Quiz Practice Questions 2 (Attendance 4) for Statistics 514
Design of Experiments
Chapters 21, 22 Neter et al. and Kuhn

These are practice questions for the quiz. The quiz (not the practice questions) is worth 5% and marked out of 5 points. One or more questions is closely, but not necessarily exactly, related to one or more of these questions will appear on the quiz. These practice questions are *not* to be handed in. Quizzes are to be done *using Vista* on the Internet **before** 4am (West Lafayette time!) of the date of the quiz. Vista will *not* allow any quiz to be done late. It is *highly* recommended that you complete this practice quiz, by hand, *before* logging onto Vista. The quiz is an **individual** one which means that each student does this quiz by themselves without help from others.

1. Applied Linear Statistical Models (Neter et al.) Questions.

Chapter	Problem(s)	hints
21, pages 885–888	21.5, 21.6, 21.7	Brainstorming
22, pages 916–923	22.6, 22.7, 22.8, 22.12, 22.17	Adjunct Professor data

(21.5) Brainstorming: qz2-21-5-brainstorming-onetreatANOVA

(a) *Treatment plots*
From SAS, the treatment means plot appears to show that the size of the group is significant, but that the type of group is not. It also appears that there is either no or only a small interaction between the two main factors. Finally, a table of means is given by

	two	three	four	five	$\overline{Y}_{i\cdot}$
Agribusiness executives	18	22	31	32	$\overline{Y}_{2\cdot} = 25.75$
Agribusiness scientists	15	23	29	33	$\overline{Y}_{3\cdot} = 25$
$\overline{Y}_{\cdot j}$	$\overline{Y}_{\cdot 1} = 16.5$	$\overline{Y}_{\cdot 2} = 22.5$	$\overline{Y}_{\cdot 3} = 30$	$\overline{Y}_{\cdot 4} = 32.5$	$\overline{Y}_{\cdot\cdot} = 25.375$

(b) *Test of main factors*
The ANOVA table is given by

Source	*df*	*SS*	*MS*
Factor A (Type of Group)	1	1.125	1.125
Factor B (Size of Group)	3	?	106.125
Error (Interaction AB)	3	6.375	2.125
Total	7	325.875	

where, notice, the interaction effect, AB, has been used as the error.

Factor A, Type of Group
$H_0 : \alpha_1 = \alpha_2 = 0$ versus
$H_a :$ at least one $\alpha_i \neq 0,\ i = 1, 2, 3.$
since p–value $=? > \alpha = 0.01$
accept null; that is, factor A effect is *not* significant

Factor B, Size of Group
$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ versus
$H_a :$ at least one $\beta_i \neq 0,\ i = 1, 2, 3, 4.$
since p–value $= 0.0046 < \alpha = 0.01$
reject null; that is, factor B effect is significant

Kimball inequality (pages 831-832): Since $\alpha = 0.01$ for both factors A and B,

$$\alpha \leq 1 - (1 - 0.01)(1 - 0.01) =?$$

(c) *Paired Bonferroni confidence intervals*

Since

$$\begin{aligned} s\left\{\hat{D}\right\} &= \sqrt{\frac{2MSE}{n}} \\ &= \sqrt{\frac{2(?)}{2}} \\ &= 1.4577 \\ t(1-\alpha/2g;(a-1)(b-1)) &= t(1-0.05/(2(3));(2-1)(4-1)) \\ &= t(0.99167;3) \\ &= ? \end{aligned}$$

then

pair	$\hat{D}$	$\hat{D} \pm t(1-\alpha/2g;(a-1)(b-1))s\left\{\hat{D}\right\}$
$(\cdot 2, \cdot 1)$	6.0	$6.0 \pm 4.857(1.4577) = (-1.08, 13.08)$
$(\cdot 3, \cdot 2)$	7.5	$7.5 \pm 4.857(1.4577) = (0.42, 14.58)$
$(\cdot 4, \cdot 3)$	2.5	$2.5 \pm 4.857(1.4577) = (?, ?)$

(d) *Most efficient confidence interval*

type	critical value
Scheffe	$\sqrt{(r-1)F(1-\alpha;b-1;(a-1)(b-1))} = \sqrt{(4-1)F(1-0.05;4-1;3)} = 5.275$
Tukey	$\frac{1}{\sqrt{2}}q\left(1-\alpha;b;(a-1)(b-1)\right) = \frac{1}{\sqrt{2}}q\left(1-0.05;4;3\right) = 4.822$
Bonferroni	$t(1-\alpha/2g;(a-1)(b-1)) = t(1-0.05/[2(3)];3) = 4.857$

the most efficient confidence interval is the ? interval because it has the smallest critical value.

