Final for Mathematics 224
Introductory Analysis II - Spring 2002
Material Covered: Chapters 5–7, B and C of Workbook and Text
29th April

This is a 2 hour final, worth 25% and marked out of 25 points. The total possible points awarded for each question is given in square brackets at the beginning of each question. Anything that can fit on two sides of an $8\frac{1}{2}$ by 11 inch piece of paper may be used as a reference during this quiz. A calculator may also be used. No other aids are permitted.

Name (please print): ______________________ . **ID Number:** __________.
last first

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1. Consider the following integration problems.

(a) [1 point] Use the method of substitution to evaluate $\int \frac{21x^6}{3x^7+4}\,dx$. Do not use the table of integrations. Circle one.

(i) $\ln(21x^6) + C$
(ii) $\ln(3x^7 + 4) + C$
(iii) $(3x^7 + 4)^{-2} + C$
(iv) $21x^6(3x^7 + 4)^{-2} + C$
(v) **none of the above**

(b) [1 point] Use the method of integration by parts to evaluate $\int x^3 \ln x^3\,dx$. Do not use the table of integrations. Circle one.

(i) $\frac{3}{4}x^4\left(\ln x^3 - 1\right)$
(ii) $\frac{1}{3}x^3\left(\ln x^3 - \frac{3}{4}\right)$
(iii) $\frac{1}{12}x^4\left(\ln x^3 - \frac{3}{4}\right)$
(iv) $\frac{1}{4}x^4\left(\ln x^3 - \frac{3}{4}\right)$
(v) **none of the above**

(c) [1 point] Use the table of integrations to evaluate $\int \frac{1}{x(4-x)}\,dx$. Circle one.

(i) $\frac{1}{4}\ln\left(\frac{x}{4-x}\right) + C$ (ii) $-\ln\left(\frac{x}{-x+4}\right) + C$ (iii) $\ln\left(\frac{x}{4x-1}\right) + C$
(iv) $-\ln\left(\frac{1}{4x-1}\right) + C$ (v) **none of the above**

2. Consider the following function.

$$f(x) = 6 - x^2$$

(a) [1 point] $\int_{-2}^{1} f(x)\,dx =$ (circle closest one) **12** / **13** / **14** / **15** / **16**.

(b) [1 point] Approximate $\int_{-2}^{1} f(x)\,dx$ with $\sum_{i=1}^{3} f(x_i)\Delta x$ where $a = x_1 = -2$, $x_2 = -1$, $x_3 = 0$ and $b = 1$.
Circle closest one. **12** / **13** / **14** / **15** / **16**.

(c) [1 point] Approximate $\int_{-2}^{1} f(x)\,dx$ with $\sum_{i=1}^{6} f(x_i)\Delta x$ where $a = x_1 = -2$, $x_2 = -1.5$, $x_3 = -1$, $x_4 = -0.5$, $x_5 = 0$, $x_6 = 0.5$ and $b = 1$.
Circle closest one. **14.125** / **14.250** / **14.375** / **14.500** / **14.625**.

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3. Consider the following questions on the normal random variable x.

(a) [1 point] Assume $\mu = 50$ and $\sigma = 20$, then $P(30 < x < 80) =$
(circle closest one) **0.715** / **0.735** / **0.775** / **0.795** / **0.815**.

(b) [1 point] Assume $\mu = 40$ and $P(x > 70) = 0.1$, then
$\sigma =$ (circle closest one) **16.38** / **19.71** / **21.11** / **23.41** / **25.25**.
(Hint: How is a nonstandard normal random variable x transformed into a standard one, z?)

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4. Consider the following questions concerning functions of several variables.

(a) [1 point] $\int_0^1 \int_{3-x}^{5+x} \int_0^{x^2} z\,dz\,dy\,dx =$ (circle closest one) $\frac{1}{12}$ / $\frac{3}{12}$ / $\frac{5}{12}$ / $\frac{7}{12}$ / $\frac{9}{12}$.

(b) [1 point] If $f(x, y) = e^x + 3x^4y^2 + 2y$, then $\frac{\partial^2 f}{\partial x \partial y} =$ (circle one)

(i) $\boldsymbol{12x^3y}$

(ii) $\boldsymbol{24x^2y^2}$

(iii) $\boldsymbol{36x^2}$

(iv) $\boldsymbol{24x^3y}$

(v) **none of the above**

5. Consider the following questions concerning trigonometric functions.

(a) [1 point] Differentiate $f(x) = \frac{1+\sin 2x}{\cos x}$. Circle one.

