Quiz Practice Questions 2 (Attendance 4) for Statistics 513
Statistical Control Theory
Material Covered: Chapter 5 Montgomery and Kuhn

These are practice questions for the quiz. The quiz (not the practice questions) is worth 5% and marked out of 5 points. One or more questions is closely, but not necessarily exactly, related to one or more of these questions will appear on the quiz. These practice questions are *not* to be handed in. Quizzes are to be done *using Vista* on the Internet **before** 4am (West Lafayette time!) of the date of the quiz. Vista will *not* allow any quiz to be done late. It is *highly* recommended that you complete this practice quiz, by hand, *before* logging onto Vista. The quiz is an individual one which means that each student does this quiz by themselves without help from others.

Introduction to Statistical Control Theory (Montgomery) Questions.

Chapter	Problem(s)	hints
5, pages 265–282	5-2	
	5-4	
	5-8	
	5-12	
	5-18	
	5-22	
	5-24	
	5-34	
	5-42	
	5-50	

(5-2) $\bar{x}$ chart and R chart, process capability: qz2-5-2-bearings-xrchart

(a) *$\bar{x}$ chart and R chart*
From SAS, the $\bar{x}$ chart and R chart for both the nominal voltage ($x_i =$ (volt $- 350) \times 10$) and actual voltage are both in statistical control with no out-of-control signals, runs, trends, or cycles.

(b) *Process capability*
From SAS[1], the (estimated) process capability is

$$\begin{aligned}
\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\
&= \frac{USL - LSL}{6(\bar{R}/d_2)} \\
&= \frac{355 - 345}{6(0.625/2.039)} \\
&\approx 5.49
\end{aligned}$$

which indicates the "natural" process limits are well inside the "prescribed" specification limits (which is "good").

(c) *Normal probability plot*
The SAS normal probability plot is more or less linear and so indicates voltage is more or less normally distributed.

[1] Or by hand, using $d_2 = 1.693$ from Appendix VI, page 761 of the text.

(5-4) more $\bar{x}$ chart and R chart, process capability: qz2-5-4-thickness-xrchart

(a) *$\bar{x}$ chart and R chart*
From SAS, the $\bar{x}$ chart for the circuit board thicknesses is out of statistical control for sample number 22
The R chart for the circuit board thicknesses is out of statistical control for sample number 14

(b) *Process standard deviation*
From SAS[2], the (estimated) process standard deviation is

$$\hat{\sigma} = \bar{R}/d_2 \approx 0.00092/1.693 \approx 0.000543553$$

(c) *Natural process limits*
From SAS, the (estimated) natural (for one individual x) process limits are

$$\bar{\bar{x}} \pm 3\hat{\sigma} = 0.062952 \pm 3(0.000543553) \approx (0.06132, 0.06456)$$

which should include most process points.

(d) *Process capability*
From SAS[3], the (estimated) process capability is

$$\begin{aligned}\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\ &= \frac{USL - LSL}{6(\bar{R}/d_2)} \\ &= \frac{0.0645 - 0.0615}{6(0.00092/1.693)} \\ &\approx 0.91987\end{aligned}$$

which indicates the "natural" process limits are a little outside the "prescribed" specification limits (which is "bad"); the process is not able to meet specifications.

[2] Or by hand, using $d_2 = 1.693$ from Appendix VI, page 761 of the text.
[3] Or by hand, using $d_2 = 1.693$ from Appendix VI, page 761 of the text.

(5-8) $\bar{x}$ chart and S chart, process capability: qz2-5-8-diameter-xschart

(a) *$\bar{x}$ chart and S chart*
From SAS, the $\bar{x}$ chart and S chart for the diameters are both in statistical control with no out-of-control signals, runs, trends, or cycles.

(b) *Process standard deviation*
Process standard deviation is approximated here using the R chart, not the S chart information (although it could be).
From SAS[4], the (estimated) process standard deviation is

$$\hat{\sigma} = \bar{R}/d_2 \approx 63.5/2.326 \approx 27.3009$$

(c) *Process capability*
Process capability is calculated using the R chart, not S chart information.
From SAS[5], the (estimated) process capability is

$$\begin{aligned}
\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\
&= \frac{USL - LSL}{6(\bar{R}/d_2)} \\
&= \frac{100 - (-100)}{6(63.5/2.326)} \\
&\approx 1.22
\end{aligned}$$

which indicates the "natural" process limits are inside the "prescribed" specification limits (which is "good"); the process is able to meet specifications.