(21.6) Brainstorming: qz2-21-6-brainstorming-mu14

(a) *Point estimate of* $\hat{\mu}_{14}$

$$\hat{\mu}_{14} = \overline{Y}_{1\cdot} + \overline{Y}_{\cdot 4} - \overline{Y}_{\cdot\cdot} = 25.75 + 32.5 - ? = ?$$

(b) *Calculate* $s^2\{\hat{\mu}_{14}\}$

The factor means version of this two–factor ANOVA,

$$Y_{ij} = \mu_{..} + \alpha_i + \beta_j + \varepsilon_{ij},$$

can be written as a multiple regression model in the following way,

$$Y_{ijk} = \mu_{..} + \alpha_1 X_{ij1} + \beta_1 X_{ij2} + \beta_2 X_{ij3} + \beta_3 X_{ij4} + \varepsilon_{ij}$$

where $i = 1, 2$; $j = 1, 2, 3, 4$ and

$$X_{ij1} = \begin{cases} 1, & \text{if case from Factor A level 1} \\ -1, & \text{if case from Factor A level 2} \end{cases}$$

$$X_{ij2} = \begin{cases} 1, & \text{if case from Factor B level 1} \\ -1, & \text{if case from Factor B level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ij3} = \begin{cases} 1, & \text{if case from Factor B level 2} \\ -1, & \text{if case from Factor B level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ij4} = \begin{cases} 1, & \text{if case from Factor B level 3} \\ -1, & \text{if case from Factor B level 4} \\ 0, & \text{otherwise,} \end{cases}$$

and so

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

where

$$\mathbf{Y} = \begin{bmatrix} 18 \\ 22 \\ \vdots \\ 29 \\ 33 \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \\ 1 & -1 & -1 & -1 & -1 \end{bmatrix}, \quad \beta = \begin{bmatrix} \mu_. \\ \alpha_1 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

and so

$$\mathbf{X}\beta = \begin{bmatrix} \mu_. + \alpha_1 + \beta_1 \\ \mu_. + \alpha_1 + \beta_2 \\ \vdots \\ \mu_. - \alpha_1 + \beta_3 \\ \mu_. - \alpha_1 - \beta_1 - \beta_2 - \beta_3 \end{bmatrix} = \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \vdots \\ \mu_{23} \\ \mu_{24} \end{bmatrix}$$

and so, from SAS,

$$s^2\{\hat{\mu}_{14}\} = MSE(\mathbf{X_h'}(\mathbf{X'X})^{-1}\mathbf{X_h}) = 1.1524^2 = ?$$

(c) *99% confidence interval of* $s^2\{\hat{\mu}_{14}\}$

$$\hat{\mu}_{14} \pm t(1-\alpha/2;(a-1)(b-1))s\{\hat{\mu}_{14}\} = 32.875 \pm t(0.995;3)(?) = (?,?)$$

(21.7) Brainstorming: qz2-21-7-brainstorming-onetreat-Tukey
A table of means is given by

	two	three	four	five	$\overline{Y}_{i\cdot}$
Agribusiness executives	18	22	31	32	$\overline{Y}_{2\cdot}$ =?
Agribusiness scientists	15	23	29	33	$\overline{Y}_{3\cdot} = 25$
$\overline{Y}_{\cdot j}$	$\overline{Y}_{\cdot 1} = ?$	$\overline{Y}_{\cdot 2} = 22.5$	$\overline{Y}_{\cdot 3} = 30$	$\overline{Y}_{\cdot 4} = 32.5$	$\overline{Y}_{\cdot\cdot} = 25.375$

and so

$$\begin{aligned} SSAB^* &= \frac{\left(\sum_i \sum_j (\overline{Y}_{i\cdot} - \overline{Y}_{\cdot\cdot})(\overline{Y}_{\cdot j} - \overline{Y}_{\cdot\cdot})Y_{ij}\right)^2}{\sum_i (\overline{Y}_{i\cdot} - \overline{Y}_{\cdot\cdot})^2 \sum_j (\overline{Y}_{\cdot j} - \overline{Y}_{\cdot\cdot})^2} \\ &= 1.4688 \end{aligned}$$

and

$$\begin{aligned} SSRem^* &= SSTO - SSA - SSB - SSAB^* \\ &= 325.875 - 1.125 - 318.375 - \ ? \\ &= \ ? \end{aligned}$$

and so
$H_0 : D = 0$ versus $H_a : \ D \neq 0$.
since $F^* = \frac{SSAB^*}{1} \div \frac{SSRem^*}{ab-a-b} = \frac{?}{1} \div \frac{?}{(2)(4)-2-4} = 0.60$ is smaller than
and $F(1-\alpha; 1, ab-a-b) = F(1-0.01; 1, 3) = ?$
accept null; that is, *no* interactions are present

