)** / **(153.5, 180.5)**.
   (Options. Edit. choose Confidence Interval 0.99, Calculate.)
   So, 99% confident population parameter μ in (147.8, 186.2).

   ii. *Using formula: degrees of freedom.*
   df $= n - 1 = 11 - 1 =$ (circle one) **10** / **11**.

   iii. *Using formula: critical value.*
   Critical value 99% $= (1 - \alpha) \cdot 100\% = (1 - 0.01) \cdot 100\%$ CI, 10 df
   $t_{\frac{\alpha}{2}} = t_{\frac{0.01}{2}} = t_{0.005} \approx$ (circle one) **1.28** / **2.23** / **3.17**.
   (Stat, Calculators, T (Not Normal!), DF: 10, Prob(X $\geq$ [?]) = 0.005 Compute.)

   iv. *Using formula.*
   The 99% CI for μ is
   $\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (circle one)

$\mathbf{20.1 \pm 20.1 \times \frac{3.17}{\sqrt{11}}}$ / $\mathbf{3.17 \pm 167 \times \frac{20.1}{\sqrt{11}}}$ / $\mathbf{167 \pm 3.17 \times \frac{20.1}{\sqrt{11}}}$.
which equals (circle one)
$\mathbf{20.1 \pm 19.21}$ / $\mathbf{3.17 \pm 19.21}$ / $\mathbf{167 \pm 19.21} \approx (147.8, 186.2)$

v. *Margin of error (half of CI length)*

$$E = t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} \approx 3.17 \times \frac{20.1}{\sqrt{11}} \approx$$

(circle one) **11** / **12.5** / **19.2**.

vi. **True** / **False** There is a 99% *chance* population average weight, μ, falls in sample interval (147.8, 186.2).

2. *Confidence interval for average length of corn cobs, raw data.*
Corn cob lengths for $n = 15 < 30$ cobs, chosen at random, are noted.

$$18, 23, 24, 20, 21, 19, 27, 24, 19, 20, 25, 20, 18, 26, 20$$

(StatCrunch: Blank data table. Relabel var1 as length. Type 15 lengths into length column. Data, Save data, 9.2 corn cob lengths.)

(a) *Point estimate.*
Point estimate of population length of *all* cobs, μ, is
$\bar{x} =$ (choose one) **2.97** / **21.6** / **15**.
Also notice population SD in cob length, σ, is *un*known and *estimated* by
$s \approx$ (choose one) **2.97** / **21.6** / **15**..
(Stat, Summary Stats, Columns, select length, Calculate.)

(b) *Check assumptions (since $n = 15 < 30$): normality and outliers.*

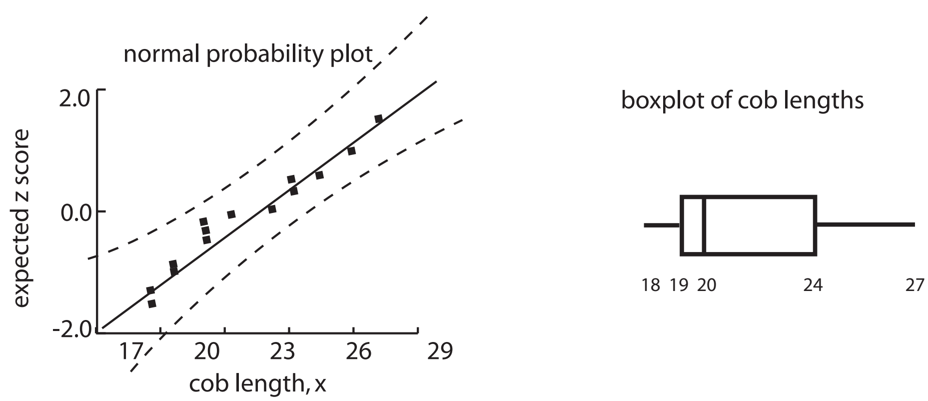

Figure 9.2 (Normal probability plot, boxplot for cob lengths.)

i. *Data normal?*
Normal probability plot indicates cob lengths **normal** / **not normal** because data within dotted bounds.
Graphics. QQ Plot, Select Columns: length, Create Graph!

ii. *Outliers?*
Boxplot indicates **outliers / no outliers**.
Graphics. Boxplot, select length, Next, Use fences to identify outliers, check Draw boxes horizontally. Create Graph!

