Final for Mathematics 223
Introductory Analysis I - Spring 2000
Material Covered: Chapters 1–6 of Workbook and text
For: 3rd May

This is a 2 hour final, worth 28% and marked out of 28 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): __________________________ . **ID Number:** _______________ .
last first

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1. A couple of chapter 1 questions.

(a) [1] If $g(h(f(x))) = \sqrt{\cos(\ln x)}$, then (circle one)

(i) $g(x) = \ln x, \quad h(x) = \cos x, \quad f(x) = \sqrt{x}$
(ii) $g(x) = \cos x, \quad h(x) = \sqrt{x}, \quad f(x) = \ln x$
(iii) $g(x) = \sqrt{x}, \quad h(x) = \cos x, \quad f(x) = \ln x$
(iv) $g(x) = \sqrt{x}, \quad h(x) = \ln x, \quad f(x) = \cos x$
(v) $g(x) = \cos x, \quad h(x) = \ln x, \quad f(x) = \sqrt{x}$

(b) [2] If $f(x) = \frac{1}{1-\frac{1}{1-x}}$. Then $\frac{f(\frac{1}{x})}{f(x)}$ = (circle one)

(i) $\frac{x}{1-x}$ (ii) $-\frac{1}{x}$ (iii)$1-x$ (iv) $-\frac{1}{1-x}$ (v) $-x$

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2. More chapter 1 questions.

(a) [1] The function $f(x) = 3x^{11} - 2x^9 + 7x$ is (*circle none, one or more!*)
odd / **even** / **cubic** / **a polynomial** / **composite**.

(b) [1] The function f is an even rational function then $f(x) + x^2 + 4$ is (*circle none, one or more!*)
odd / **even** / **rational** / **a polynomial** / **composite**.

3. A couple of chapter 2 questions.

(a) [2] Consider the following function

$$f(x) = \begin{cases} x+2 & \text{if } x \le 1 \\ 2x & \text{if } x > 1 \end{cases}$$

Then $\lim_{x\to 1^-} f(x) =$ ______________________________,

and $\lim_{x\to 1^+} f(x) =$ ______________________________,

and $\lim_{x\to 1} f(x) =$ ______________________________,

and $\lim_{x\to -1} f(x) =$ ______________________________.

(b) [1] If $\lim_{x\to a} f(x) = 4$, then $\lim_{x\to a}[\sqrt{f(x)} + f(x)]^2 =$
(circle one) **32** / **33** / **34** / **35** / **36**.

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4. More chapter 2 questions.

(a) [1] If $f(x) = (x^2 + x - 4)(x + \sqrt[3]{x})$, then $f'(2) =$
(circle closest one) **17.65** / **18.72** / **19.18** / **20.92** / **21.32**.

(b) [1] If $f(x) = \frac{2x^2+x-3}{x}$, then $f'(2) =$ (circle one) **0.75** / **1.25** / **1.75** / **2.25** / **2.75**.

(c) [1] If $f(x) = [g(x)]^2 + g(x)$, then $f'(x) =$ (circle one)

(i) $[g'(x)]^2$ (ii) $g'(x)[2g(x)+1]$ (iii) $2g(x)g'(x)$
(iv) $g(x) + 2g'(x)$ (v) $[g'(x)]^2 + g'(x)$

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5. A couple of chapter 3 questions.

(a) [1] The following physical process is (circle one) **continuous** / **discrete** for all $t > 0$: *Your height t hours after you were born.* The following physical process is (circle one) **continuous** / **discrete** for all $t > 0$: *The number of people inside the city limits of Chicago t hours from now.*

(b) [1] The function $f(x) = \frac{x^2-2x-8}{3x-12}$ has a removable discontinuity at $a = 4$. How should $f(a)$ be defined to remove this discontinuity?
Circle one. $\boldsymbol{f(a) = 1}$ / $\boldsymbol{f(a) = 2}$ / $\boldsymbol{f(a) = 3}$ / $\boldsymbol{f(a) = 4}$ / $\boldsymbol{f(a) = 5}$.

