Quiz Practice Questions 7 (Attendance 14) for Statistics 514
Design of Experiments
Chapter 32 Neter et al. and Kuhn

These are practice questions for the quiz. The quiz (not the practice questions) is worth 5% and marked out of 5 points. One or more questions is closely, but not necessarily exactly, related to one or more of these questions will appear on the quiz. These practice questions are *not* to be handed in. Quizzes are to be done *using Vista* on the Internet **before** 4am (West Lafayette time!) of the date of the quiz. Vista will *not* allow any quiz to be done late. It is *highly* recommended that you complete this practice quiz, by hand, *before* logging onto Vista. The quiz is an **individual** one which means that each student does this quiz by themselves without help from others.

Applied Linear Statistical Models (Neter et al.) Questions.

Chapter	Problem(s)	hints
32, pages 1306–1309	32.7	
	32.13	Whipped topping

(32.7) Five–factor response surface

(a) *Number of regression coefficients*
The response function is given by

$$\begin{aligned}
Y_i = \beta_0 X_0 &+ \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_5 X_{i5} \\
&+ \beta_{11} X_{i1}^2 + \beta_{22} X_{i2}^2 + \beta_{33} X_{i3}^2 + \beta_{44} X_{i4}^2 + \beta_{55} X_{i5}^2 \\
&+ \beta_{12} X_{i1} X_{i2} + \beta_{13} X_{i1} X_{i3} + \beta_{14} X_{i1} X_{i4} + \beta_{15} X_{i1} X_{i5} \\
&+ \beta_{23} X_{i2} X_{i3} + \beta_{24} X_{i2} X_{i4} + \beta_{25} X_{i2} X_{i5} \\
&+ \beta_{34} X_{i3} X_{i4} + \beta_{35} X_{i3} X_{i5} + \beta_{45} X_{i4} X_{i5} + \varepsilon_{ijk}
\end{aligned}$$

and so there are ? regression coefficients.

(b) *Linear, quadratic and two–factor terms?*
There are
? linear main effects
? quadratic main effects
? two–factor effects

(c) *Number of design points*
Need at least 21 design points to estimate the 21 regression coefficients. Need at least ? design points for a five–factor central composite design that is based on a 2_V^{5-1} fractional factorial design because

$$n_T = 2^{k-f} n_c + 2kc_s + n_0 = \,?$$

where there are $n_c = c_s = n_0 = 1$ replications at the corner points, star points and center points, respectively.

(32.13) Whipped topping: qz7-32-13-topping-response

(a) *Two–factor rotatable central composite design*
The second–order response function is

$$\begin{aligned} Y_i = \beta_0 X_0 &+ \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_{12} X_{i1} X_{i2} \\ &+ \beta_{11} X_{i1}^2 + \beta_{22} X_{i2}^2 + \varepsilon_{ijk} \end{aligned}$$

where, from SAS, the estimated coefficients are

coefficient	b_q	p–value
b_0	189.75	< 0.0001
b_1	28.247	0.0008
b_2	-0.772	0.87
b_{12}	?	0.0748
b_{11}	-18.128	0.0115
b_{22}	-6.875	0.223

It looks like an appropriate hierarchical model would be

$$Y_i = \beta_0 X_0 + \beta_1 X_{i1} + \beta_{11} X_{i1}^2 + \varepsilon_{ijk}$$

since b_1 is significant (active), but b_2 is not significant.

(b) *Residuals*
Look at the SAS output.

(c) *Lack of fit test*

From SAS, the ANOVA table is

Source	Sum Of Squares	Degrees of Freedom	Mean Squares
Regression	9316.05569	5	1863.2114
Error	978.860975	6	163.1435
Lack of Fit	?	3	?
Pure Error	?	3	?
Total	10295	11	

$H_0: \ \mu = \beta_0 X_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_{12} X_{i1} X_{i2} + \beta_{11} X_{i1}^2 + \beta_{22} X_{i2}^2$ versus

$H_a: \ \mu \neq \beta_0 X_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_{12} X_{i1} X_{i2} + \beta_{11} X_{i1}^2 + \beta_{22} X_{i2}^2$

(In other words, $H_o:$ no lack of fit versus $H_a:$ lack of fit)

The test statistic is

$$\begin{aligned} F^* &= \frac{SSE - SSPE}{df_E - df_{PE}} \div \frac{SSPE}{df_{PE}} \\ &= \frac{SSLF}{df_{LF}} \div \frac{SSPE}{df_{PE}} \\ &= \frac{?}{3} \div \frac{?}{3} \\ &= 3.24 \end{aligned}$$

The critical value is $F(1-\alpha; df_{LF}, df_{PE}) = F(0.99; 3, 3) = 29.5$

since $F^* = 3.84 < F = 29.5$

accept null; that is, there is *not* lack of fit

(In other words, the model appears to be a good one.)

(d) *Three–dimensional plot and contour plot for response surface*

Look at the SAS output.

(e) *Calculation of maximum for the response surface*

$$\mathbf{B} = \begin{bmatrix} -18.128 & 13.75/2 \\ 13.75/2 & -6.875 \end{bmatrix}, \quad \mathbf{b}^* = \begin{bmatrix} 28.247 \\ -0.772 \end{bmatrix}$$

and so

$$\mathbf{X_s} = -\frac{1}{2}\mathbf{B^{-1}}\mathbf{b}^* = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

(f) *Confidence interval for mean response*

From SAS,

$\hat{Y}_h = ?$

$s\{\hat{Y}_h\} = 13.70$

$t(1-\alpha/2; n-p) = t(1-0.05/2; 12-6) = 2.447$

and so the 95% confidence interval is

$\hat{Y}_h \pm t(1-\alpha/2; n-p)s\{\hat{Y}_h\} = ? \pm 13.70(2.447) = (173.1, 240.1)$