(22.6) Adjunct professor: qz2-22-6-prof-unequaltreatment

(a) *ANOVA model*

The factor effects ANOVA model is given by,

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk},$$

and the equivalent regression model is

$$\begin{aligned} Y_{ijk} &= \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} \\ &+ (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} \\ &+ (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5} \\ &+ (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \varepsilon_{ijk}, \end{aligned}$$

where

$$X_{ijk1} = \begin{cases} 1, & \text{if case from Factor A level 1} \\ -1, & \text{if case from Factor A level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1, & \text{if case from Factor A level 2} \\ -1, & \text{if case from Factor A level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1, & \text{if case from Factor A level 3} \\ -1, & \text{if case from Factor A level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk4} = \begin{cases} 1, & \text{if case from Factor B level 1} \\ -1, & \text{if case from Factor B level 3} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk5} = \begin{cases} 1, & \text{if case from Factor B level 2} \\ -1, & \text{if case from Factor B level 3} \\ 0, & \text{otherwise,} \end{cases}$$

(b) $\mathbf{X}$ *and* β

Using matrix notation, the linear regression version of this two–factor ANOVA is given by

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

where, in particular,

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

where the twelve columns are for $(\mu_{..}, X_1, X_2, X_3, X_4, X_5, X_1X_4, X_1X_5, X_2X_4, X_2X_5, X_3X_4, X_3X_5)$ respectively. In addition,

$$\beta = \begin{bmatrix} \mu_{..} \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ (\alpha\beta)_{11} \\ (\alpha\beta)_{12} \\ (\alpha\beta)_{21} \\ (\alpha\beta)_{22} \\ (\alpha\beta)_{31} \\ (\alpha\beta)_{32} \end{bmatrix}$$

(c) $\mathbf{X}\beta$

The entries are

$$\mathbf{X}\beta = \begin{bmatrix} \mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11} \\ \mu_{..} + \alpha_1 + \beta_1 + (\alpha\beta)_{11} \\ \cdots \\ \mu_{..} - \alpha_1 - \alpha_2 - \alpha_3 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22} + (\alpha\beta)_{31} + (\alpha\beta)_{32} \\ \mu_{..} - \alpha_1 - \alpha_2 - \alpha_3 - \beta_1 - \beta_2 + (\alpha\beta)_{11} + (\alpha\beta)_{12} + (\alpha\beta)_{21} + (\alpha\beta)_{22} + (\alpha\beta)_{31} + (\alpha\beta)_{32} \end{bmatrix}$$

(d) *Residuals*

From SAS, some residuals are, for example,
$e_{111} = -0.10$
$e_{312} = ?$

(e) *Normal probability plot*

From SAS, the residual plot is more or less straight and so indicates normality.

(22.7) Adjunct professor: qz2-22-7-prof-unequal,regression

(a) *Treatment means plot*
From SAS, a table of the treatments, $\overline{Y}_{ij\cdot}$, is given below.

1.80	1.95	2.70
2.45	2.52	3.45
2.75	?	3.74
2.55	2.55	3.42

The treatment plot indicates little, if any, interaction.

(b) *Reduced model*
The reduced model for testing for interaction effects is given by,

$$Y_{ijk} = \mu_{\cdot\cdot} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5}$$

where, notice, no interaction effects are present.

(c) *Test for interaction*
From SAS, the *full* model is given by,

$$\begin{aligned}\hat{Y} &= 2.72750 - 0.57750X_1 + 0.07917X_2 - 0.38583X_3 - 0.34000X_4 - 0.26000X_5 \\ &\quad -0.01000X_1X_4 + \,?X_1X_5 - 0.01667X_2X_4 - 0.2667X_2X_5 \\ &\quad -0.02333X_3X_4 - 0.00333X_3X_5\end{aligned}$$

and the *reduced* model is given by,

$$\hat{Y} = 2.72074 - 0.59611X_1 + 0.08412X_2 + \,?X_3 - 0.33756X_4 - 0.26317X_5$$

