**Practice Final for Mathematics 224**
**Introductory Analysis II - Spring 2001**
**Material Covered: Chapters 5–7, B and C of Workbook and Text**
**For: 2nd May**

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

**Name (please print):** _____ . **ID Number:** _____.
last first

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**1.** [1 point] Assuming that the equation $x^2y + \cos y = 4$ defines $y$ as a differentiable function of $x$, what is the slope of the tangent line to the graph of the curve which is the graph of the equation?

**(i)** $\frac{-2xy}{\sin y}$

**(ii)** $\frac{-2xy}{x^2-\sin y}$

**(iii)** $\frac{2xy}{x^2+\sin y}$

**(iv)** $\frac{-x^2}{\sin y}$

**(v)** none of the above

[Hint: Use implicit differentiation to find $\frac{dy}{dx}$.]

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**2.** Infestation of crops by insects has long been of great concern to farmers and agricultural scientists. Below, is data on the age of a cotton plant (days), $x$, and percent damaged squares, $y$.

| $x$ | 9 | 12 | 12 | 15 | 18 | 18 | 21 | 21 | 27 | 30 | 30 | 33 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y$ | 11 | 12 | 23 | 30 | 29 | 52 | 41 | 65 | 60 | 72 | 84 | 93 |

**(a)** [1 point] The least squares line

is _____.

**(b)** [1 point] The predicted percentage of damaged squares when the age is 20 days is (circle closest one) **46** / **47** / **48** / **49** / **50**

**3.** Use the method of substitution to solve the following integration problems.

**(a)** [1 point] $\int (3x-1)^{-4}\,dx =$

**(i)** $-12(3x-1)^{-5}+C$

**(ii)** $-\frac{1}{9}(3x-1)^{-3}+C$

**(iii)** $(3x-1)^{-3}+C$

**(iv)** $-\frac{1}{3}(3x-1)^{-3}+C$

**(v)** none of the above

**(b)** [1 point] $\int e^{3-2x}\,dx =$

**(i)** $-2e^{3-2x}+C$

**(ii)** $-\frac{1}{2}e^{3-2x}+C$

**(iii)** $\frac{e^{4-2x}}{4-2x}+C$

**(iv)** $\frac{1}{3}e^{3-2x}+C$

**(v)** none of the above

**(c)** [2 points] $\int \frac{3x}{(x+1)(x-2)}\,dx =$

**(i)** $\ln(|x-2|)+ln(|x+1|)+C$

**(ii)** $-2\ln(|x-2|)+ln(|x+1|)+C$

**(iii)** $\ln(|x-2|)-2ln(|x+1|)+C$

**(iv)** $2\ln(|x-2|)+ln(|x+1|)+C$

**(v)** none of the above

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**4.** [2 points] What is the volume of the solid figure obtained by revolving the region bounded by the graphs of the curves $y=x^2$ and $y=2-x^2$ about the $x$–axis?

**(i)** $8\pi$

**(ii)** $\frac{16\pi}{3}$

**(iii)** $\frac{32\pi}{3}$

**(iv)** $\frac{136\pi}{3}$

**(v)** none of the above

**5.** Try the following integration problems.

**(a)** [2 points] $\int x \ln(x^2)\, dx =$

**(i)** $x^2 \ln x - \frac{x^2}{2} + C$

**(ii)** $\frac{x^2}{2} \ln x - \frac{x^2}{2} + C$

**(iii)** $x^2 \ln x - \frac{x^3}{6} + C$

**(iv)** $x \ln x^2 - \frac{1}{x} + C$

**(v)** none of the above

**(b)** [1 point] $\int_1^2 x \ln(x^2)\, dx =$

**(i)** $-0.30$

**(ii)** 1.27

**(iii)** 2.27

**(iv)** 0.77

**(v)** none of the above

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**6.** Try the following problems related to the probability density function

$$f(x) = \begin{cases} c(3-x) & \text{if } 0 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

**(a)** [1 point] For what value of $c$ is $\int_0^\infty f(x)\, dx = 1$?

**(i)** $c = -\frac{2}{3}$

**(ii)** $c = -\frac{1}{3}$

**(iii)** $c = \frac{1}{9}$

**(iv)** $c = \frac{2}{9}$

**(v)** none of the above

**(b)** [1 point] The expected value is
$\mu =$ (circle one) **0.5** / **1.0** / **1.5** / **2.0** / **2.5**

**(c)** [1 point] The standard deviation is
$\sigma^2 =$ (circle one) **0.5** / **1.0** / **1.5** / **2.0** / **2.5**

**7.** Try the following problems.

**(a)** [1 point] A calculator manufacturer estimates that $t$ months from now that consumers will be buying 1000 calculators per month at a price of $20 + 3\sqrt{t}$ per calculator. What is the total revenue the manufacturer can expect from the sale of the calculators over the next four months?