(c) *95% CI*

i. *Using StatCrunch.* The 95% CI for μ is (choose one)
$\mathbf{(17.96, 21.24)}$ / $\mathbf{(19.96, 22.24)}$ / $\mathbf{(19.96, 23.25)}$.
(Stat, T statistics, One sample, with data, Next, choose Confidence Interval 0.95, Calculate.)

ii. *Using formula: degrees of freedom (df).*
df $= n - 1 =$ (circle one) **15** / **14**.

iii. *Using formula: critical value.*
Critical value 95% $= (1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI, 14 df
$t_{\frac{\alpha}{2}} = t_{\frac{0.05}{2}} = t_{0.025} \approx$ (circle one) **1.76** / **2.15**.
(Stat, Calculators, T (Not Normal!), DF: 14, Prob(X $\geq$?) = 0.025 Compute.)

iv. *Using formula.*
The 95% CI for μ is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (choose one)
$\mathbf{21.6 \pm 2.15 \times \frac{2.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 2.15 \times \frac{3.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 3.15 \times \frac{2.97}{\sqrt{15}}}$.

(d) *99% CI*

i. *Using StatCrunch.* The 99% CI for μ is (choose one)
$\mathbf{(19.23, 23.45)}$ / $\mathbf{(19.96, 23.24)}$ / $\mathbf{(19.32, 23.88)}$.
(Options. Edit. choose Confidence Interval 0.99, Calculate.)

ii. *Using formula: degrees of freedom (df).*
The df, here, for 99% CI is (choose one) **same as** / **different from** degrees of freedom calculated for 95% CI above because same sample size is used in both cases.

iii. *Using formula: critical value.*
Critical value 99% $= (1-\alpha)\cdot 100\% = (1-0.01)\cdot 100\%$ CI, 14 df
$t_{\frac{\alpha}{2}} = t_{\frac{0.01}{2}} = t_{0.005} \approx$ (circle one) **1.76** / **2.98**.
(Stat, Calculators, T, DF: 14, Prob(X $\geq$?) = 0.005 Compute.)

iv. *Using formula.*
Thus, the 99% CI for μ is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} =$ (choose one)
$\mathbf{21.6 \pm 2.15 \times \frac{2.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 2.15 \times \frac{3.97}{\sqrt{15}}}$ / $\mathbf{21.6 \pm 2.98 \times \frac{2.97}{\sqrt{15}}}$.
which equals (circle one)
$\mathbf{21.6 \pm 1.29}$ / $\mathbf{21.6 \pm 2.29}$ / $\mathbf{21.6 \pm 3.29} \approx (19.32, 23.88)$.

(e) *Some comments*

i. **True** / **False**. Long 99% CI better than shorter 95% CI in the sense we are more confident 99% contains or "captures" unknown parameter μ.

However, 95% CI better than longer 99% CI in the sense, if unknown parameter μ is 95% interval estimate, we are more certain of location of this unknown parameter.

ii. Since sample size is small, we **can** / **cannot** use central limit theorem.

iii. Match columns.

terms	corn example
(a) population	**(a)** average length of 15 plants, X
(b) sample	**(b)** average length of all plants, μ
(c) statistic	**(c)** lengths of all plants
(d) parameter	**(d)** observed lengths of 15 plants

terms	(a)	(b)	(c)	(d)
corn example				

3. *Population, sample, statistic and parameter: CI for average corn cob length.* Simple random sample of 15 corn cobs is taken. Assume sample SD in length is $s = 2.97$ and, although we typically don't know it, *population* (not *sample*) length is $\mu = 22$ inches. Assume normality.

(a) *Population* $\mu = 22$ *length*
Population $\mu = 22$ is a **statistic** / **parameter**.
Population μ **changes** / **remains same** for every *random* sample.
Population μ (usually) **known** / **unknown** to us,
(although we are pretending for this question we do know it.)

(b) *Sample* $\bar{x}$ *length*
Sample $\bar{x}$ is a **statistic** / **parameter**.
Sample $\bar{x}$ **changes** / **remains same** for every *random* sample.
Sample $\bar{x}$ (usually) **known** / **unknown** to us:
it may be $\bar{x} = 21.6$ for one sample, but $\bar{x} = 29.8$ for another sample.

(c) A 95% CI for μ, if $\bar{x} = 21.6$, is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 21.6 \pm 1.96 \frac{2.97}{\sqrt{15}} =$ (circle one)
(19.95, 23.24) / **(23.45, 27.80)** / **(28.16, 31.44)**.
(Stat, T statistics, One sample, with summary, Sample mean: 21.6 Sample std. dev.: 2.97 sample size: 15, Next, choose Confidence Interval 0.95, Calculate.)
This 95% CI (circle one) **contains** / **does not contain** $\mu = 22$.

