Course Review Information
Mathematics 224

This course covers chapters 6-12 (except 6.1-6.3,7.3,7.6) in both my lecture notes and the text. There are four main parts to this course. What they are and where they are located in both the lecture notes and text are given below.

- *Differential equations* accounts for about 20% of course
  and is found in chapters 6 and 10 (lecture notes 1, 9, 10).
- *Integration* accounts for about 50% of the course
  and is found in chapters 7, 8 and 9 (lecture notes 2, 3, 4, 5, 6, 7, 8).
- *Probability* accounts for about 10% of the course
  and is found in chapter 11 (lecture notes 11).
- *Series* accounts for about 20% of the course
  and is found in chapter 12 (lecture notes 12, 13 and 14).

Chapter 6. Applications of the Derivative (notes 1)

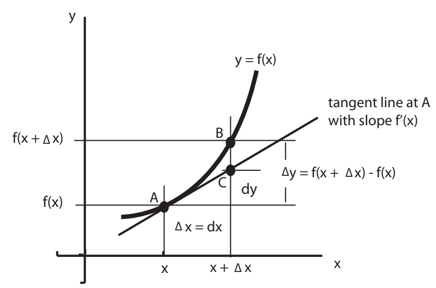
- *basic rules of differentiation*
  - *notation*: $f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}[f(x)]$, $D_x[f(x)]$
  - *constant rule*: if $f(x) = k$, k real, $f'(x) = 0$
  - *power rule*: if $f(x) = x^n$, n real, $f'(x) = nx^{n-1}$
  - *constant times function rule*: if $f(x) = k \cdot g$, k real, $f' = kg'$
  - *sum or difference rule*: if $f(x) = u \pm v$, $f'(x) = u' \pm v'$
  - *product rule*: if $f(x) = u \cdot v$, $f'(x) = v \cdot u' + u \cdot v'$
  - *quotient rule*: if $y = \frac{u}{v}$, $f'(x) = \frac{v \cdot u' - u \cdot v'}{[v]^2}$
  - *chain rule*: if $y = g[f(x)]$, $\frac{dy}{dx} = f'[g] \cdot g'$
- *special cases of differentiation*

$$\frac{d}{dx}e^x = e^x, \qquad \frac{d}{dx}a^x = (\ln a)a^x$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x} = x^{-1}, \qquad \frac{d}{dx}[\log_a|x|] = \frac{1}{(\ln a)x} = ((\ln a)x)^{-1}$$

- *implicit differentiation: finding $\frac{dy}{dx}$ without explicitly expressing y in terms of x*
  - differentiate both sides of equation
  - place all terms of $\frac{dy}{dx}$ on one side of equation; all other terms on other side
  - factor out $\frac{dy}{dx}$, solve for $\frac{dy}{dx}$
- *related rates: implicit differentiation, where all variables depend on time, t*
- *differentials and linear approximation*: point B *approximated* by point C

$$f(x + \Delta x) \approx f(x) + dy = f(x) + f'(x)dx.$$



Chapter 7. Integration (lecture notes 2, 3)

- *basic rules of integration, indefinite integrals*
  - *antiderivative* $F(x)$ is the *integral* of $f(x)$, $\int f(x)\,dx = F(x) + C$
  - *power rule* $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
  - *constant multiple rule* $\int k \cdot f(x)\,dx = k \int f(x)\,dx + C$
  - *sum or difference rule* $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
  - *exponential functions*
    * $\int e^{kx}\,dx = \frac{e^{kx}}{k} + C$, $k \neq 0$
    * $\int a^{kx}\,dx = \frac{a^{kx}}{k(\ln a)} + C$, $a > 0, a \neq 1$
  - $\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \int \frac{dx}{x} = \ln|x| + C$

  use boundary conditions to determine constant of integration, C

- *method of substitution* after substituting $u = f(x)$ (and so $du = f'(x)dx$),
  - $\int [f(x)]^n f'(x)\,dx$ becomes $\int u^n\,du = \frac{u^{n+1}}{n+1} + C$, $n \neq 1$
  - $\int e^{f(x)} f'(x)\,dx$ becomes $\int e^u\,du = e^u + C$

  - $\int \frac{f'(x)}{f(x)}\,dx$ becomes $\int \frac{1}{u}\,du = \int u^{-1}\,du = \ln|u| + C$

