

Quiz Practice Questions 4 (Attendance 8) for Statistics 514
Design of Experiments
Chapter 26 Neter et al. and Kuhn

These are practice questions for the quiz. The quiz (not the practice questions) is worth 5% and marked out of 5 points. One or more questions is closely, but not necessarily exactly, related to one or more of these questions will appear on the quiz. These practice questions are *not* to be handed in. Quizzes are to be done *using Vista* on the Internet **before** 4am (West Lafayette time!) of the date of the quiz. Vista will *not* allow any quiz to be done late. It is *highly* recommended that you complete this practice quiz, by hand, *before* logging onto Vista. The quiz is an **individual** one which means that each student does this quiz by themselves without help from others.

1. Applied Linear Statistical Models (Neter et al.) Questions.

Chapter	Problem(s)	hints
26, pages 1064–1070	26.6	Productivity improvement
	26.9	two-factor study
	26.12, 26.31	Programmer requirements
	26.18	
	26.24	Premium distribution

(26.6) Productivity improvement, single factor ANOVA, Table B.11

Since, assume (guess) parameter $\sigma = 0.9$,

and assume (guess) parameters $\mu_1 = 7.0$, $\mu_2 = 8.0$ and $\mu_3 = 9.0$,

$$\mu. = \frac{\sum n_i \mu_i}{n_T} = \frac{9(7.0) + 12(8.0) + 6(9.0)}{27} = ?,$$

$$\phi = \frac{1}{\sigma} \sqrt{\frac{\sum n_i (\mu_i - \mu.)^2}{r}} = \frac{1}{0.9} \sqrt{\frac{9(?)^2 + \dots + 6(?)^2}{3}} \approx ?$$

$$\nu_1 = r - 1 = 3 - 1 = 2$$

$$\nu_2 = n_T - r = 27 - 3 = 24$$

$$\alpha = 0.05$$

and so $1 - \beta = ?$, using Table B.11, page 1356

In other words, there is a ?% chance that the mean differences will be detected by this test, when, in fact, the means are different in the assumed way.

(26.9) two-factor study, Table B.11

(a) factor A

Since, assume (guess) parameter $\sigma = 20$,

and assume (guess) parameters $\alpha_1 = -10$, $\alpha_2 = 7$, $\alpha_3 = 3$ and $\alpha_4 = 0$

$$\phi = \frac{1}{\sigma} \sqrt{\frac{nb \sum \alpha_i^2}{a}} = \frac{1}{20} \sqrt{\frac{(6)(3)[(-10)^2 + \dots + 0^2]}{?}} = ?$$

$$\nu_1 = a - 1 = ? - 1 = ?$$

$$\nu_2 = ab(n - 1) = (?)(3)(6 - 1) = ?$$

$$\alpha = 0.05$$

and so $1 - \beta = 0.56$, using Table B.11 (and averaging), pp 1356–1360

In other words, there is a 56% chance that the factor A effects will be detected by this test when these effects are different in the assumed way.

(b) factor B

Since, assume (guess) parameter $\sigma = 20$,

and assume (guess) parameters $\beta_1 = ?$, $\beta_2 = ?$, $\beta_3 = ?$

$$\phi = \frac{1}{\sigma} \sqrt{\frac{na \sum \beta_i^2}{b}} = \frac{1}{20} \sqrt{\frac{(6)(4)[(?^2 + \dots + (?^2)]}{4}} = 1.386$$

$$\nu_1 = b - 1 = 3 - 1 = 2$$

$$\nu_2 = ab(n - 1) = (4)(3)(6 - 1) = 60$$

$$\alpha = 0.05$$

and so $1 - \beta = 0.64$, using Table B.11, pp 1356–1360

In other words, there is a 64% chance that the factor B effects will be detected by this test when these effects are different in the assumed way.