(d) A 95% CI for μ, if $\bar{x} = 29.8$, is
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} = 29.8 \pm 1.96 \frac{2.97}{\sqrt{15}} =$ (circle one)
(19.60, 23.60) / **(23.45, 27.80)** / **(28.16, 31.44)**.
(Options. Edit. Back. Sample mean: 29.8, Calculate.)
This 95% CI (circle one) **contains** / **does not contain** $\mu = 22$.

(e) If sample average length, $\bar{x}$, changes, corresponding 95% CI,
$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$, **changes** / **remains the same**. More than this, (circle one)

i. *all* possible 95% CIs contain $\mu = 22$.
ii. *none* of all possible 95% CIs contain $\mu = 22$.
iii. ninety–nine percent of all possible 95% CIs contain $\mu = 22$, and so one percent of all possible 95% CIs do not contain $\mu = 22$.
iv. ninety–five percent of all possible 95% CIs contain $\mu = 22$, and so five percent of all possible 95% CIs do not contain $\mu = 22$.

This is demonstrated in figure below.

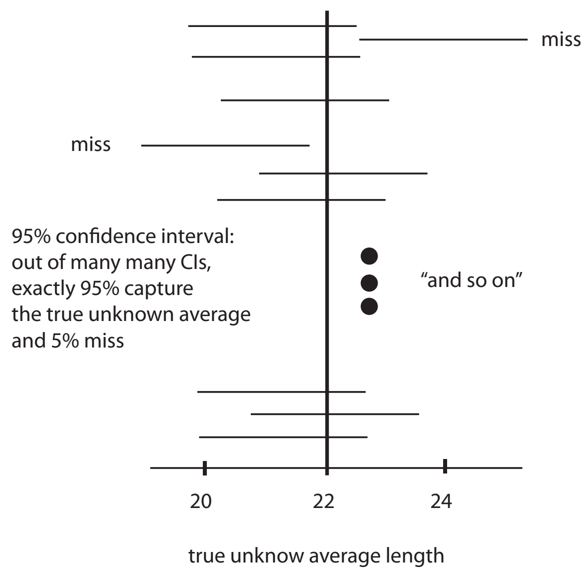

Figure 9.3 (Interpreting confidence intervals.)

(f) Choose true or false.

i. **True** / **False**. 95% *chance* (19.95, 23.24) contains μ.
ii. **True** / **False**. 95% *chance* (19.95, 23.24) contains $\bar{x} = 21.6$.
iii. **True** / **False**. 95% *confident* (19.95, 23.24) contains μ.
iv. **True** / **False**. 95% *confident* (19.95, 23.24) contains $\bar{x} = 21.6$.

4. *Sample size*[4] *given margin of error and level of confidence: corn cob lengths.* Sample size necessary to achieve a required margin of error, E, with a given level of confidence in a confidence interval determined using formula

$$n = \left(\frac{z_{\frac{\alpha}{2}} s}{E} \right)^2.$$

[4]The way this formula is used in the text is incorrect because you have to know n to know s; that is, n is in the formula for s. Having said this, we will assume we know s at least approximately, without knowing n, and then use this formula as given in the text. Notice, also, $z_{\frac{\alpha}{2}}$ replaces $t_{\frac{\alpha}{2}}$.

(a) What sample size, n, required to estimate average corn cob length , μ, to within margin of error $E = 0.08$ with 95% confidence? Assume $s = 0.25$.

$$n = \left(\frac{z_{\frac{\alpha}{2}} s}{E}\right)^2 = \left(\frac{z_{0.025} s}{E}\right)^2 = \left(\frac{1.96 \cdot 0.25}{0.08}\right)^2 \approx 37.7 \approx$$

(circle one) **37** / **38** / **39**.

(Since 95% = $(1-\alpha) \cdot 100\% = (1-0.05) \cdot 100\%$, $z_{\frac{\alpha}{2}} = z_{\frac{0.05}{2}} = z_{0.025} \approx 1.96$ since Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ [?]) = 0.025 Compute.)

(b) *Increase margin of error, E.*
What sample size, n, required to estimate average corn cob length , μ, to within margin of error $E = 0.16$ with 95% confidence? Assume $s = 0.25$.

$$n = \left(\frac{z_{\frac{\alpha}{2}} s}{E}\right)^2 = \left(\frac{z_{0.025} s}{E}\right)^2 = \left(\frac{1.96 \cdot 0.25}{0.16}\right)^2 \approx 9.38 \approx$$

(circle one[5]) **9** / **10** / **11**.
When margin of error doubled, from $E = 0.08$ to $E = 0.16$, sample size (choose one) **quartered** / **halved** / **doubled** from $n = 38$ to $n = 10$.