- *Fundamental Theorem of calculus, definite integrals*
  - *theorem:* $\int_a^b f(x)\,dx = F(b) - F(a) = F(x)|_a^b$
  - $\int_a^a f(x)\,dx = 0$
  - $\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx$, for real k
  - $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
  - $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
  - $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

- *area* between two functions, where $f(x) \geq g(x)$ on $[a,b]$: $\int_a^b [f(x) - g(x)]\,dx$

- *economic's applications*
  - *consumer's surplus:* $\int_0^{q_0} [D(q) - p_0]\,dq$
  - *producer's surplus:* $\int_0^{q_0} [p_0 - S(q)]\,dq$

Chapter 8. Further Techniques Integration (4, 5)

- *integration by parts:* $\int u\,dv = uv - \int v\,du$

- *table of various integrations*

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$

2. $\int e^{kx}\,dx = \frac{1}{k} \cdot e^{kx} + C$

3. $\int \frac{a}{x}\,dx = a \ln|x| + C$

4. $\int \ln|ax|\,dx = x(\ln|ax| - 1) + C$

5. $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln|x + \sqrt{x^2 + a^2}| + C$

6. $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln|x + \sqrt{x^2 - a^2}| + C$

7. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a} \cdot \ln\left|\frac{a+x}{a-x}\right| + C$, $a \neq 0$

8. $\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a} \cdot \ln\left|\frac{x-a}{x+a}\right| + C$, $a \neq 0$

9. $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx = -\frac{1}{a} \ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$, $0 < x < a$

10. $\int \frac{1}{x\sqrt{a^2+x^2}}\,dx = -\frac{1}{a} \ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$, $a \neq 0$

11. $\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2} \ln|ax + b| + C$, $a \neq 0$

12. $\int \frac{x}{(ax+b)^2}\,dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \cdot \ln|ax + b| + C$, $a \neq 0$

13. $\int \frac{1}{x(ax+b)}\,dx = \frac{1}{b} \cdot \ln\left|\frac{x}{ax+b}\right| + C,\ b \neq 0$

14. $\int \frac{1}{x(ax+b)^2}\,dx = \frac{1}{b(ax+b)} + \frac{1}{b} \cdot \ln\left|\frac{x}{ax+b}\right| + C,\ b \neq 0$

15. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2} \cdot \ln|x + \sqrt{x^2+a^2}| + C$

16. $\int x^n \ln x\,dx = x^{n+1}\left[\frac{\ln|x|}{n+1} - \frac{1}{(n+1)^2}\right] + C,\ n \neq -1$

17. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \cdot \int x^{n-1}e^{ax} + C,\ a \neq 0$

- *volume of a solid of revolution:* $V = \lim_{\Delta \to 0} \sum_{i=1}^{n} \pi[f(x_i)]^2 \Delta x = \int_a^b \pi[f(x)]^2\,dx$
- *average value of a function* $f(x)$ on interval $[a,b]$: $\frac{1}{b-a}\int_a^b f(x)\,dx$
- *rate of money flow (change in money per unit time)*
  - *present value of money flow:* $P = \int_0^T f(t)e^{-rt}\,dt$
  - *accumulated amount of money flow at time* T*:* $A = e^{rT}\int_0^T f(t)e^{-rt}\,dt$
- *improper integrals*
  - $\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$
  - $\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$
  - $\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx$

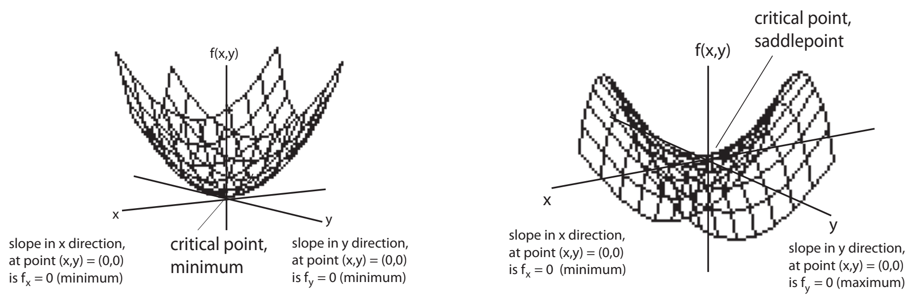
Chapter 9. Multivariate Calculus (notes 6, 7 and 8)