(26.12) Programmer requirements, two factor ANOVA, Table B.11

Since, assume (guess) parameter $\sigma = 9.0$,

and assume (guess) parameters $\beta_1 = 15$, $\beta_2 = -5$ and $\beta_3 = -10$,

$$\phi = \frac{1}{\sigma} \sqrt{\frac{na \sum \beta_j^2}{b}} = \frac{1}{9.0} \sqrt{\frac{4(2)(15^2 + (-5)^2 + (-10)^2)}{3}} \approx ?$$

$$\nu_1 = b - 1 = 3 - 1 = 2$$

$$\nu_2 = ab(n - 1) = ?$$

$$\alpha = 0.05$$

and so $1 - \beta = ?$, using Table B.11, page 1356

In other words, there is a ?% chance that the mean effects will be detected by this test, when, in fact, these effects are different in the assumed way.

(26.18) single factor ANOVA, Table B.12

(a) since $r = 5$, $\alpha = 0.01$, $\beta = 0.05$ (and so $1 - \beta = 0.95$), $\sigma = 10$,

Δ	10	15	20	30
$\frac{\Delta}{\sigma}$	1.0	1.5	2.0	3.0
n (Table B.12)	51	23	?	?

In other words, as the range of the treatments means, Δ , becomes bigger, the required sample size, n , necessary to achieve α and β becomes smaller.

(b) as in (a), only $\alpha = 0.05$,

Δ	10	15	20	30
$\frac{\Delta}{\sigma}$	1.0	1.5	2.0	3.0
n (Table B.12)	?	?	?	?

In other words, as the range of the treatments means, Δ , becomes bigger, the required sample size, n , necessary to achieve α and β becomes ?.

(26.24) Premium distribution, single factor ANOVA, Tables B.12, CI and Table B.13

(a) Table B.12

since $r = 5$, $\alpha = 0.10$, $1 - \beta = 0.95$, $\sigma = 10$, $\Delta = 3.75$,

so $\frac{\Delta}{\sigma} = 1.25$

and so $n = ?$, using Table B.12

In other words, a sample size of $n = ?$ is required to achieve the various requirements of this test.

(b) using confidence interval width to determine sample size n

since, $g = 4$, $\alpha = 0.10$, $n_T = 100$, $r = 5$,

for Bonferroni, $t(1 - \frac{\alpha}{2g}; n_T - r) = ?$

and since $\sigma = 3.0$,

and so, for example, for $L_1 = \mu_1 - \mu_2$, and assuming $n = 90$,

$\sigma\{\hat{L}_1\} = \sqrt{\frac{\sigma^2}{n} \sum c_i^2} = \sqrt{\frac{3^2}{90}((-1)^2 + (1)^2)} \approx 0.4472$

(where, remember, c_i are the coefficients of the contrast)

Trying $n = 90, 92, 100$,

contrast	$\sigma\{\hat{L}\}, n = 90$	$\sigma\{\hat{L}\}, n = 92$	$\sigma\{\hat{L}\}, n = 100$
$\mu_1 - \mu_2$	0.4472	?	?
$\mu_3 - \mu_4$	0.4472	?	?
$\frac{\mu_1 + \mu_2}{2} - \mu_5$	0.3873	?	?
$\frac{\mu_1 + \mu_2}{2} - \frac{\mu_3 + \mu_4}{2}$	0.3162	0.3128	?
maximum	0.4472	0.4423	?
maximum width of CI	$\pm(0.4472)(2.27748) = \pm 1.018$	± 1.0078	$\pm ?$

In other words, the width of the CI is closest to ± 1.000 when $n = ?$

(c) Table B.13

since $r = 5$, $\lambda = 1.0$, $1 - \alpha = 0.90$, $\sigma = 3.0$,

so $\frac{\lambda\sqrt{n}}{\sigma} = \frac{(1.0)\sqrt{n}}{3} = 2.5997$

(where 2.5997 comes from Table B.13)

and so, rearranging, $n \approx ?$

In other words, a sample size of $n = ?$ is required to achieve the various requirements of this test.

(26.31) Premium distribution, single factor ANOVA, Tables B.12, CI and Table B.13

since $r = ?$, $\lambda = 8.0$, $1 - \alpha = 0.95$, $\sigma = 9.1$,

so $\frac{\lambda\sqrt{n}}{\sigma} = \frac{(8.0)\sqrt{n}}{9.1} = ?$

(where ? comes from Table B.13)

and so, rearranging, $n \approx 13$

In other words, a sample size of $n = ?$ is required to achieve the various requirements of this test.