(c) *Increase confidence,* $(1-\alpha) \cdot 100\%$.
What sample size, n, required to estimate average corn cob length , μ, to within margin of error $E = 0.16$ with 99% confidence? Assume $s = 0.25$.

$$n = \left(\frac{z_{\frac{\alpha}{2}} s}{E}\right)^2 = \left(\frac{z_{0.005} s}{E}\right)^2 = \left(\frac{2.58 \cdot 0.25}{0.16}\right)^2 \approx 16.2 \approx$$

(circle one) **16** / **17** / **18**.
When confidence increased, from 95% to 99%, sample size (choose one) **decreases** / **remains same** / **increases** from $n = 10$ to $n = 17$.

(Since 99% = $(1-\alpha) \cdot 100\% = (1-0.01) \cdot 100\%$, $z_{\frac{\alpha}{2}} = z_{\frac{0.01}{2}} = z_{0.005} \approx 2.58$ since Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≥ [?]) = 0.005 Compute
OR (more accurately): Stat, T Stats, One Sample, Width/Sample Size, Confidence level: 0.99, Std. dev.: 0.25, Width: 0.32 (notice: 2 × 0.16 = 0.32) Compute gives Sample size: 20

5. *Percentages for t distribution: temperatures.*
Assume temperature, T, follows a t distribution with 4 degrees of freedom.

[5] Always round *up*, never down, to satisfy margin of error and confidence conditions.

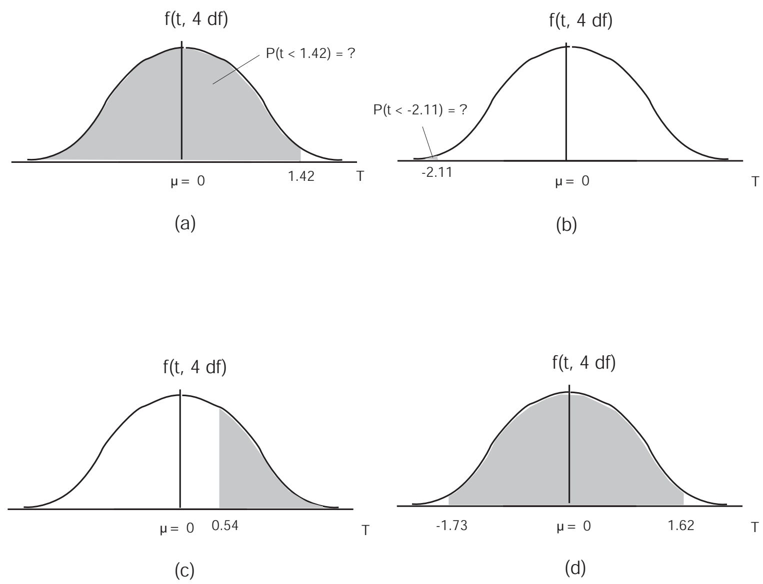

Figure 9.4 (Probabilities for t with 4 df.)

(a) Probability temperature is less than 1.42^o, is
$P(t < 1.42) =$ (circle one) **0.786** / **0.834** / **0.886** / **0.905**
(Stat, Calculators, T (Not Normal!), DF: 4, Prob(X $\leq$ 1.42) = [?] Compute.)

(b) $P(t < -2.11) =$ (circle one) **0.023** / **0.051** / **0.124** / **0.243**.
(Stat, Calculators, T, DF: 4, Prob(X $\leq$ -2.11) = [?] Compute.)

(c) $P(t > 0.54) =$ (circle one) **0.309** / **0.356** / **0.435** / **0.470**.
(Stat, Calculators, T, DF: 4, Prob(X $\geq$ 0.54) = [?] Compute.)

(d) $P(-1.73 < t < 1.62) =$ (circle one) **0.647** / **0.734** / **0.801** / **0.830**.
(Stat, Calculators, T, Between, DF : 4, Prob(-1.73 $\leq$ X $\leq$ 1.62) = [?] Compute.)

(e) t–distributions are (pick one) **skewed right** / **symmetric** / **skewed left**.

(f) Total area (probability) under any t–distribution curve is
(circle one) **50%** / **75%** / **100%** / **150%**.

(g) Shape of t–distribution curve is
(circle one) **triangular** / **bell–shaped** / **rectangular**.

(h) The t–distribution is centered at (circle one) $\boldsymbol{\mu = 0^o}$ / $\boldsymbol{\mu = 1.42^o}$.