Consequently, a test of the interaction is given by
$H_0:$ all $(\alpha\beta)_{ij} = 0$ versus $H_a:$ not all $(\alpha\beta)_{ij} = 0$
Since

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{? - 0.7180}{39 - 33} \div \frac{0.7180}{33} = 0.34$$

is smaller than
$F(1-\alpha; df_R - df_F, df_F) = F(1 - 0.01; 6, 33) = 3.41$
accept null; that is, interaction effect is *not* significant

p–value $= P(F > F^*; 6, 33) = P(P > 0.34; 6, 33) = 0.91$

(d) *Tests of main effects*

Subject matter.
The reduced model is given by,

$$
\begin{aligned}
Y_{ijk} &= \mu_{\cdot\cdot} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} \\
&+ (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} \\
&+ (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5} \\
&+ (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \varepsilon_{ijk} \\
\hat{Y} &\approx 2.75121 - 0.34885 X_4 - 0.19441 X_5 \\
&\quad + ?X_1 X_4 + 0.19481 X_1 X_5 \\
&\quad -0.01178 X_2 X_4 - 0.08433 X_2 X_5 \\
&\quad -0.22413 X_3 X_4 - 0.00731 X_3 X_5
\end{aligned}
$$

where, notice, no subject matter main effects are present.
$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ versus
$H_a :$ at least one $\alpha_i \neq 0, \ i = 1, 2, 3, 4.$
Since

$$
F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{4.95061214 - ?}{36 - 33} \div \frac{?}{33} = 64.845
$$

is smaller than
and $F(1 - \alpha; df_R - df_F, df_F) = F(1 - 0.01; 3, 33) = 4.437$
reject null; that is, subject matter effect *is* significant
also, p–value $= P(F > F^*; 3, 33) = P(P > 64.845; 3, 33) \approx 0$

Degree
The reduced model is given by,

$$
\begin{aligned}
Y_{ijk} &= \mu_{\cdot\cdot} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} \\
&+ (\alpha\beta)_{11} X_{ijk1} X_{ijk4} + (\alpha\beta)_{12} X_{ijk1} X_{ijk5} \\
&+ (\alpha\beta)_{21} X_{ijk2} X_{ijk4} + (\alpha\beta)_{22} X_{ijk2} X_{ijk5} \\
&+ (\alpha\beta)_{31} X_{ijk3} X_{ijk4} + (\alpha\beta)_{32} X_{ijk3} X_{ijk5} + \varepsilon_{ijk} \\
\hat{Y} &= 2.88451 - 0.44871 X_1 - 0.09702 X_2 - 0.36160 X_3 \\
&\quad -0.06779 X_1 X_4 + 0.08939 X_1 X_5 \\
&\quad -0.05349 X_2 X_4 - 0.03742 X_2 X_5 \\
&\quad + ?X_3 X_4 - 0.10851 X_3 X_5
\end{aligned}
$$

where, notice, no degree main effects are present.
$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$ versus

$H_a:$ at least one $\beta_i \neq 0,\ i = 1, 2, 3.$
Since

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{? - 0.7180}{35 - 33} \div \frac{0.7180}{33} = 189.10$$

is smaller than
and $F(1 - \alpha; df_R - df_F, df_F) = F(1 - 0.01; 2, 33) = 5.321$
reject null; that is, degree effect *is* significant
also, p–value $= P(F > F^*; 2, 33) = P(P > 189.10; 2, 33) \approx 0$