- **(i)** $\frac{\mathbf{2\cos x\cos 2x+\sin x\sin 2x+\sin x}}{\mathbf{\cos^2 x}}$
- **(ii)** $\frac{\mathbf{2\cos x\cos 2x-\sin x\sin 2x+\sin x}}{\mathbf{\cos^2 x}}$
- **(iii)** $\frac{\mathbf{2\cos x\cos 2x-\cos x\sin 2x+\sin x}}{\mathbf{\cos^2 x}}$
- **(iv)** $\frac{\mathbf{2\sin x\cos 2x-\sin x\sin 2x+\sin x}}{\mathbf{\cos^2 x}}$
- **(v) none of the above**

(b) [1 point] If $f(x, y) = \sin 2xy$, then $f_{xy} =$ (circle one)

- **(i)** $\mathbf{\cos 2xy(2 - xy)}$
- **(ii)** $\mathbf{\cos 2xy(2 - 2xy)}$
- **(iii)** $\mathbf{\cos 2xy(2 - 3xy)}$
- **(iv)** $\mathbf{\cos 2xy(2 - 4xy)}$
- **(v) none of the above**

(c) [1 point] Integrate by parts $\int 2x \cos x \, dx$. Circle one.

- **(i)** $\mathbf{2x\sin x - 2\cos x + C}$
- **(ii)** $\mathbf{2x\sin x + \cos x + C}$
- **(iii)** $\mathbf{x\sin x + 2\cos x + C}$
- **(iv)** $\mathbf{2x\sin x + 2\cos x + C}$
- **(v) none of the above**

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6. Consider the multivariate function (surface)

$$f(x, y) = 4x^2 + y^2 - 3xy$$

(a) [1 point] The surface $f(x, y)$ has a minimum at the point $(a, b) =$ (circle closest one) $\mathbf{(-1, 0)}$ / $\mathbf{(-1, 1)}$ / $\mathbf{(0, -1)}$ / $\mathbf{(0, 1)}$ / $\mathbf{(0, 0)}$.

(b) [1 point] The surface $f(x, y)$ has a minimum at point (a, b) where the D–test value, $D =$ (circle closest one) **4** / **5** / **6** / **7** / **8**.

(c) [1 point] The surface $f(x, y)$ subject to the constraint $xy = 1$ has the Lagrange multiplier, $\lambda =$ (circle closest one) $\mathbf{-1}$ / **0** / **1** / **2** / **3**.

7. Consider the following differential equation,

$$(3x^2+5)\frac{dy}{dx} = (2y+1)6x$$

(a) [1 point] Determine the general solution to this differential equation using the separation of variables method. Circle one.

(i) $\frac{1}{2}\ln(2y+1) - \ln(3x^2+5) + C = 0$

(ii) $\ln(2y+1) - \ln(3x^2+5) + C = 0$

(iii) $\frac{3}{2}\ln(2y+1) - \ln(3x^2+5) + C = 0$

(iv) $2\ln(2y+1) - \ln(3x^2+5) + C = 0$

(v) none of the above

(b) [1 point] The constant of integration for the particular solution to this differential equation at $(x, y) = (1, 1)$ is
$C =$ (circle closest one) $\mathbf{-1.53}$ / $\mathbf{-0.53}$ / $\mathbf{1.53}$ / $\mathbf{2.53}$ / $\mathbf{3.53}$.

(c) [1 point] This differential equation is a first order linear differential equation, $a_1(x)y' + a_0(x)y = g(x)$, where $P(x) = \frac{a_0(x)}{a_1(x)}$ and so
$u(x) = e^{\int P(x)\,dx} =$ (circle one)

(i) $(3x^2+5)^{-2}$

(ii) $(3x^2+5)^{-1}$

(iii) $(3x^2+5)$

(iv) $(3x^2+5)^2$

(v) none of the above

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8. Consider the following questions on the volume of the solid of revolution.

(a) [1 point] $\int_0^1 \pi\left(\sqrt{xe^{-x}}\right)^2 dx =$ (circle closest one)
$\pi\left(1-\frac{2}{e^2}\right)$ / $\pi\left(1+\frac{2}{e}\right)$ / $\pi\left(1-\frac{2}{e}\right)$ / $\pi\left(1-\frac{1}{e}\right)$ / **divergent**.