(i) Since t–distribution is symmetric, (circle one) **25%** / **50%** / **75%** of temperatures are above (to right) of $\mu = 0^o$.

(j) **True** / **False** Probability temperature is *exactly* 1.42^o, say, is *zero*.
(Stat, Calculators, T, Between, DF : 4, Prob(1.42 $\leq$ X $\leq$ 1.42) = [?] Compute.)

(k) *Comparing t to Z*

i. $P(t \leq 1.42^o) \approx 0.886$ **equals** / **does not equal** $P(Z < 1.42^o) \approx 0.922$ where t has 4 df and "Z" stands for the "standard normal".

(Is Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≤ 1.42) = [?] Compute ≈ 0.922 equal to Stat, Calculators, T, DF: 4, Prob(X ≤ 1.42) = [?] Compute ≈ 0.886 ?)

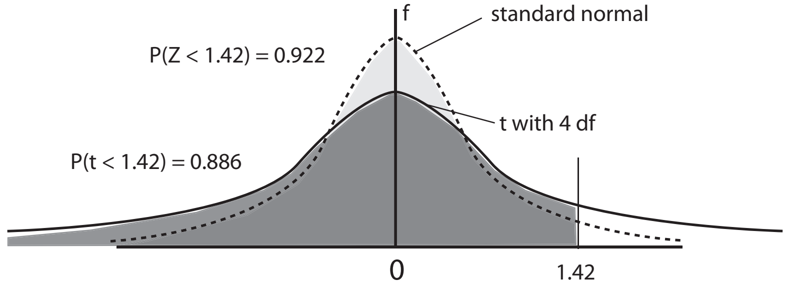

Figure 9.5 (Compare t to Z.)

ii. **True** / **False** $P(Z < 1.42^o) \approx P(t \leq 1.42^o)$ where t has 1000 df.
(Is Stat, Calculators, Normal, Mean: 0, Std. Dev.: 1, Prob(X ≤ 1.42) = [?] Compute ≈ 0.922 equal to Stat, Calculators, T, DF: 1000, Prob(X ≤ 1.42) = [?] Compute ≈ 0.922 ?)

iii. **True** / **False**. The t distribution is a "flatter" version of standard normal. The larger the sample size, n, (or degrees of freedom, df) the less flat the t distribution becomes, the more like the standard normal it becomes.

6. *Percentiles for t distribution: temperatures.*
Assume temperature, T, follows a t distribution with 4 degrees of freedom.

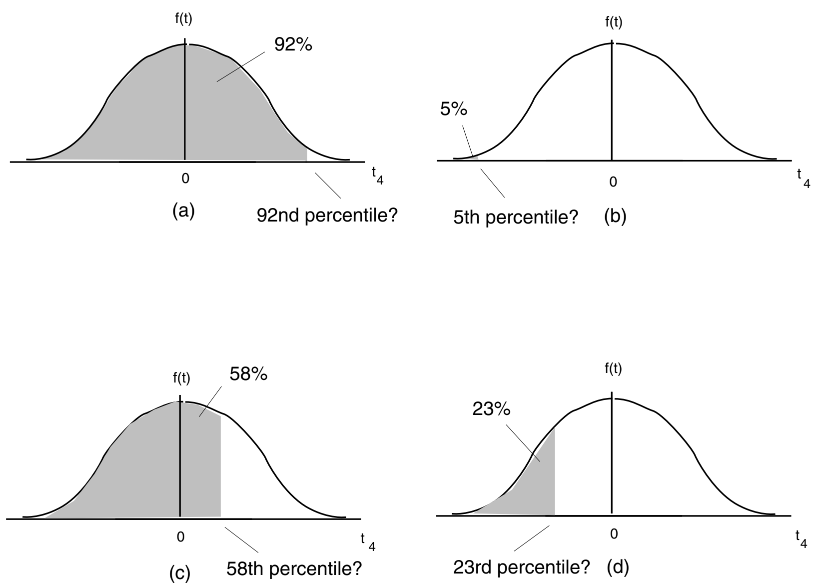

Figure 9.6 (Percentiles for t with 4 df.)

(a) The 92nd percentile is (circle one) $\mathbf{0.95^o}$ / $\mathbf{1.23^o}$ / $\mathbf{1.72^o}$ / $\mathbf{2.21^o}$.
(Stat, Calculators, T, DF: 4, Prob(X ≤ [?]) = 0.92 Compute.)