**(i)** 8,000 dollars

**(ii)** 16,000 dollars

**(iii)** 96,000 dollars

**(iv)** 192,000 dollars

**(v)** none of the above

**(b)** [1 point] An object moves so that its velocity after $t$ minutes is given by $20e^{-0.01t}$. What is the distance the object travels during the tenth minute?

**(i)** $\int_0^{10} 20e^{-0.01t}\,dt$

**(ii)** $\int_9^{10} -20e^{-0.01t}\,dt$

**(iii)** $\int_0^{10} -20e^{-0.01t}\,dt$

**(iv)** $\int_9^{10} 20e^{-0.01t}\,dt$

**(v)** none of the above

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**8.** Try the following problems.

**(a)** [1 point] Records indicate that $t$ hours past midnight, the temperature in Westville was given by $T(t) = -0.3t^2 + 4t + 10$ degrees Celsius. What is the average temperature at the airport between 2 am and 7 am?

**(i)** 3.4 degrees

**(ii)** 15.4 degrees

**(iii)** 21.3 degrees

**(iv)** 35.7 degrees

**(v)** none of the above

**(b)** [1 point] The region $R$ is bounded by the graph of $y = 1 + x^2$ and the $x$–axis over the interval $0 \le x \le 2$. The approximate value of the area of $R$ given by a Reimann sum with four subintervals of equal length is
(circle one) $\frac{15}{4}$ / $\frac{20}{4}$ / $\frac{25}{4}$ / $\frac{30}{4}$ / $\frac{35}{4}$

**9.** A manufacturer has found that the marginal cost is $6q - 1$ dollars per unit when $q$ units have been produced. The total cost of producing the first unit is 130 dollars. What is the total cost of producing the first 10 units?

**(a)** [1 point] Using the separation of variables technique, the solution is (circle one) **310** / **418** / **436** / **498** / **566**

**(b)** [2 points] Solve, using Euler's method; in particular, complete the following table.

| $n$ | $a_n = x_0 + nh$ | $b_n = b_{n-1} + hg(a_{n-1}, b_{n-1})$ |
|---|---|---|
| 1 | $a_1 =$ _____ | $b_1 =$ _____ |
| 2 | $a_2 =$ _____ | $b_2 =$ _____ |
| 3 | $a_3 =$ _____ | $b_3 =$ _____ |
| 4 | $a_4 =$ _____ | $b_4 =$ _____ |
| 5 | $a_5 =$ _____ | $b_5 =$ _____ |

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**10.** Consider $z = (3x^2 + 4)(2xy - x^2y^2)$.

**(a)** [1 point] $z_x =$ (circle one)

**(i)** $(3x^2 - 4)(2y - 2xy^2) + (2xy - x^2y^2)(6x)$

**(ii)** $(3x^2 + 4)(2y - 2xy^2) + (2xy + x^2y^2)(6x)$

**(iii)** $(3x^2 - 4)(2y - 2xy^2) + (2xy + x^2y^2)(6x)$

**(iv)** $(3x^2 + 4)(2y + 2xy^2) + (2xy - x^2y^2)(6x)$

**(v)** $(3x^2 + 4)(2y - 2xy^2) + (2xy - x^2y^2)(6x)$

**(b)** [1 point] $z_y =$ _____

**(c)** [1 point] $z_{yx} =$ (circle one)

**(i)** $18x^2 + 24x^3y + 8 - 16xy$

**(ii)** $18x^2 - 24x^3y + 8 + 16xy$

**(iii)** $18x^2 - 24x^3y - 8 - 16xy$

**(iv)** $18x^2 - 24x^3y - 8 + 16xy$

**(v)** $18x^2 - 24x^3y + 8 - 16xy$

**1.** (ii)

**2.** (a) $\boldsymbol{y = -19.676 + 3.285x}$; (b) **46**.

**3.** (a) (ii); (b) (ii) (c) (iv)

**4.** (ii).

**5.** (a) (i); (b) (ii)

**6.** (a) (iv) (b) **1.0** (c) **0.5**.

**7.** (a) (iii) (b) (iv)

**8.** (a) (iii) (b) $\frac{15}{4}$

**9.** (a) **418** (b) 2.8, 4.6, 6.4, 8.2, 10; 139, 167.4, 215.3, 282.6, 369.4

**10.** (a) (v); (b) $\boldsymbol{6x^3 - 6x^4y + 8x - 8x^2y}$ (c) (v)