(b) [1 point] $\int_{-1}^{\infty} \pi\left(\frac{1}{(x+2)^{3/5}}\right)^2 dx =$ (circle closest one)
-5π / -3π / 5π / 6π / **divergent**.

9. We wish to approximate $f(10)$ for the differential equation

$$y' = (y - x)(y + x), \quad y(0) = 1$$

in $N = 5$ steps.

(a) [1 point] In this case, $a_0 = x_0 = 0$, $a_N = a_5 = x_1 = 10$, $b_0 = y_0 = 1$, $g(a_{n-1}, b_{n-1}) = (b_{n-1} - a_{n-1})(b_{n-1} + a_{n-1})$ and $h =$ (circle closest one) **0.5 / 1 / 1.5 / 2 / 2.5**.

(b) [1 point] Use Euler's method to complete the following table.

n	$a_n = x_0 + nh$	$b_n = b_{n-1} + hg(a_{n-1}, b_{n-1})$
1	$a_1 =$ ____________	$b_1 =$ ____________
2	$a_2 =$ ____________	$b_2 =$ ____________
3	$a_3 =$ ____________	$b_3 =$ ____________
4	$a_4 =$ ____________	$b_4 =$ ____________
5	$a_5 =$ ____________	$b_5 =$ ____________

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10. Consider the differential equation

$$\frac{dy}{dx} = x(x-2)(x-3)$$

(a) [1 point] The slope of the tangent to the curve of the differential equation is positive (increasing) if (circle none, one or more)
$\boldsymbol{x < 0}$ **/** $\boldsymbol{0 < x < 1}$ **/** $\boldsymbol{1 < x < 2}$ **/** $\boldsymbol{2 < x < 3}$ **/** $\boldsymbol{x > 3}$

(b) [1 point] The curve of the differential equation

is concave *down* if ________________________________

(1) (a) (ii) $\mathbf{\ln(3x^7+4)+C}$
(b) (iv) $\mathbf{\frac{1}{4}x^4\left(\ln x^3-\frac{3}{4}\right)}$
(c) (i) $\mathbf{\frac{1}{4}\ln\left(\frac{x}{x-4}\right)+C}$

(2) (a) **15** (b) **13** (c) **14.125**

(3) (a) **0.775** (b) **23.41**

(4) (a) $\mathbf{\frac{5}{12}}$ (closest to $\frac{11}{30}$) (b) (iv) $\mathbf{24x^3y}$

(5) (a) (i) $\mathbf{\frac{2\cos x\cos 2x+\sin x\sin 2x+\sin x}{\cos^2 x}}$
(b) (iv) $\mathbf{\cos 2xy(2-4xy)}$
(c) (iv) $\mathbf{2x\sin x+2\cos x+C}$

(6) (a) $\mathbf{(0,0)}$ (b) **7** (c) **1**

(7) (a) (i) $\mathbf{\frac{1}{2}\ln(2y+1)-\ln(3x^2+5)+C=0}$
(b) **1.53** (using the form of the answer given in part (a))
(c) (i) $\mathbf{(3x^2+5)^{-2}}$

(8) (a) $\mathbf{\pi\left(1-\frac{2}{e}\right)}$ (b) $\mathbf{5\pi}$

(9) (a) **2**
(b)

n	$a_n = x_0 + nh$ $a_n = 0 + 2n$	$b_n = b_{n-1} + hg(a_{n-1}, b_{n-1})$ $b_n = b_{n-1} + 2(b_{n-1} - a_{n-1})(b_{n-1} + a_{n-1})$
1	$a_1 = 0 + 2(1) = 2$	$b_1 = b_0 + 2(b_0 - a_0)(b_0 + a_0) = 1 + 2(1-0)(1+0) = 3$
2	$a_2 = 4$	$b_2 = 13$
3	$a_3 = 6$	$b_3 = 319$
4	$a_4 = 8$	$b_4 = 203769$
5	$a_5 = 10$	$b_5 = 8.3 \times 10^{10}$

To use the calculator, use MODE Seq, then
$nMin = 0$
$u(n) = 0 + 2n$
$u(nMin) = \{0\}$
$v(n) = v(n-1) + 2(v(n-1) - u(n-1))(v(n-1) + u(n-1))$
$v(nMin) = 1$

(10) (a) $\mathbf{0<x<1}$, $\mathbf{1<x<2}$, $\mathbf{x>3}$
(b) $\mathbf{0.784<x<2.549}$