6. Consider the function $f(x) = x^3 - 3x^2 - 24x$.

(a) [1] Function f is increasing on the intervals (circle one)
$\mathbf{(-\infty, -4) \cup (2, \infty)}$ / $\mathbf{(-\infty, -3) \cup (4, \infty)}$ / $\mathbf{(-\infty, -2) \cup (2, \infty)}$ / $\mathbf{(-\infty, -4) \cup (4, \infty)}$ / $\mathbf{(-\infty, -2) \cup (4, \infty)}$.

(b) [1] Function f is concave up on the interval(s) (*circle none, one or more*)
$\mathbf{(1, \infty)}$ / $\mathbf{(-\infty, 1)}$ / $\mathbf{(-\infty, -1)}$ / $\mathbf{(-2, \infty)}$ / $\mathbf{(0, \infty)}$.

(c) [1] The point of inflection occurs at $x =$ (circle closest one) **1** / **2** / **3** / **4** / **5**.

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7. More chapter 3 questions.

(a) [1] The

$\lim_{x \to -\infty} \frac{2x^{3/2}+1}{x^{3/2}+5} =$ ____________________________________.

(b) [1] The vertical asymptote(s) for $f(x) = \frac{x^2-4}{x^2-2x-3}$, is/are (*circle none, one or more*)
$\boldsymbol{x = -3}$ / $\boldsymbol{x = -1}$ / $\boldsymbol{x = 1}$ / $\boldsymbol{x = 2}$ / $\boldsymbol{x = 3}$.

(c) [1] The horizontal asymptote(s) for $f(x) = \frac{x^2-4}{x^2-2x-3}$, is/are (*circle none, one or more*)
$\boldsymbol{y = -3}$ / $\boldsymbol{y = -1}$ / $\boldsymbol{y = 0}$ / $\boldsymbol{y = 1}$ / $\boldsymbol{y = 3}$.

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8. [3] A closed cylindrical can is to have a volume of 750 cubic centimeters. Find the height, h, and radius, r, of the can that will minimize the amount of sheet metal required to make it (assuming no waste). [Hint: $S = 2\pi r^2 + 2\pi rh$, where $V = 750 = \pi r^2 h$.]

9. A few chapter 5 questions.

(a) [1] Consider the equation $y = (2x+1)e^x$.

Then $\frac{dy}{dx} =$ ____.

(b) [1] Consider the equation $y = \frac{4}{(\ln x)^3}$.

Then $\frac{dy}{dx} =$ ____.

(c) [1] If $(3.2)^x = (2.8)^{x+1}$, then $x =$
(circle closest one) **6.54** / **6.78** / **7.01** / **7.32** / **7.71**.

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10. [3] Welsh Pembroke Corgis can only grow to a maximum length of 3.5 feet long. If Obie (a Welsh Pembroke Corgi) is now 2.3 feet long, how long will she be 2.7 years from now, if $k = 0.15$? Use the simple bounded model. (Circle closest one)

(i) 2.3 (ii) 2.4 (iii) 2.5 (iv) 2.6 (v) 2.7

1. (a) **(iii)**
(b) **(v)**

2. (a) **odd**, **a polynomial**
(b) **even**, **rational**, **a polynomial**.

3. (a) 3,
2,
does not exist,
1
(b) **(v) 36**

4. (a) **(ii) 18.72**
(b) **(v) 2.75**
(c) **(ii)** $g'(x)[2g(x) + 1]$

5. (a) **continuous**, **discrete**
(ii) (b) $\boldsymbol{f(a) = 2}$

6. (a) **(v)** $\boldsymbol{(-\infty, -2) \cup (4, \infty)}$
(b) **(i)** $\boldsymbol{(1, \infty)}$
(c) **(i) 1**

7. (a) does not exist
(b) **(ii), (v)** $\boldsymbol{x = -1}$, $\boldsymbol{x = 3}$
(c) **(iv)** $\boldsymbol{y = 1}$

8. $\boldsymbol{r \approx 4.92}$ and $\boldsymbol{h \approx 9.85}$

9. (a) $\boldsymbol{(2x + 1)e^x + e^x(2)}$
(b) $\boldsymbol{-\frac{12}{x(\ln x)^4}}$
(c) **(v) 7.71**

10. **(v) 2.7**