- *first order partial derivative* For $z = f(x,y)$,

$$\frac{\partial z}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}, \quad \frac{\partial z}{\partial y} = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$

- *second–order partial derivatives:* $\frac{\partial^2 z}{\partial x \partial x}$, $\frac{\partial^2 z}{\partial x \partial y}$, $\frac{\partial^2 z}{\partial y \partial x}$, $\frac{\partial^2 z}{\partial y \partial y}$ which can also be written as: $f_{xx}(x,y) = z_{xx}$, $f_{yx}(x,y) = z_{yx}$, $f_{xy}(x,y) = z_{xy}$, $f_{yy}(x,y) = z_{yy}$
  Notice reversal in order of x and y between, for example, notation $\frac{\partial^2 z}{\partial x \partial y}$ and notation $f_{yx}(x,y) = z_{yx}$.
- *discriminant test* identifies relative minimum, maximum or saddlepoint
  - find f_x, f_y, f_{xx}, f_{yy}, f_{xy}
  - find (a,b) such that $f_x(a,b) = 0$ and $f_y(a,b) = 0$
  - find discriminant $D = f_{xx}(a,b) \cdot f_{yy}(a,b) - [f_{xy}(a,b)]^2$
  - then
    - f (relative) maximum at (a,b) if $D > 0$ and $f_{xx}(a,b) < 0$

- $*$ f (relative) minimum at (a, b) if $D > 0$ and $f_{xx}(a, b) > 0$
- $*$ f saddlepoint at (a, b) if $D < 0$
- $*$ test not applicable, gives no information, if $D = 0$



(critical point, extremum and saddlepoint)

- *Lagrange multipliers method* used to solve *constrained optimization problems*

$$\text{optimize } f(x, y), \qquad \text{subject to } g(x, y) = 0,$$

  - create Lagrange function: $F(x, y, \lambda) = f(x, y) - \lambda \cdot g(x, y)$
    a constraint such as $r(x, y) = c$ must be rewritten as $g(x, y) = r(x, y) - c = 0$
  - determine partial derivatives: $F_x(x, y, \lambda)$, $F_y(x, y, \lambda)$, $F_\lambda(x, y, \lambda)$
  - solve system: $F_x(x, y, \lambda) = 0$, $\quad F_y(x, y, \lambda) = 0$, $\quad F_\lambda(x, y, \lambda) = 0$
    for critical points (which may be minima, maxima or saddlepoints)

- *total differential* of $z = f(x, y)$: $\quad dz = f_x(x, y) \cdot dx + f_y(x, y) \cdot dy$
  if differentials dx and dy are *small*,

$$\begin{aligned} f(x + \Delta x, y + \Delta y) &= f(x, y) + \Delta z \\ &\approx f(x, y) + dz \\ &= f(x, y) + f_x(x, y) \cdot dx + f_y(x, y) \cdot dy. \end{aligned}$$

  more general $z = f(x, y, z) : dz = f_x(x, y, z) \cdot dx + f_y(x, y, z) \cdot dy + f_z(x, y, z) \cdot dz$
  dz is sometimes written df

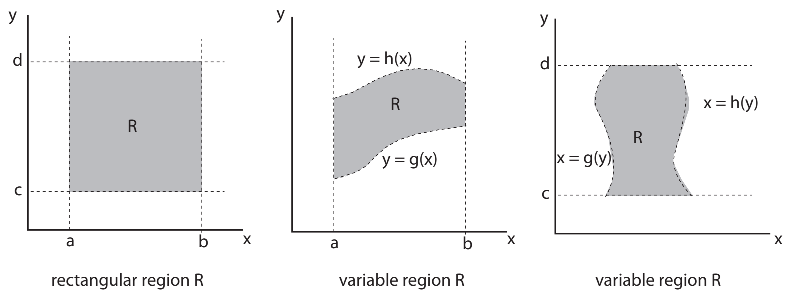
- *double integration*

  - rectangular region R in $a \le x \le b$, $c \le y \le d$,

$$\iint_R f(x, y)\, dy\, dx = \int_a^b \int_c^d f(x, y)\, dy\, dx = \int_c^d \int_a^b f(x, y)\, dx\, dy$$

– *variable region R*

$$\int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx, \quad \text{or} \quad \int_{g(y)}^{h(y)} \int_c^d f(x,y)\,dy\,dx$$



(double integrals over rectangular and variable regions)

if $z = f(x,y)$ never negative, double integration is *volume*.