(e) *Pairwise comparisons, subject matter*
From SAS,
$\mu_{1\cdot} = 2.15,\ \mu_{2\cdot} = 2.81,\ \mu_{3\cdot} = 3.11,\ \mu_{4\cdot} = 2.84$
and furthermore,
$\hat{D}_1 = \hat{\mu}_{1\cdot} - \hat{\mu}_{2\cdot} = 2.15 - 2.81 = -0.6567$
$\hat{D}_2 = \hat{\mu}_{1\cdot} - \hat{\mu}_{3\cdot} = 2.15 - 3.1133 = -0.9633$
$\hat{D}_3 = \hat{\mu}_{1\cdot} - \hat{\mu}_{4\cdot} = 2.15 - 2.84 = -0.6900$
$\hat{D}_4 = \hat{\mu}_{2\cdot} - \hat{\mu}_{3\cdot} = ?$
$\hat{D}_5 = \hat{\mu}_{2\cdot} - \hat{\mu}_{4\cdot} = -0.0333$
$\hat{D}_6 = \hat{\mu}_{3\cdot} - \hat{\mu}_{4\cdot} = 0.2733$
and also,
$s\left\{\hat{D}_1\right\} = \sqrt{\frac{1}{a^2} MSE \sum_i \left(\frac{1}{n_{ij}} + \frac{1}{n_{ij'}}\right)} = 0.06642$
$s\left\{\hat{D}_2\right\} = 0.07083$
$s\left\{\hat{D}_3\right\} = 0.07497$
$s\left\{\hat{D}_4\right\} = ?$
$s\left\{\hat{D}_5\right\} = 0.06777$
$s\left\{\hat{D}_6\right\} = 0.07209$
and so since $\frac{1}{\sqrt{2}} q(0.95, 4, 33) = \frac{3.825}{\sqrt{2}} = 2.705$ the various Tukey CIs are
$\hat{D}_1$: $-0.6567 \pm 2.705(0.06642) = (-0.836, -0.477)$
$\hat{D}_2$: $-0.9633 \pm 2.705(0.07083) = (?, ?)$
$\hat{D}_3$: $(-0.893, -0.487)$
$\hat{D}_4$: $(-0.477, -0.136)$
$\hat{D}_5$: $(-0.217, 0.150)$
$\hat{D}_6$: $(0.078, 0.468)$

(f) *Pairwise comparisons, degree*

From SAS,
$\mu_{\cdot 1} = 2.39$, $\mu_{\cdot 2} = 2.47$, $\mu_{\cdot 3} = 3.33$,
and furthermore,
$\hat{D}_1 = \hat{\mu}_{1\cdot} - \hat{\mu}_{2\cdot} = 2.3875 - 2.4675 = -0.0800$
$\hat{D}_2 = \hat{\mu}_{1\cdot} - \hat{\mu}_{3\cdot} = 2.3875 - 3.3350 = -0.9475$
$\hat{D}_3 = \hat{\mu}_{2\cdot} - \hat{\mu}_{3\cdot} = ?$
and also,
$s\left\{\hat{D}_1\right\} = 0.06597$
$s\left\{\hat{D}_2\right\} = 0.05860$
$s\left\{\hat{D}_3\right\} = 0.05501$
and so since $\frac{1}{\sqrt{2}} q(0.95, 3, 33) = \frac{3.470}{\sqrt{2}} = 2.4537$ the various Tukey CIs are
$\hat{D}_1$: $-0.0800 \pm 2.4537(0.06597) = (-0.242, 0.082)$
$\hat{D}_2$: $(-1.091, -0.804)$
$\hat{D}_3$: $(?, ?)$

(22.8) Adjunct professor: qz2-22-8-prof-unequal,nointeract
In both cases, the full model is given by,

$$\begin{aligned} Y_{ijk} &= \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \varepsilon_{ijk} \\ \hat{Y} &\approx 2.72074 - 0.59611 X_1 + 0.08412 X_2 + 0.33964 X_3 - 0.33756 X_4 - 0.26317 X_5 \end{aligned}$$

where, notice, no interaction effects are present.

(a) *Test of subject matter*
The reduced model is given by,

$$\begin{aligned} Y_{ijk} &= \mu_{..} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \varepsilon_{ijk} \\ \hat{Y} &\approx 2.72494 - 0.32494 X_4 - 0.18648 X_5 \end{aligned}$$

where, notice, no subject matter main effects are present.
$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = 0$ versus
$H_a :$ at least one $\alpha_i \neq 0,\ i = 1, 2, 3.$
Since

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{6.741678 - 0.7762425}{42 - ?} \div \frac{0.7762425}{?} = 101.95$$

is larger than
$F(1 - \alpha; df_R - df_F, df_F) = F(1 - 0.05; 3, 39) = 2.845$
reject null; that is, subject matter effect *is* significant
also, p–value $= P(F > F^*; 3, 39) = P(P > 101.95; 3, 39) \approx 0$

(b) *Test of degree*
The reduced model is given by,

$$\begin{aligned} Y_{ijk} &= \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \varepsilon_{ijk} \\ \hat{Y} &= \ ? \ - 0.44483 X_1 - 0.08521 X_2 - 0.36654 X_3 \end{aligned}$$

where, notice, no degree main effects are present.
$H_0 : \beta_1 = \beta_2 = 0$ versus
$H_a :$ at least one $\beta_i \neq 0,\ i = 1, 2.$
Since

