Final for Statistics 225
Introduction To Probability Models–Spring 2002
Material Covered: Chapters 1-9.2 of Workbook and Text
29th April

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): _____ . **ID Number:** _____.
last first

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1. Consider the following joint probability density:

$$f(x) = \begin{cases} \frac{e^{-x/y}e^{-y}}{y} & \text{if } x > 0, y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

[Hint: $\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$.]

(a) [1 point] $\int xf(x,y)\,dx =$
(circle one) $\boldsymbol{y}$ / $\boldsymbol{e^{-y}}$ / $\boldsymbol{ye^{-y}}$ / $\boldsymbol{y^{-1}e^{-y}}$ / $\boldsymbol{y^{-2}e^{-y}}$

(b) [1 point] $f_{X|Y}(x|y) =$
(circle one) $\boldsymbol{y^{-1}}$ / $\boldsymbol{e^{-x/y}}$ / $\boldsymbol{ye^{-x/y}}$ / $\boldsymbol{y^{-1}e^{-x/y}}$ / $\boldsymbol{y^{-2}e^{-x/y}}$

(c) [1 point] $E(X|Y=3) =$
(circle closest one) $\boldsymbol{\frac{1}{9}}$ / $\boldsymbol{\frac{1}{3}}$ / **3** / **6** / **9**

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2. For every five flips of a coin, a head comes up three (3) times and a tail comes up two (2) times.

(a) [1 point] The chance of flipping seven heads in a row is
(circle closest one) $\boldsymbol{\frac{2185}{78125}}$ / $\boldsymbol{\frac{2186}{78125}}$ / $\boldsymbol{\frac{2187}{78125}}$ / $\boldsymbol{\frac{2188}{78125}}$ / $\boldsymbol{\frac{2189}{78125}}$.

(b) [1 point] The chance of flipping three heads, followed by four tails, is
(circle closest one) $\boldsymbol{\frac{432}{78125}}$ / $\boldsymbol{\frac{433}{78125}}$ / $\boldsymbol{\frac{434}{78125}}$ / $\boldsymbol{\frac{435}{78125}}$ / $\boldsymbol{\frac{436}{78125}}$.

3. Try the following combinatorial questions.

(a) [1 point] The number of ways that 3 students and 4 faculty members can sit in a row if all students must be separated by one faculty member is
(circle closest one) **144** / **288** / **432** / **576** / **720**.

(b) [1 point] The number of different arrangements that can be made from the letters PURDUE is
(circle closest one) **360** / **720** / **1080** / **1440** / **1800**.

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4. [1 point] The random variable X has the following probability density.

$$f(x) = \begin{cases} \frac{1}{3}, & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Draw a graph of the cumulative distribution of X.

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5. A STAT 225 student estimates she will achieve an A grade with an 80 percent chance if she studies 10 hours per week, with a 60 percent chance if she studies 7 hours per week and with a 40 percent chance if she studies 4 hours per week. She further believes that she will study 10 hours, 7 hours and 4 hours per week with probability 0.1, 0.2 and 0.7, respectively.

(a) [1 point] What is the chance she will achieve an A grade?
(circle closest one) **0.18** / **0.28** / **0.38** / **0.48** / **0.58**.

(b) [1 point] Given that she achieves an A grade, what is the chance she studied for four (4) hours?
(circle closest one) $\mathbf{\frac{6}{12}}$ / $\mathbf{\frac{7}{12}}$ / $\mathbf{\frac{8}{12}}$ / $\mathbf{\frac{9}{12}}$ / $\mathbf{\frac{10}{12}}$.

(c) [1 point] Given that she does *not* achieve an A grade, what is the chance she studied for four (4) hours?
(circle closest one) $\mathbf{\frac{17}{26}}$ / $\mathbf{\frac{18}{26}}$ / $\mathbf{\frac{19}{26}}$ / $\mathbf{\frac{20}{26}}$ / $\mathbf{\frac{21}{26}}$.

6. A small store owner buys loaves of bread at \$1.05 per loaf wholesale, and sells it for \$1.45 per loaf. His daily demand is a binomial random variable where $n = 5$ and $p = \frac{2}{3}$.

(a) [1 point] The expected number of loaves sold per week is:
(circle closest one) $\mathbf{\frac{10}{3}}$ / $\mathbf{\frac{11}{3}}$ / $\mathbf{\frac{12}{3}}$ / $\mathbf{\frac{13}{3}}$ / $\mathbf{\frac{14}{3}}$.

(b) [1 point] Complete the following probability distribution.

X	0	1	2	3	4	5
$P(X = x)$	_____	_____	_____	_____	_____	_____

(c) [1 point] How many loaves should be sold so that the store owner maximizes his *expected* profit? (circle closest one) **1** / **2** / **3** / **4** / **5**.

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7. Consider the following questions on the normal random variable X.

(a) [1 point] Assume $\mu = 50$ and $\sigma = 20$, then $P(30 < X < 80) =$
(circle closest one) **0.715** / **0.735** / **0.775** / **0.795** / **0.815**.

(b) [1 point] Assume $\mu = 40$ and $P(X > 70) = 0.1$, then $\sigma =$
(circle closest one) **16.38** / **19.71** / **21.11** / **23.41** / **25.25**.