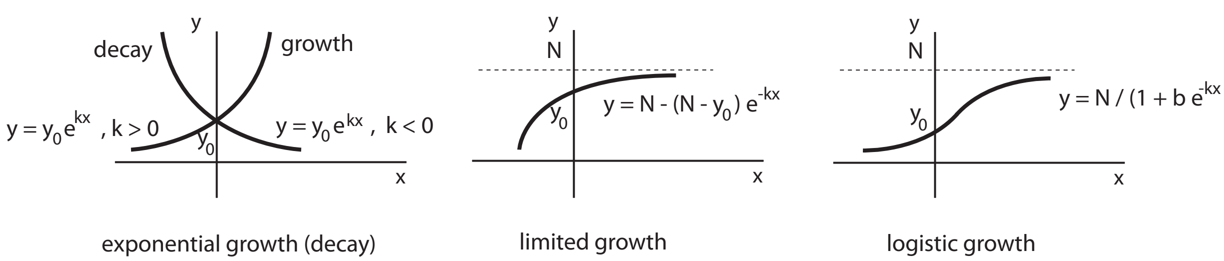
Chapter 10. Differential Equations (notes 9 and 10)

- *elementary differential equation:* $\frac{dy}{dx} = g(x)$
  - *general solution:* $y = \int g(x)\,dx = G(x) + C$

With addition of a *initial (boundary) condition*, $y(x_0)$ at $x = x_0$, elementary differential equation becomes *initial value problem* which has a *particular solution* where a "particular" constant C can be identified.

- *separable differential equations* has (general) solution

$$\int q(y)\,dy = \int p(x)\,dx, \quad \text{or} \quad Q(y) = P(x) + C,$$



(examples of separable differential equations)

separable differential equations	differential equation, initial condition	solution
exponential growth (decay)	$\frac{dy}{dx} = ky,\ y(0) = y_0$	$y = y_0e^{ky}$
limited growth	$\frac{dy}{dx} = k(N-y),\ y(0) = y_0$	$y = N - (N-y_0)e^{-kt}$
logistic growth	$\frac{dy}{dx} = k\left(1 - \frac{y}{N}\right)y,\ y(0) = y_0$	$y = \frac{N}{1+be^{-kt}}, b = \frac{N-y_0}{y_0}$

- *linear first-order differential equation* $\frac{dy}{dx} + P(x)y = Q(x)$
  has *integrating factor* $I(x) = e^{\int P(x)\,dx}$ and is solved using the following steps:
  - rewrite given equation in form $\frac{dy}{dx} + P(x)y = Q(x)$
  - multiply result by integrating factor, $I(x)$
  - replace terms on left of result with $D_x[I(x)y]$
  - integrate result, solve for y
- *Euler's method* numerical method to solve differential equations:
  Let $y = f(x)$ be the solution to the differential equation

$$\frac{dy}{dx} = g(x, y), \quad \text{with} \quad y(x_0) = y_0$$

  for $x_0 \leq x \leq x_n$ and let $x_{i+1} = x_i + h$, where $h = \frac{x_n - x_0}{n}$ and

$$y_{i+1} = y_i + g(x_i, y_i)h, \quad \text{for } 0 \leq i \leq n-1, \text{ then} \quad f(x_{i+1}) \approx y_{i+1}$$

Chapter 11. Probability and Calculus (notes 11)

- *(cumulative) distribution function* for *random variable* X

$$F(x) = P\left(X \leq x\right), \quad -\infty < x < \infty,$$

  has properties
  - $\lim_{x \to -\infty} F(x) = 0$,
  - $\lim_{x \to \infty} F(x) = 1$,
  - if $x_1 < x_2$, then $F(x_1) \leq F(x_2)$; that is, F is nondecreasing.
- *(probability) density function*, $f(x)$

$$f(x) = \frac{dF(x)}{dy} = F'(x), \quad \text{and so, also,} \quad F(x) = \int_{-\infty}^{x} f(t)\,dt$$

  had properties
  - $f(x) \geq 0$, for all x, $-\infty < x < \infty$,
  - $\int_{-\infty}^{\infty} f(x)\,dx = 1$
- *probability*

$$P(a \leq X \leq b) = P(X \leq b) - P(X \leq a) = F(b) - F(a) = \int_{a}^{b} f(x)\,dy$$