(b) The 5th percentile is (circle one) $\mathbf{-2.31^o}$ / $\mathbf{-2.13^o}$ / $\mathbf{-1.76^o}$ / $\mathbf{-0.76^o}$.
(Stat, Calculators, T, DF: 4, Prob(X ≤ ?) = 0.05 Compute.)

(c) The 58th percentile is (circle one) $\mathbf{0.22^o}$ / $\mathbf{0.97^o}$ / $\mathbf{1.21^o}$ / $\mathbf{1.35^o}$.
(Stat, Calculators, T, DF: 4, Prob(X ≤ ?) = 0.58 Compute.)

(d) The 23rd percentile is (circle one) $\mathbf{-1.58^o}$ / $\mathbf{-1.23^o}$ / $\mathbf{-0.82^o}$ / $\mathbf{-0.56^o}$.
(Stat, Calculators, T, DF: 4, Prob(X ≤ ?) = 0.23 Compute.)

(e) *Percentiles and critical values*

i. The 95th percentile for t is equal to critical value $t_{0.05}$.
The 5th percentile for t is (circle *two*!) $\boldsymbol{t_{0.025}}$ / $\boldsymbol{-t_{0.05}}$ / $\boldsymbol{t_{0.95}}$.
The 99th percentile for t is (circle *two*!) $\boldsymbol{-t_{0.01}}$ / $\boldsymbol{t_{0.01}}$ / $\boldsymbol{-t_{0.99}}$.

ii. Probability (area) between $-t_{0.10}$ and $t_{0.10}$ is
(circle one) **0.80** / **0.09** / **0.88**.

9.3 Estimating a Population Standard Deviation

After looking at chi–square distribution, we use it for $(1-\alpha)\cdot 100\%$ CI for σ^2:

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right)$$

and used when underlying distribution is normal with no outliers and sample is obtained using simple random sampling[6].

Exercise 9.3 (Confidence Intervals for a Population Standard Deviation)

1. *Estimation for variance: car door and jamb.*
In a simple random sample of 28 cars, variance in gap between door and jamb is $s^2 = 0.7$ mm^2. Calculate 95% CI. Assume normality with no outliers.

(a) *Using StatCrunch.* The 95% CI for σ^2 is (choose one)
$\mathbf{(0.39, 1.22)}$ / $\mathbf{(0.41, 1.25)}$ / $\mathbf{(0.44, 1.30)}$.
(Stat, Variance, One sample, with summary, Sample variance: 0.7 Sample size: 28, Next, choose Confidence Interval 0.95, Calculate.)

(b) *Upper* critical value for $95\% = (1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI is
$\chi^2_{\frac{\alpha}{2}} = \chi^2_{\frac{0.05}{2}} = \chi^2_{0.025} =$ (circle one) **8.7** / **40.1** / **43.2**
(Stat, Calculators, Chi-square, DF: 27, Prob(X ≥ ?) = 0.025 Compute.)

(c) *Lower* critical value for $95\% = (1-\alpha)\cdot 100\% = (1-0.05)\cdot 100\%$ CI is
$\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{1-\frac{0.05}{2}} = \chi^2_{0.975} =$ (circle one) **14.6** / **40.1** / **43.2**
(Stat, Calculators, Chi-square, DF: 27, Prob(X ≥ ?) = 0.975 Compute.)

[6]This CI for σ^2 can be converted to CI for σ by taking square-root. CI is sensitive to non-normal data which is not always fixed by large sample size. There is no preset menu in TI-84+ for this CI.

(d) So, 95% CI for variance σ^2 is[7]

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right) = \left(\frac{(28-1)0.7}{43.2}, \frac{(28-1)0.7}{14.6}\right) =$$

(circle one) $\mathbf{(0.61, 1.65)}$ / $\mathbf{(0.59, 1.29)}$ / $\mathbf{(0.43, 1.29)}$.

(e) Since 95% CI (0.43, 1.29) does *not* include 0.40, this *indicates*[8] variance in distance between door and jamb (circle one) **is** / **is not** 0.4 mm^2.

(f) *Population, parameter, sample and statistic.* Match columns.

terms	jamb example
(a) population	**(a)** variance in jamb–door distance, of 28 cars, s^2
(b) sample	**(b)** variance in jamb–door distance, of all cars, σ^2
(c) statistic	**(c)** jamb–door distances, of all cars
(d) parameter	**(d)** jamb–door distances, of 28 cars

terms	(a)	(b)	(c)	(d)
jamb example				

2. *Estimation for variance: machine parts.*
In a simple random sample of 18 machine parts, variance in lengths is $s^2 = 12^2$. Calculate 90% CI. Assume normality with no outliers.

