**Final for Mathematics 224**
**Introductory Analysis II - Spring 2001**
**Material Covered: Chapters 5–7, B and C of Workbook and Text**
**For: 2nd May**

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

**Name (please print):** _____ . **ID Number:** _____.
last first

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**1.** [1 point] We wish to approximate $f(5)$ for the differential equation

$$y' = xy(y - x), \quad y(0) = 1$$

Use Euler's method to complete the following table.

| $n$ | $a_n = x_0 + nh$ | $b_n = b_{n-1} + hg(a_{n-1}, b_{n-1})$ |
|---|---|---|
| 1 | $a_1 =$ _____ | $b_1 =$ _____ |
| 2 | $a_2 =$ _____ | $b_2 =$ _____ |
| 3 | $a_3 =$ _____ | $b_3 =$ _____ |
| 4 | $a_4 =$ _____ | $b_4 =$ _____ |
| 5 | $a_5 =$ _____ | $b_5 =$ _____ |

**2.** Try the following differential equation problems.

**(a)** [1 point] What is the general solution of the differential equation $y' = 5x^9 + x^9 y$? Circle one.

**(i)** $\boldsymbol{y = Ce^{\frac{1}{9}x^9} - 5}$

**(ii)** $\boldsymbol{y = Ce^{\frac{1}{10}x^{10}} - 5}$

**(iii)** $\boldsymbol{y = \frac{1}{10}x^{10} + C}$

**(iv)** $\boldsymbol{\ln y = Ce^{\frac{1}{10}x^{10}} - 5}$

**(v) none of the above**

**(b)** [2 points] What is the general solution of the differential equation $3xy' + 4y = 7$? Circle one.

**(i)** $\boldsymbol{y = C(x)^{-4/3} + \frac{7}{4}}$

**(ii)** $\boldsymbol{y = Cx^{4/3} - \frac{7}{4}}$

**(iii)** $\boldsymbol{y = Cx^{-4/3} - \frac{7}{4}}$

**(iv)** $\boldsymbol{y = C(3x)^{4/3} + \frac{7}{4}}$

**(v) none of the above**

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**3.** Try the following integration problems involving trigonometric functions.

**(a)** [1 point] $\int \sin(x+3)\,dx =$ (circle one)

**(i)** $\boldsymbol{-3\cos(x-3) + C}$

**(ii)** $\boldsymbol{-\cos(x-3) + C}$

**(iii)** $\boldsymbol{\cos(x+3) + C}$

**(iv)** $\boldsymbol{3\cos(x+3) + C}$

**(v) none of the above**

**(b)** [1 point] Evaluate $F(x) = \int \sec^2(6x)\,dx$ when $F(0) = 1$. Circle one.

**(i)** $\boldsymbol{\frac{1}{6}\tan(6x)}$

**(ii)** $\boldsymbol{\frac{1}{6}\tan(6x) + 1}$

**(iii)** $\boldsymbol{\tan(6x)}$

**(iv)** $\boldsymbol{\tan(6x) + 1}$

**(v) none of the above**

**4.** Try the following integration problems.

**(a)** [1 point] An appliance company determines that the marginal cost of the $x$th appliance is given by (in thousands of dollars)

$$C'(x) = -0.002x + 1.75, \;\; C(0) = 500$$

Find the cost of producing 750 appliances. Circle one.

**(i) $\$750,000$**

**(ii) $\$1,000,000$**

**(iii) $\$1,250,000$**

**(iv) $\$1,500,000$**

**(v) none of the above**

**(b)** [1 point] A particle is initially at the origin. Its velocity, in meters per second, at any time $t$, $t \geq 0$, is given by $v(t) = 3t^2 + 4t$. Find the distance that the particle travels in the first 5 seconds (from $t = 0$ to $t = 5$). Circle one.

**(i) 125**

**(ii) 150**

**(iii) 175**

**(iv) 200**

**(v) none of the above**

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**5.** Use the integration by parts method to solve the following problems.