- *expected value, variance and standard deviation*

$$E(X) = \sum_x xP(X = x), \quad E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$$

$$\text{Var}(X) = \sigma^2 = E[(X - \mu)^2] = E(X^2) - [E(X)]^2 = E(X^2) - \mu^2$$

  with associated *standard deviation*, $\sigma = \sqrt{\sigma^2}$

- *median*: m that satisfies $P(X \leq m) \geq \frac{1}{2}$ *and* $P(X \geq m) \geq \frac{1}{2}$

- *special distributions*

  - *uniform*

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b, \\ 0, & \text{elsewhere,} \end{cases}$$

$$\mu = E(X) = \frac{a+b}{2}, \quad \sigma^2 = \text{Var}(X) = \frac{(b-a)^2}{12}, \quad \sigma = \sqrt{\text{Var}(X)}.$$

  - *exponential*

$$f(x) = \begin{cases} ae^{-ax}, & 0 \leq x < \infty, \\ 0, & \text{elsewhere,} \end{cases}$$

$$\mu = E(X) = \frac{1}{a}, \quad \sigma^2 = V(Y) = \frac{1}{a^2}, \quad \sigma = \frac{1}{a}.$$

  - *normal* density with parameters μ and σ,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(x-\mu)/\sigma]^2}, -infty < x < \infty$$

$$E(X) = \mu, \quad \text{Var}(X) = \sigma^2, \quad \sigma = \sqrt{\text{Var}(X)}.$$

    may be transformed to a *standard* normal, Z ($\mu = 0$ and $\sigma = 1$)

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \quad \text{using} \quad Z = \frac{X - \mu}{\sigma}$$

Chapter 12. Sequences and Series (12, 13 and 14)

- *basic definitions*

  - *sequence* is a function whose domain is set of natural numbers
  - *series*: *sum* of elements of a sequence

- *geometric sequence and series*, ratio of any two consecutive terms is r

$$a_n = ar^{n-1} = a_{n-1}r, \quad \text{where} \quad r = \frac{a_{n+1}}{a_n}, \quad n \geq 1, \quad \text{and} \quad S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

- *annuities*
  - *(accumulated future) amount S of an annuity R*

$$S = R\left[\frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}}\right] = R \cdot s_{\overline{mt}|\frac{r}{m}} = R\left[\frac{(1+i)^n - 1}{i}\right] = R \cdot s_{\overline{n}|i}$$

  - *annuity payments required for a (accumulated future) sinking fund*

$$R = S\left[\frac{\left(\frac{r}{m}\right)}{\left(1 + \frac{r}{m}\right)^{mt} - 1}\right] = S\left[\frac{i}{(1+i)^n - 1}\right]$$

  - *present value of a sequence of annuity payments*

$$P = R\left[\frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}}\right] = R \cdot a_{\overline{mt}|\frac{r}{m}} = R\left[\frac{1 - (1+i)^{-n}}{i}\right] = R \cdot a_{\overline{n}|i}$$

  - *amortization, annuity payments required to retire a present loan*

$$R = P\left[\frac{\left(\frac{r}{m}\right)}{1 - \left(1 + \frac{r}{m}\right)^{-mt}}\right] = P\left[\frac{i}{1 - (1+i)^{-n}}\right]$$

- *Taylor polynomial of degree n* for differentiable function f at $x = 0$

$$P_n(x) = f(0) + \frac{f^{(1)}(0)}{1!}x + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots + \frac{f^{(n)}(0)}{n!}x^n = \sum_{i=0}^{n} \frac{f^{(n)}(0)}{i!}x^i.$$

for values of x *close* to 0 or large n, $P_n(x) \approx f(x)$

- *infinite series*
  - *definitions:* define infinite series $a_1 + a_2 + a_3 + \cdots + a_n \cdots = \sum_{i=1}^{\infty} a_i$,
    then if $S_n = a_1 + a_2 + a_3 + \cdots + a_n$ and $\lim_{n\to\infty} S_n = L$
    then infinite series *converges* if L exists, otherwise it diverges
  - *geometric series* $\sum_{i=1}^{\infty} ar^{i-1} = a + ar + ar^2 + ar^3 + \cdots$
    converges if r is in $(-1, 1)$ and has sum $\frac{a}{1-r}$, otherwise it diverges