[4] Or by hand, using $d_2 = 2.326$ from Appendix VI, page 761 of the text.
[5] Or by hand, using $d_2 = 2.326$ from Appendix VI, page 761 of the text.

(5-12) $\bar{x}$ chart and R chart, new observations: qz2-5-12-dimension-xrchart,newobs

(a) *$\bar{x}$ chart and R chart*
From SAS, the $\bar{x}$ chart and R chart for the dimensions are both in statistical control with no out-of-control signals, runs, trends, or cycles.

(b) *Previous $\bar{x}$ chart and R chart, new observations*
From SAS, the new observations are all above the upper control limit of the $\bar{x}$ chart; in other words, the process has shifted upwards.

(c) *Previous $\bar{x}$ chart and R chart, adjusted new observations*
From SAS, the adjusted new observations are now all below the center line and one is, in fact, now below the lower control limit of the $\bar{x}$ chart; in other words, the adjustment has overcompensated for the upward shift.

(5-18) $\bar{x}$ chart and S chart, probability in control

(a) *Process standard deviation*
The (estimated) process standard deviation is

$$\hat{\sigma} = \bar{S}/c_4 \approx 1.5/0.9400 \approx 1.60$$

(b) *Control limits*
The (estimated) control limits for $\bar{x}$ are

$$\bar{\bar{x}} \pm A_3\bar{S} = 20.0 \pm 1.427(1.5) \approx (17.86, 22.14)$$

The (estimated) control limits for S are

$$(B_3\bar{S}, B_4\bar{S}) = (0(1.5), 2.089(1.5)) \approx (0, 3.13)$$

(c) *Probability in control*
The probability the process is in control is equal to the probability $\bar{x}$ falls between the upper control limit and the lower control limit,

$$\begin{aligned} P\,(\text{in control}) &= P\,(\text{LCL } \leq \bar{x} \leq \text{ UCL}) \\ &= P(17.86 \leq \bar{x} \leq 22.14; \mu = 22, \hat{\sigma} \approx 1.60/\sqrt{5}) \\ &= 0.5776 \end{aligned}$$

Assuming normality, it is 2nd DISTR normalcdf(17.86, 22.14, 22, 1.60/$\sqrt{5}$)

(5-22) $\bar{x}$ chart and R chart, probability of not detecting mean process shift

(a) *Control limits*
The (estimated) control limits for $\bar{x}$ are

$$\bar{\bar{x}} \pm A_2\bar{S} = 20.0 \pm 0.577(4.56) \approx (17.37, 22.63)$$

The (estimated) control limits for R are

$$(D_3\bar{S}, D_4\bar{S}) = (0(4.56), 2.115(4.56)) \approx (0, 9.64)$$

(b) *Process standard deviation*
The (estimated) process standard deviation is

$$\hat{\sigma} = \bar{R}/d_2 \approx 4.56/2.326 \approx 1.96$$

(c) *Process capability*
The (estimated) process capability is

$$\begin{aligned}\hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\ &= \frac{USL - LSL}{6(\bar{R}/d_2)} \\ &= \frac{5 - (-5)}{6(4.56/2.326)} \\ &\approx 0.85\end{aligned}$$

which indicates the "natural" process limits are outside the "prescribed" specification limits (which is "bad"); the process is not able to meet specifications.

(d) *Probability of not detecting mean process shift*
The probability of not detecting a mean process shift to $\mu = 24$ is equal to the probability $\bar{x}$ falls between the upper control limit and the lower control limit when $\mu = 24$,

$$\begin{aligned}P\,(\text{in control}) &= P\,(\text{LCL } \leq \bar{x} \leq \text{ UCL}) \\ &= P(17.37 \leq \bar{x} \leq 22.63; \mu = 24, \hat{\sigma} \approx 1.96/\sqrt{5}) \\ &= 0.0590\end{aligned}$$

Assuming normality, it is 2nd DISTR normalcdf(17.86, 22.14, 22, 1.96/$\sqrt{5}$)