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{? - 0.7762425}{41 - 39} \div \frac{0.7762425}{39} = 214.39$$

is larger than
$F(1 - \alpha; df_R - df_F, df_F) = F(1 - 0.05; 2, 39) = 3.238$
reject null; that is, degree effect *is* significant
also, p–value $= P(F > F^*; 2, 39) = P(P > 214.39; 2, 39) \approx 0$

(22.12) Adjunct professor: qz2-22-12-prof-nointeract,nocellsample

(a) *ANOVA model*
The factor effects ANOVA model, without interaction, is given by,

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + \varepsilon_{ijk},$$

and so the equivalent *full* regression model is

$$\begin{aligned} Y_{ijk} &= \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \varepsilon_{ijk} \\ \hat{Y} &\approx 2.71932 + \ ?\ X_1 + 0.08546 X_2 + 0.40036 X_3 + \ ? X_4 - 0.26218 X_5 \end{aligned}$$

and the *reduced* regression model for testing factor A is,

$$\begin{aligned} Y_{ijk} &= \mu_{..} + \beta_1 X_{ijk4} + \beta_2 X_{ijk5} + \varepsilon_{ijk} \\ \hat{Y} &\approx 2.77494 - 0.22494 X_4 - 0.23648 X_5 \end{aligned}$$

and the *reduced* regression model for testing factor B is,

$$\begin{aligned} Y_{ijk} &= \mu_{..} + \alpha_1 X_{ijk1} + \alpha_2 X_{ijk2} + \alpha_3 X_{ijk3} + \varepsilon_{ijk} \\ \hat{Y} &\approx 2.90108 - 0.35108 X_1 + 0.11646 X_2 + 0.33529 X_3 \end{aligned}$$

where

$$X_{ijk1} = \begin{cases} 1, & \text{if case from Factor A level 1} \\ -1, & \text{if case from Factor A level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk2} = \begin{cases} 1, & \text{if case from Factor A level 2} \\ -1, & \text{if case from Factor A level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk3} = \begin{cases} 1, & \text{if case from Factor A level 3} \\ -1, & \text{if case from Factor A level 4} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk4} = \begin{cases} 1, & \text{if case from Factor B level 1} \\ -1, & \text{if case from Factor B level 3} \\ 0, & \text{otherwise,} \end{cases}$$

$$X_{ijk5} = \begin{cases} 1, & \text{if case from Factor B level 2} \\ -1, & \text{if case from Factor B level 3} \\ 0, & \text{otherwise,} \end{cases}$$

(b) *Tests of subject matter (A) and degree (B)*

Test of subject matter, A
$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = 0$ versus
$H_a :$ at least one $\alpha_i \neq 0,\ i = 1, 2, 3.$
Since

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{?\ - 0.7419}{40 - 37} \div \frac{0.7419}{37} = 84.45$$

is larger than
$F(1 - \alpha; df_R - df_F, df_F) = F(1 - 0.01; 3, 37) = 4.360$
reject null; that is, subject matter effect *is* significant
also, p–value $= P(F > F^*; 3, 37) = P(P > 84.45; 3, 37) \approx 0$

Test of degree, B
$H_0 : \beta_1 = \beta_2 = 0$ versus
$H_a :$ at least one $\beta_i \neq 0,\ i = 1, 2.$
Since

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{8.18737762 - 0.7419}{39 - \ ?} \div \frac{0.7419}{?} = 185.66$$

is larger than
$F(1 - \alpha; df_R - df_F, df_F) = F(1 - 0.01; 2, 37) = 5.229$
reject null; that is, degree effect *is* significant
also, p–value $= P(F > F^*; 2, 37) = P(P > 185.66; 2, 37) \approx 0$

(22.17) Adjunct professor: qz2-22-17-prof-proportionalsample

Test of subject matter, A
$H_0 : \alpha_1 = \alpha_2 = \alpha_3 = 0$ versus
$H_a :$ at least one $\alpha_i \neq 0, \; i = 1, 2, 3.$
Since[1]

$$F^* = \frac{SSE(R) - SSE(F)}{df_R - df_F} \div \frac{SSE(F)}{df_F} = \frac{4.167567}{36 - 33} \div \frac{?}{33} = ?$$

is larger than
$F(1-\alpha; df_R - df_F, df_F) = F(1 - 0.05; 3, 33) = 2.89$
reject null; that is, subject matter effect *is* significant
also, p–value $= P(F > F^*; 3, 33) = P(P > ?; 3, 33) \approx 0$

[1]From SAS, notice $SSE(R) - SSE(F) = SSA = 4.167567$.