- *(infinite) Taylor series* for differentiable function f at $x = 0$

$$f(0) + \frac{f^{(1)}(0)}{1!}x + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots$$

  - example $f(x)$, corresponding Taylor series and interval of convergence:
    * $f(x) = e^x, \quad 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots, \quad (-\infty, \infty)$
    * $f(x) = \ln(1+x), \quad x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^n x^{n+1}}{n+1} + \cdots, \quad (-1, 1]$
    * $f(x) = \frac{1}{1-x}, \quad 1 + x + x^2 + x^3 + \cdots + x^n + \cdots, \quad (-1, 1)$
  - let f and g be functions with Taylor series

$$\begin{aligned} f(x) &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots \\ g(x) &= b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots + b_n x^n + \cdots \end{aligned}$$

    and so Taylor series of
    * $f + g: \quad (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + \cdots + (a_n + b_n)x^n + \cdots$
    * $c \cdot f(x): \quad c \cdot a_0 + c \cdot a_1 x + c \cdot a_2 x^2 + \cdots + c \cdot a_n x^n + \cdots$
    * $x^k \cdot f(x): \quad a_0 x^k + a_1 x^{k+1} + a_2 x^{k+2} + \cdots + a_n x^{k+n} + \cdots$
    * composition $f[g(x)]$, where $g(x) = cx^k$, is

$$a_0 + a_1[g(x)] + a_2[g(x)]^2 + a_3[g(x)]^3 + \cdots + a_n[g(x)]^n + \cdots$$

- *Newton's method* numerical method to find x such that $f(s) = 0$

$$c_{n+1} = c_n - \frac{f(c_n)}{f'(c_n)}$$

- *L'Hospital's rule* if

$$\lim_{x\to a} f(x) = 0, \quad \lim_{x\to a} g(x) = 0, \quad \text{or} \quad \lim_{x\to a} f(x) = \pm\infty, \quad \lim_{x\to a} g(x) = \pm\infty$$

  then

$$\lim_{x\to a} \frac{f'(x)}{g'(x)} = L \quad \Rightarrow \quad \lim_{x\to a} \frac{f(x)}{g(x)} = L$$

  applies to infinite limits as well

Notes	Chapter	Topics	Description
1	6	Graphing Implicit Functions (example)	$Y_1 = \sqrt{1 - X^2}$, $Y_2 = -\sqrt{1 - X^2}$, GRAPH, WINDOW
1	6	Evaluating Functions	Y = function, VAR Y–VAR ENTER ENTER (function) ENTER
3	7	Definite (Numerical) Integration	MATH fnInt(Y_1, X, lower bound, upper bound)
4	8	Volume of solid of revolution	MATH fnInt($Y_1 = \pi f(x)^2$, X, lower bound, upper bound)
4	8	Average value	$\frac{1}{b-a}$ MATH fnInt(Y_1, X, lower bound, upper bound)
4	8	Money flow (example)	$Y_1 = (3x + 5)e^{-0.07x}$ MATH fnInt(Y_1, X, lower bound, upper bound)
10	10	Euler's Method (example)	For example, $X + 0.1 \to X : Y + Y_1 \times 0.1 \to Y$ ENTER
11	11	Normal Distribution	2nd DISTR 2:normalcdf(lower bound, upper bound, mean, SD)
12	12	Geometric Series (example)	2nd LIST OPS seq$\left(7 * \left(\frac{3}{2}\right)^{X-1}, X, 1, 6\right)$
13	12	Taylor Series (example)	$Y_1 = 1 + X, \quad Y_2 = Y_1 + \frac{X^2}{2}, \ldots$ 2nd TBLSET -1 1 Ask Auto 2nd TABLE
14	12	Newton's Method (example)	$Y_1 = -3X^2 + 2X + 1$, $Y_2 = -6X + 2$, then $2 \to X$ and $X - Y_1/Y_2 \to X$