(5-24) $\bar{x}$ chart and R chart, probability of detection of mean shift

Since $\sigma = 6$,

$$\bar{R} = d_2\sigma = 2.059(6) = 12.354,$$

the (estimated) control limits for $\bar{x}$ are

$$\bar{\bar{x}} \pm A_2\hat{R} = 100 \pm 0.729(12.354) \approx (90.994, 109.006)$$

and so the probability of detecting a mean process shift to $\mu = 92$ is equal to one minus the probability $\bar{x}$ falls between the upper control limit and the lower control limit when $\mu = 92$,

$$\begin{aligned} P\,(\text{detecting shift}) &= 1 - P\,(\text{LCL } \leq \bar{x} \leq \text{ UCL}) \\ &= 1 - P(90.994 \leq \bar{x} \leq 109.006; \mu = 92, \hat{\sigma} \approx 6/\sqrt{5}) \\ &= 0.354 \end{aligned}$$

2nd DISTR normalcdf(90.994, 109.006, 92, $6/\sqrt{5}$)

(5-34) ARL_1

Since $\mu = 98$ and $\sigma = 8$, the probability of not detecting the first point out of control is

$$\begin{aligned} P\,(\text{not detecting first point}) &= P\,(\text{LCL } \leq \bar{x} \leq \text{ UCL}) \\ &= P(96 \leq \bar{x} \leq 104; \mu = 100, \hat{\sigma} \approx 8/\sqrt{5}) \\ &= 0.6651 \end{aligned}$$

2nd DISTR normalcdf(96, 104, 100, $8/\sqrt{5}$)
and so the ARL_1 is

$$\text{ARL}_1 = \frac{1}{1-\beta} = \frac{1}{1-0.6651} = 2.986$$

(5-42) type I error, probability of accidently rejecting null

(a) *Type I error, α*
The (estimated) process standard deviation is

$$\hat{\sigma} = \bar{R}/d_2 \approx 8.91/2.970 \approx 3.000$$

The probability of being out of control, assuming $\mu = 360$, is

$$\begin{aligned} P\,(\text{out of control}) &= 1 - P\,(\text{LCL } \leq \bar{x} \leq \text{ UCL}) \\ &= 1 - P(357 \leq \bar{x} \leq 363; \mu = 360, \hat{\sigma} \approx 3/\sqrt{9}) \\ &= 0.0026 \end{aligned}$$

2nd DISTR normalcdf(357, 363, 360, $3/\sqrt{9}$)

(b) *Process capability*
The (estimated) process capability is

$$\begin{aligned} \hat{C}_p &= \frac{USL - LSL}{6\hat{\sigma}} \\ &= \frac{USL - LSL}{6(\bar{R}/d_2)} \\ &= \frac{6 - (-6)}{6(8.91/2.970)} \\ &\approx 0.667 \end{aligned}$$

which indicates the "natural" process limits are outside the "prescribed" specification limits; the process is not able to meet specifications (which is "bad").

(c) *Probability of detecting shift on first point*
The probability of not detecting the first point out of control, if shift from $\mu = 360$ to $\mu = 357$, is

$$\begin{aligned} P\,(\text{not detecting first point}) &= P\,(\text{LCL } \leq \bar{x} \leq \text{ UCL}) \\ &= P(357 \leq \bar{x} \leq 363; \mu = 357, \hat{\sigma} \approx 3/\sqrt{9}) \\ &= 0.5 \end{aligned}$$

2nd DISTR normalcdf(357, 363, 357, $3/\sqrt{9}$)

(d) *Control limits if $\alpha = 0.01$*
If $\alpha = 0.01$, then $z_{0.01/2} = 2.576$.
invNorm(0.995)
and so the control limits for $\bar{x}$ are

$$\bar{\bar{x}} \pm z_{0.01/2} \frac{\sigma}{\sqrt{n}} = 360 \pm 2.576 \left(\frac{3}{\sqrt{9}} \right) \approx (357.424, 362.576)$$

(5-50) Shewhart control chart individual measurements: qz2-5-50-oxide-irchart

(a) *individual chart and R chart*
From SAS, the individual chart and R chart for the thicknesses are both in statistical control with no out-of-control signals, runs, trends, or cycles. Also, because the normal probability plot is more or less linear, this indicates the normality assumption is reasonable.

(b) *Previous individual chart and R chart, new observations*
From SAS, the new observations are all above the center line of the individual chart and one is above the upper control limit; in other words, the process is out of control.

(c) *Previous individual chart and R chart, adjusted new observations*
From SAS, the adjusted new observations are now all both in statistical control with no out-of-control signals, runs, trends, or cycles.