(a) *Using StatCrunch.* The 90% CI for σ^2 is (choose one)
$\mathbf{(88.1, 281.3)}$ / $\mathbf{(88.7, 282.3)}$ / $\mathbf{(88.2, 282.3)}$.
(Stat, Variance, One sample, with summary, Sample variance: 144 Sample size: 18, Next, choose Confidence Interval 0.90, Calculate.)

(b) *Upper* critical value for 90% = $(1-\alpha)\cdot 100\% = (1-0.10)\cdot 100\%$ CI is
$\chi^2_{\frac{\alpha}{2}} = \chi^2_{\frac{0.10}{2}} = \chi^2_{0.05} =$ (circle one) **8.7** / **27.6** / **43.2**
(Stat, Calculators, Chi-square, DF: 17, Prob(X $\geq$ [?]) = 0.05 Compute.)

(c) *Lower* critical value for 90% = $(1-\alpha)\cdot 100\% = (1-0.10)\cdot 100\%$ CI is
$\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{1-\frac{0.10}{2}} = \chi^2_{0.95} =$ (circle one) **8.7** / **40.1** / **43.2**
(Stat, Calculators, Chi-square, DF: 17, Prob(X $\geq$ [?]) = 0.95 Compute.)

(d) So, 90% CI for variance σ^2 is (there may round-off error)
$\left(\frac{(n-1)s^2}{\chi^2_U}, \frac{(n-1)s^2}{\chi^2_L}\right) = \left(\frac{(18-1)12^2}{27.6}, \frac{(18-1)12^2}{8.7}\right) =$
(circle one) $\mathbf{(80.5, 101.4)}$ / $\mathbf{(100.5, 104.2)}$ / $\mathbf{(88.7, 281.4)}$.

(e) Since 90% CI (88.7, 281.4) includes test statistic $13^2 = 169$, this *indicates* variance in lengths (circle one) **is** / **is not** $\sigma^2 = 13^2$ mm^2.

[7] Answer here might be different from StatCrunch answer due to round-off error.
[8] Population variance may be between 0.43 and 1.29, but we are 95% *confident* it is not.

(f) Also, 90% CI for *standard deviation* σ is
$\left(\sqrt{\frac{(n-1)s^2}{\chi^2_U}}, \sqrt{\frac{(n-1)s^2}{\chi^2_L}}\right) = \left(\sqrt{\frac{(18-1)12^2}{27.6}}, \sqrt{\frac{(18-1)12^2}{8.7}}\right) =$
(circle one) **(9.4, 16.8)** / **(10.5, 14.2)** / **(88.7, 281.4)**.

3. *Probabilities for chi–square: waiting time to order*
At McDonalds in Westville, waiting time to order (in minutes), X, follows a *chi–square*, χ^2, distribution. Consider following figure with two χ^2 distributions, each with different shaded areas (probabilities).

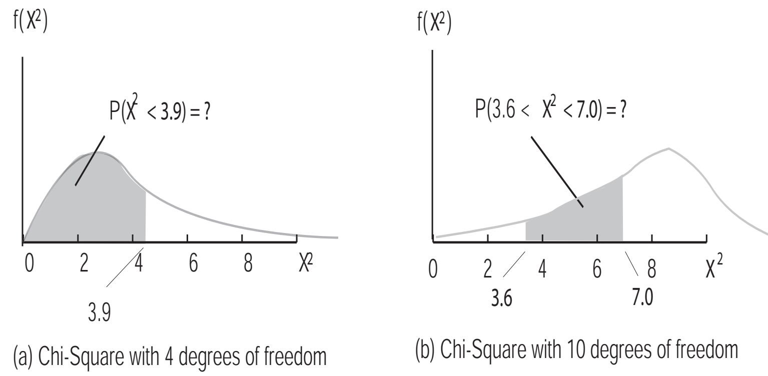

Figure 9.7 (Probabilities for χ^2 distribution.)

(a) For a χ^2 with 4 df, probability of waiting less than 3.9 minutes
$P(\chi^2 < 3.9) =$ (circle one) **0.35** / **0.45** / **0.58** / **0.66**
(Stat, Calculators, Chi-square, DF: 4, Prob(X $\leq$ 3.9) = ? Compute.)

(b) For a χ^2 with 10 df,
$P(3.6 < \chi^2 < 7.0) =$ (circle one) **0.24** / **0.34** / **0.42** / **0.56**.
(Stat, Calculators, Chi-Square, Between, DF: 10, P(3.6 $\leq X \leq$ 7) = ? Compute.)