**(a)** [1 point] $\int xe^{7x}\,dx =$ (circle one)

**(i)** $\frac{1}{7}e^{7x} - \frac{1}{49}e^{7x} + C$

**(ii)** $\frac{1}{7}x^2e^{7x} - \frac{1}{49}e^{7x} + C$

**(iii)** $\frac{1}{7}xe^{7x} - \frac{1}{49}e^{7x} + C$

**(iv)** $\frac{1}{7}xe^{7x} - \frac{1}{7}e^{7x} + C$

**(v)** $\frac{1}{7}xe^{7x} - \frac{1}{49}x^2e^{7x} + C$

**(b)** [2 points] $\int x^5 \ln x^6\,dx =$ _____

**6.** Try the following integration problems.

**(a)** [1 point] $\int_{-2}^{y}(5x-4x^3)\,dx =$ (circle one)

**(i)** $\frac{5}{2}y^2 - y^4 - 6$

**(ii)** $\frac{5}{2}y^2 - y^4 - 3$

**(iii)** $\frac{5}{2}y^2 - y^4$

**(iv)** $\frac{5}{2}y^2 - y^4 + 3$

**(v)** $\frac{5}{2}y^2 - y^4 + 6$

**(b)** [1 point] If

$$f(x) = \begin{cases} 2 & \text{if } x > 1 \\ 3x^2 - 5 & \text{if } x \leq 1 \end{cases}$$

then $\int_0^2 f(x)\,dx =$ (circle one)
(circle closest one) $\mathbf{-3}$ / $\mathbf{-2}$ / $\mathbf{-1}$ / $\mathbf{0}$ / $\mathbf{1}$.

**(c)** [1 point] Find the area of the region bounded by $y = x$, $x = 0$, $y = x^5$ and $x = 1$
(circle closest one) $\mathbf{-\frac{2}{3}}$ / $\mathbf{-\frac{1}{3}}$ / $\mathbf{\frac{1}{3}}$ / $\mathbf{\frac{2}{3}}$ / $\mathbf{1}$.

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**7.** Consider the following demand and supply functions

$$p = D(x) = (x-6)^2; \quad p = S(x) = x^2 + 5x$$

**(a)** [1 point] The equilibrium point is (circle closest one)

**(i)** $\mathbf{(2.12, 15.07)}$

**(ii)** $\mathbf{(3.12, 14.07)}$

**(iii)** $\mathbf{(4.12, 13.07)}$

**(iv)** $\mathbf{(5.12, 12.07)}$

**(v)** $\mathbf{(6.12, 11.07)}$

**(b)** [1 point] The consumer's surplus at the equilibrium point is
(circle closest one) **13.12** / **13.96** / **14.82** / **17.64** / **20.58**.

**(c)** [1 point] The producer's surplus at the equilibrium point is
(circle closest one) **11.02** / **14.26** / **14.72** / **16.63** / **17.54**.

**8.** Try the following improper integration problems.

**(a)** [1 point] $\int_2^\infty 2x^4 e^{x^5}\,dx =$ (circle one)

**(i)** $\frac{1}{5}\left[\lim_{b\to\infty} e^{b^5} - e^{2^5}\right]$

**(ii)** $\frac{2}{5}\left[\lim_{b\to\infty} e^{b^5} - e^{3^5}\right]$

**(iii)** $\frac{3}{5}\left[\lim_{b\to\infty} e^{b^5} - e^{2^5}\right]$

**(iv)** $\frac{2}{5}\left[\lim_{b\to\infty} e^{b^5} - e^{1}\right]$

**(v)** $\frac{2}{5}\left[\lim_{b\to\infty} e^{b^5} - e^{2^5}\right]$

**(b)** [1 point] Use your calculator to approximate the integral to within two decimal points of accuracy:

$$\int_{-\infty}^{\infty} \frac{4}{x^2+6}\,dx$$

Circle one. **5.12** / **5.13** / **5.14** / **5.15** / **5.16**.

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**9.** Try the following double integration problems.