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8. The joint density of X and Y is

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) [1 point] $f_X(x) =$ (circle closest one)
$\mathbf{\frac{2}{5}(2x + \frac{1}{2})}$ / $\mathbf{\frac{2}{5}(2x + \frac{2}{2})}$ / $\mathbf{\frac{2}{5}(2x + \frac{3}{2})}$ / $\mathbf{\frac{2}{5}(2x + \frac{4}{2})}$ / $\mathbf{\frac{2}{5}(2x + \frac{5}{2})}$.

(b) [1 point] $F_X(x) =$ (circle closest one)
$\mathbf{\frac{2}{5}(x^2 + \frac{1}{2}x)}$ / $\mathbf{\frac{2}{5}(x^2 + \frac{2}{2}x)}$ / $\mathbf{\frac{2}{5}(x^2 + \frac{3}{2}x)}$ / $\mathbf{\frac{2}{5}(x^2 + \frac{4}{2}x)}$ / $\mathbf{\frac{2}{5}(x^2 + \frac{5}{2}x)}$.

(c) [1 point] $f_{Y|X}(y|x) =$ (circle closest one)
$\mathbf{\frac{4x+6y}{5x+3}}$ / $\mathbf{\frac{4x+6y}{4x+3}}$ / $\mathbf{\frac{4x+6y}{3x+3}}$ / $\mathbf{\frac{4x+6y}{2x+3}}$ / $\mathbf{\frac{4x+6y}{x+3}}$.

9. A gambler places a \$1 bet on the second section of a roulette table coming up on the roulette wheel. There is a $\frac{26}{38}$ chance the gambler will lose the dollar, $x = -\$1$, and a $\frac{12}{38}$ chance the gambler will keep the dollar and win $x = \$2$ more. The gambler's utility is given by

$$f(x) = 3x^3 - 2x^2 + 5x$$

(a) [1 point] The gambler's utility is convex when
(circle closest one) $\boldsymbol{x < \frac{4}{18}}$ / $\boldsymbol{x < \frac{6}{18}}$ / $\boldsymbol{x > \frac{2}{18}}$ / $\boldsymbol{x > \frac{4}{18}}$ / $\boldsymbol{x > \frac{6}{18}}$

(b) [1 point] $E[f(x)] =$
(circle closest one) $\boldsymbol{\frac{22}{19}}$ / $\boldsymbol{\frac{23}{19}}$ / $\boldsymbol{\frac{24}{19}}$ / $\boldsymbol{\frac{25}{19}}$ / $\boldsymbol{\frac{26}{19}}$

(c) [1 point] $f[E(x)] =$
(circle closest one) $\mathbf{-0.369}$ / $\mathbf{-0.269}$ / $\mathbf{-0.169}$ / $\mathbf{-0.069}$ / $\mathbf{0.169}$

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10. Consider the following exponential density:

$$f(x) = \begin{cases} 3e^{-3x} & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) [1 point] For $t < 3$, $M(t) =$
(circle closest one) $\boldsymbol{e^{3t}}$ / $\boldsymbol{3e^{3t}}$ / $\boldsymbol{(3-t)^{-1}}$ / $\boldsymbol{3(3-t)^{-1}}$ / $\boldsymbol{3(3-t)^{-2}}$

(b) [1 point] $M'''(t) =$
(circle closest one) $\boldsymbol{27e^{3t}}$ / $\boldsymbol{81e^{3t}}$ / $\boldsymbol{9(3-t)^{-1}}$ / $\boldsymbol{12(3-t)^{-3}}$ / $\boldsymbol{18(3-t)^{-4}}$

(c) [1 point] $E[X^3] =$
(circle closest one) $\boldsymbol{\frac{2}{9}}$ / $\boldsymbol{\frac{3}{9}}$ / $\boldsymbol{\frac{4}{9}}$ / $\boldsymbol{\frac{5}{9}}$ / $\boldsymbol{\frac{6}{9}}$

(1) (a) $\boldsymbol{ye^{-y}}$ (b) $\boldsymbol{y^{-1}e^{-x/y}}$ (c) $\mathbf{3}$

(2) (a) $\boldsymbol{\frac{2187}{78125}}$ (b) $\boldsymbol{\frac{432}{78125}}$

(3) (a) $\mathbf{432}$ (b) $\mathbf{360}$

(4) $F(x) = \frac{1}{3}x$, $0 \leq x \leq 3$, and zero elsewhere

(5) (a) $\mathbf{0.48}$ (b) $\boldsymbol{\frac{7}{12}}$

(6) (a) $\boldsymbol{\frac{10}{3}}$
(b) 1/243, 10/243, 40/243, 80/243, 80/243, 32/243
(c) $\mathbf{4}$

(7) (a) $\mathbf{0.775}$ (b) $\mathbf{23.41}$

(8) (a) (iii) $\boldsymbol{\frac{2}{5}(2x + \frac{3}{2})}$
(iii) $\boldsymbol{\frac{2}{5}(x^2 + \frac{3}{2}x)}$
(ii) $\boldsymbol{\frac{4x+6y}{4x+3}}$

(9) (a) $\boldsymbol{x > \frac{4}{18}}$ (b) $\boldsymbol{\frac{26}{19}}$ (c) $\mathbf{-0.269}$

(10) (a) $\boldsymbol{3(3-t)^{-1}}$ (b) $\boldsymbol{18(3-t)^{-4}}$ (c) $\boldsymbol{\frac{2}{9}}$