(c) Like t distribution, different χ^2 distributions indexed by degrees of freedom (df), which equal df $= n - 1$. For $n = 5$, df $= n - 1 = 5 - 1 =$ **3** / **4** / **5**.

(d) The χ^2 distribution with sample of size $n = 5$ (df $= 4$), in (a) of figure above, has mode (high point) at $n - 2 =$ (circle one) **1** / **2** / **3**.

(e) The χ^2 distribution is (choose one) **symmetric** / **asymmetric** but becomes more symmetric as df increase.

(f) Total area (probability) under chi-square is **50%** / **75%** / **100%** / **150%**.

(g) **True** / **False** Values of χ^2 are greater than or equal to zero.

4. *Percentiles and critical values for chi–square: waiting time to order*
At McDonalds in Westville, waiting time to order (in minutes), X, follows a *chi–square*, χ^2, distribution. Consider following figure with two χ^2 distributions, each with 72nd percentile waiting time.

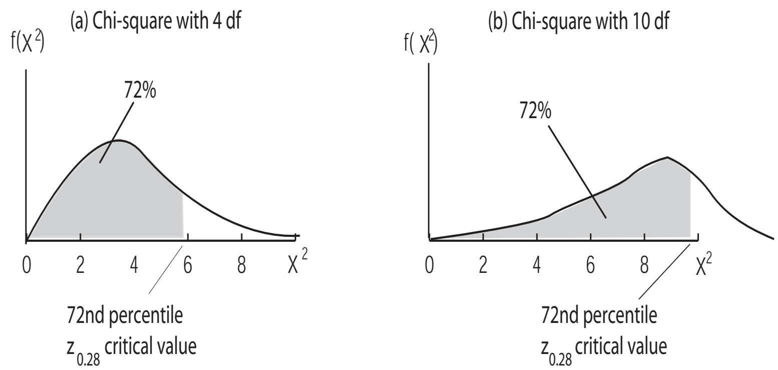

Figure 9.8 (Percentiles for χ^2 distribution.)

(a) The 72nd percentile waiting time for a χ^2 with 4 df, is (circle one) **3.1** / **5.1** / **8.3** / **9.1**.
(Stat, Calculators, Chi-square, DF: 4, Prob(X $\leq$ [?]) = 0.72 Compute.)

(b) 72nd percentile or critical value $\chi^2_{0.28}$ waiting time with 10 df, is (circle one) **2.5** / **10.5** / **12.1** / **20.4**.
(Stat, Calculators, Chi-square, DF: 10, Prob(X $\leq$ [?]) = 0.72 Compute
OR Prob(X $\geq$ [?]) = 0.28 Compute.)

(c) The 32nd percentile or $\chi^2_{0.68}$ critical value for a χ^2 with 18 df, is (circle one) **2.5** / **10.5** / **14.7** / **20.4**.
(Stat, Calculators, Chi-square, DF: 18, Prob(X $\leq$ [?]) = 0.32 Compute
OR Prob(X $\geq$ [?]) = 0.68 Compute.)
The 32nd percentile is that waiting time such that 32% of waiting times are less than this waiting time and 68% are more than this time.

(d) If $\alpha = 0.20$ and $n = 19$,
$\chi^2_{\frac{\alpha}{2}} = \chi^2_{\frac{0.20}{2}} = \chi^2_{0.10} =$ (circle one) **20.5** / **21.5** / **24.7** / **26.0**.
(Stat, Calculators, Chi-square, DF: 18, Prob(X $\geq$ [?]) = 0.10 Compute.)

(e) If $\alpha = 0.20$ and $n = 19$,
$\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{1-\frac{0.20}{2}} = \chi^2_{0.90} =$ (circle one) **10.9** / **11.5** / **14.7** / **19.4**.
(Stat, Calculators, Chi-square, DF: 18, Prob(X $\geq$ [?]) = 0.90 Compute.)

9.4 Putting it Together: Which Procedure Do I Use?

As will soon be discovered (or already has been discovered), the main difficulty with confidence intervals is not so much to do with calculating them, as much as to do with deciding which one to calculate. Following table summaries confidence intervals given in this chapter and under what circumstances to calculate any one of these confidence intervals; other confidence intervals are given in later chapters. A table similar to this one will be used in later chapters to summarize tests of hypotheses.

CONFIDENCE INTERVALS		mean μ	variance σ^2	proportion p
	one	σ: $\bar{x} \pm t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}}\right)$	$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right)$	$\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
sample	two	chapter 11	chapter 11	chapter 11
	multiple	not covered for confidence intervals	not covered for CIs	not covered for CIs

9.5 Estimating with Bootstrapping

Not covered.