**(a)** [1 point] $\int_0^3 \int_0^2 (4x^3 - 3y^2)\,dy\,dx =$ (circle closest one)

**(i) 98**

**(ii) 108**

**(iii) 118**

**(iv) 128**

**(v) 138**

**(b)** [1 point] $\int_0^3 \int_0^x (4x^3 - 3y^2)\,dy\,dx =$ (circle closest one)

**(i) 174.15**

**(ii) 184.15**

**(iii) 194.15**

**(iv) 204.15**

**(v) 214.15**

**10.** Consider the function $f(x, y) = 2xy - 4x^2 + y^2$.

**(a)** [1 point] This function has a saddlepoint at $(x, y) =$ (circle one)

**(i)** $\mathbf{(-1, -1)}$

**(ii)** $\mathbf{(0, 0)}$

**(iii)** $\mathbf{(1, 1)}$

**(iv)** $\mathbf{(2, 2)}$

**(v)** $\mathbf{(3, 3)}$

**(b)** [2 points] Using the *D–test*, $D =$ (circle one)

**(i)** $\mathbf{-20}$

**(ii)** $\mathbf{-10}$

**(iii)** $\mathbf{0}$

**(iv)** $\mathbf{10}$

**(v)** $\mathbf{20}$

**(c)** [1 point] Use the Lagrange method to determine the maximum value of this function, subject to the constraint $x - y = 40$.

**1.** Euler's method

| $n$ | $a_n = x_0 + nh$<br>$a_n = 0 + n$ | $b_n = b_{n-1} + hg(a_{n-1}, b_{n-1})$<br>$b_n = b_{n-1} + (a_{n-1}b_{n-1}(b_{n-1} - a_{n-1}))$ |
|---|---|---|
| 1 | $a_1 = 1$ | $b_1 = 1$ |
| 2 | $a_2 = 2$ | $b_2 = 1$ |
| 3 | $a_3 = 3$ | $b_3 = -1$ |
| 4 | $a_4 = 4$ | $b_4 = 11$ |
| 5 | $a_5 = 5$ | $b_5 = 319$ |

To use the calculator, use MODE Seq, then
$nMin = 0$
$u(n) = 0 + 1n$
$u(nMin) = \{0\}$
$v(n) = v(n-1) + u(n-1)v(n-1)(v(n-1) - u(n-1))$
$v(nMin) = 1$

**2.** (a) $\boldsymbol{(ii)\ y = Ce^{\frac{1}{10}x^{10}} - 5}$; (b) $\boldsymbol{(i)\ \frac{7}{4} + Cx^{-4/3}}$.

**3.** (a) **(v) none of the above** $(-\cos(x+3) + C)$; (b) $\boldsymbol{(ii)\ \frac{1}{6}\tan(6x) + 1}$

**4.** (a) $\boldsymbol{(iii)\ \$1,250,000}$; (b) $\boldsymbol{(iii)\ 175}$.

**5.** (a) $\boldsymbol{(iii)\ \frac{1}{7}xe^{7x} - \frac{1}{49}e^{7x} + C}$; (b) $\boldsymbol{\frac{1}{6}x^6 \ln x^6 - \frac{1}{6}x^6 + C}$

**6.** (a) $\boldsymbol{(v)\ \frac{5}{2}y^2 - y^4 + 6}$ (b) $\boldsymbol{-2}$ (c) $\boldsymbol{\frac{1}{3}}$.

**7.** (a) $\boldsymbol{(i)\ (2.12, 15.07)}$ (b) **20.58** (c) **17.54**

**8.** (a) $\boldsymbol{(v)\ \frac{2}{5}\left[\lim_{b\to\infty} e^{b^5} - e^{2^5}\right]}$ (b) **5.13**

**9.** (a) $\boldsymbol{(v)\ 138}$ (b) $\boldsymbol{(i)\ 174.15}$

**10.** (a) $\boldsymbol{(ii)\ (0,0)}$; (b) $\boldsymbol{(i)\ -20}$; (c) $\boldsymbol{(x,y) = (-80, -120)}$.