Practice Final II for Mathematics 224
Introductory Analysis II - Spring 2001
Material Covered: Chapters 5–7, B and C of Workbook and Text
For: 2nd May

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): _____ . **ID Number:** _____.
last first

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1. Try the following derivative questions.

(a) [1 point] What is the derivative of the function defined by $f(x) = \frac{xe^x}{\sin x}$?

(i) $\frac{e^x}{\cos x}$

(ii) $\frac{e^x(x\sin x)+x\cos x+\sin x}{\sin^2 x}$

(iii) $\frac{e^x(x\sin x)-x\cos x+\sin x}{\sin^2 x}$

(iv) $\frac{e^x(x\sin x)+x\cos x}{\sin^2 x}$

(v) none of the above

(b) [1 point] What is the derivative of the function defined by $f(x) = \cos^2(\sqrt{x})$?

(i) $\frac{2\cos(\sqrt{x})}{2\sqrt{x}}$

(ii) $\frac{2\cos(\sqrt{x})\sin(\sqrt{x})}{2\sqrt{x}}$

(iii) $\frac{-2\cos(\sqrt{x})\sin(\sqrt{x})}{2\sqrt{x}}$

(iv) $\frac{-2\sin(\sqrt{x})}{2\sqrt{x}}$

(v) none of the above

2. Try the following integration problems.

(a) [1 point] $\int \left(\frac{2}{x} - \sqrt{x}\right)\, dx =$

(i) $\ln(|x|) - \frac{2}{\sqrt{x}} + C$

(ii) $-\frac{2}{x^2} - \frac{1}{2\sqrt{x}} + C$

(iii) $2\ln(|x|) - \frac{2x^{3/2}}{3} + C$

(iv) $-\frac{2}{x^2} - \frac{2x^{3/2}}{3} + C$

(v) none of the above

(b) [1 point] $\int \frac{w}{(1-w^2)^{3/2}}\, dw =$

(i) $\frac{1}{\sqrt{1-w^2}} + C$

(ii) $\frac{1}{2\sqrt{1-w^2}} + C$

(iii) $\frac{-1}{\sqrt{1-w^2}} + C$

(iv) $\frac{-1}{2\sqrt{1-w^2}} + C$

(v) none of the above

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3. [2 points] $\int_0^{\pi/2} x\cos x\, dx =$

(i) $1 - \frac{\pi}{2}$

(ii) $\frac{\pi}{2}$

(iii) $\frac{\pi}{2} - 1$

(iv) 1

(v) none of the above

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4. [1 point] For what value of k is $\int \sin(3x)\, dx = k\cos(3x) + C$ true?

(i) $k = 3$

(ii) $k = -3$

(iii) $k = \frac{1}{3}$

(iv) $k = -\frac{1}{3}$

(v) none of the above

5. Try the following integration problems.

(a) [2 points] Find an expression which represents a function whose tangent line has slope $x\sqrt{5-x^2}$ for each value of x and whose graph passes through the point $(2, 10)$.

(i) $-\frac{1}{3}(5-x^2)^{3/2}$

(ii) $\frac{2}{3}(5-x^2)^{3/2}+\frac{28}{3}$

(iii) $\frac{1}{3}(5-x^2)^{3/2}+\frac{29}{3}$

(iv) $-\frac{1}{3}(5-x^2)^{3/2}+\frac{31}{3}$

(v) none of the above

(b) [2 points] What is the general solution of the differential equation $\frac{dy}{dx}=2y+1$?

(i) $x=y^2+y+C$

(ii) $2y+1=Ce^{2x}$

(iii) $y=2xy+x+C$

(iv) $y=Ce^{2x}-2y-1$

(v) none of the above

(c) [2 points] What is the solution of the differential equation $xy'-3y=x^2$ at $(x, y)=(1, 0)$?

(i) $y=\frac{1}{4}\left(x-x^{-2}\right)$

(ii) $y=\frac{1}{4}\left(x-x^{-3}\right)$

(iii) $y=\frac{1}{4}\left(x-x^{-4}\right)$

(iv) $y=\frac{2}{4}\left(x-x^{-4}\right)$

(v) $y=\frac{3}{4}\left(x-x^{-4}\right)$

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6. [2 points] What is the area of the region bounded by the curves which are the graphs of $y=1+x^2$ and $y=3x+5$?

(i) $\frac{125}{6}$

(ii) $\frac{56}{3}$

(iii) $\frac{27}{2}$

(iv) $\frac{25}{6}$

(v) none of the above

7. Try the following integration problems.

(a) [1 point] It is estimated that t years from now, the population of a certain city will be changing at a rate of $5+3t^{2/3}$ hundred people per year. If the population is presently 100,000, by how many people will the population increase over the next eight years?

(i) 100

(ii) 9,760

(iii) 6,200

(iv) 109,760

(v) none of the above

(b) [1 point] The Purdue University North Central mutual fund grows at a rate equal to 13.5% of its value at any time t. What is an expression for the value of a PU/NC mutual fund account that has an initial investment of 1500 dollars?

(i) $1500e^{-0.135t}$ dollars

(ii) $1500 + e^{-0.135t}$ dollars

(iii) $1500e^{0.135t}$ dollars

(iv) $1500(1 + 0.135t)$ dollars

(v) none of the above

(c) [1 point] If there are 10,000 people in a community and, if at time t, M of them have the measles, write a differential equation describing the following situation: The rate at which measles spreads in the community is jointly proportional to the number of people who have the measles and the number of people in the community who do not as yet have the measles.

(i) $\frac{dM}{dt} = k(M - 10,000)$

(ii) $\frac{dM}{dt} = k(10,000 - M)$

(iii) $\frac{dM}{dt} = 10,000kM$

(iv) $\frac{dM}{dt} = kM(10,000 - M)$

(v) none of the above

8. [2 points] $\int_0^\infty xe^{-x^2}\,dx =$

(i) $-\frac{1}{2}$

(ii) 1

(iii) $\frac{1}{2}$

(iv) the integral diverges

(v) none of the above

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9. Consider the function $z = 4y + 4x - 3x^2 + y^2$.

(a) [1 point] This function is *optimal* (either a maximum or a minimum) at $(x, y) =$
(circle one) $\left(\frac{1}{3}, 2\right)$ / $\left(\frac{2}{3}, 2\right)$ / $\left(\frac{3}{3}, 2\right)$ / $\left(\frac{4}{3}, 2\right)$ / $\left(\frac{5}{3}, 2\right)$

(b) [2 points] At the optimal point,
$z =$ (circle one) $\frac{53}{3}$ / $\frac{54}{3}$ / $\frac{55}{3}$ / $\frac{56}{3}$ / $\frac{57}{3}$,
and is (circle one) **maximum** / **minimum**.

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10. [2 points] $\int_1^4 \int_x^{2x} (3xy + 4y^2)\,dy\,dx =$
(circle closest one) **672.5** / **689.5** / **697.5** / **705.5** / **722.5**,

1. (a) (iii); (b) (iii).

2. (a) (iii); (b) (i).

3. (iii).

4. (iv).

4. (a) (iv); (b) (ii) (c) (ii).

6. (i).

7. (a) (ii); (b) (iii) (c) (iv).

8. (iii).

9. (a) $\left(\frac{2}{3}, 2\right)$; (b) $\frac{56}{3}$, **minimum**

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